

MATHEMATICS 1

Chapter: Complex Numbers

Short and Broad Questions Mixed | Exam-Oriented Practice Set

This is a mixed set of short (2 marks) and broad (6 marks) questions of the chapter Complex Numbers. Every topic starts with short questions, followed by broad questions of the same topic, exactly as they can appear together in an exam paper. Short questions are labelled S1, S2, ... and broad questions are labelled B1, B2, All questions are new and different from the earlier documents.

Topics covered:

- Topic 1: Algebra of complex numbers (S1–S4, B1–B2)
- Topic 2: Modulus, argument and polar form (S5–S7, B3)
- Topic 3: De Moivre's theorem (S8–S9, B4)
- Topic 4: Multiple angles and powers of $\sin\theta$, $\cos\theta$ (S10, B5–B6)
- Topic 5: Roots of complex numbers (S11–S13, B7–B8)
- Topic 6: Cube roots of unity (S14–S16, B9)
- Topic 7: Locus in the Argand plane (S17–S19, B10)

Topic 1: Algebra of Complex Numbers

Short Questions (2 Marks)

S1. Evaluate $[i^{18} + (1/i)^{25}]^3$. [2 Marks]

Solution:

1. $i^{18} = i^{4 \times 4 + 2} = i^2 = -1$. $(1/i)^{25} = 1/i^{25}$; since $25 = 4 \times 6 + 1$, $i^{25} = i$, so $1/i = -i$.
2. Bracket $= -1 - i = -(1 + i)$.
3. $(1 + i)^2 = 2i$, so $(1 + i)^3 = 2i(1 + i) = 2i + 2i^2 = -2 + 2i$. Hence $[-(1 + i)]^3 = -(-2 + 2i) = 2 - 2i$.

Answer: $2 - 2i$

S2. Find the conjugate of $(1 + i)^2/(3 - i)$. [2 Marks]

Solution:

1. $(1 + i)^2 = 2i$, so the number is $2i/(3 - i)$.
2. Multiply by $(3 + i)$: $2i(3 + i)/[(3 - i)(3 + i)] = (6i + 2i^2)/10 = (-2 + 6i)/10 = (-1 + 3i)/5$.
3. Conjugate: change the sign of the imaginary part.

Answer: $(-1 - 3i)/5$

S3. Find the complex number z if $(1 + i)z = 2i$. [2 Marks]

Solution:

1. $z = 2i/(1 + i)$.

2. Multiply numerator and denominator by $(1 - i)$: $z = 2i(1 - i)/2 = i(1 - i) = i - i^2$.

Answer: $z = 1 + i$

S4. Find the real numbers x and y if $(2 + 3i)x + (1 - 2i)y = 5 + 4i$. [2 Marks]

Solution:

1. Group parts: $(2x + y) + i(3x - 2y) = 5 + 4i$.
2. Equate real and imaginary parts: $2x + y = 5 \dots(1)$ and $3x - 2y = 4 \dots(2)$.
1. Multiply (1) by 2 and add to (2): $4x + 2y + 3x - 2y = 10 + 4$, so $7x = 14$ and $x = 2$. Then $y = 5 - 4 = 1$.

Answer: $x = 2, y = 1$

Broad Questions (6 Marks)

B1. If $x + iy = (a + ib)/(a - ib)$, find x and y and prove that $x^2 + y^2 = 1$. [6 Marks]

Solution:

1. Multiply numerator and denominator by $(a + ib)$: $x + iy = (a + ib)^2/[(a - ib)(a + ib)]$.
2. Numerator: $(a + ib)^2 = a^2 - b^2 + 2iab$. Denominator: $a^2 + b^2$.
3. So $x + iy = (a^2 - b^2)/(a^2 + b^2) + i \times 2ab/(a^2 + b^2)$.
4. Equate real and imaginary parts: $x = (a^2 - b^2)/(a^2 + b^2)$ and $y = 2ab/(a^2 + b^2)$.
5. $x^2 + y^2 = [(a^2 - b^2)^2 + 4a^2b^2]/(a^2 + b^2)^2 = (a^4 - 2a^2b^2 + b^4 + 4a^2b^2)/(a^2 + b^2)^2 = (a^2 + b^2)^2/(a^2 + b^2)^2 = 1$.

Answer: $x = (a^2 - b^2)/(a^2 + b^2), y = 2ab/(a^2 + b^2); x^2 + y^2 = 1$ (proved)

Note: Shortcut: $|x + iy| = |a + ib|/|a - ib| = 1$, so $x^2 + y^2 = 1$.

B2. If $z_1 + z_2$ and $z_1 z_2$ are both real numbers, prove that either z_1 and z_2 are both real or z_1 and z_2 are conjugates of each other. [6 Marks]

Solution:

1. Let $z_1 = a + ib$ and $z_2 = c + id$, where a, b, c, d are real.
2. $z_1 + z_2 = (a + c) + i(b + d)$ is real, so its imaginary part is zero: $b + d = 0$, i.e. $d = -b$.
3. $z_1 z_2 = (ac - bd) + i(ad + bc)$ is real, so $ad + bc = 0$.
4. Substitute $d = -b$: $-ab + bc = 0$, i.e. $b(c - a) = 0$.
5. Hence $b = 0$ or $c = a$.
6. Case 1: if $b = 0$, then $d = -b = 0$, so $z_1 = a$ and $z_2 = c$ are both real.
7. Case 2: if $c = a$, then $z_2 = a + id = a - ib = \bar{z}_1$, so z_2 is the conjugate of z_1 .

Answer: Hence proved: z_1, z_2 are both real or conjugates of each other.

Topic 2: Modulus, Argument and Polar Form

Short Questions (2 Marks)

S5. Find the modulus and argument of $(1 + i)/(\sqrt{3} - i)$. [2 Marks]

Solution:

1. $|1 + i| = \sqrt{2}$ and $|\sqrt{3} - i| = 2$, so the modulus $= \sqrt{2}/2 = 1/\sqrt{2}$.
2. $\arg(1 + i) = \pi/4$ and $\arg(\sqrt{3} - i) = -\pi/6$, so the argument $= \pi/4 - (-\pi/6) = 5\pi/12$.

Answer: Modulus = $1/\sqrt{2}$, argument = $5\pi/12$

S6. Find the modulus and principal argument of $(1 + i)(1 - i\sqrt{3})$. [2 Marks]

Solution:

1. $|1 + i| = \sqrt{2}$ and $|1 - i\sqrt{3}| = 2$, so the modulus $= 2\sqrt{2}$.
2. $\arg(1 + i) = \pi/4$ and $\arg(1 - i\sqrt{3}) = -\pi/3$, so the argument $= \pi/4 - \pi/3 = -\pi/12$.

Answer: Modulus = $2\sqrt{2}$, argument = $-\pi/12$

S7. Find the modulus and principal argument of $(1 + i)^{10}$. [2 Marks]

Solution:

1. $(1 + i)^2 = 2i$, so $(1 + i)^{10} = (2i)^5 = 32i^5 = 32i$.
2. $32i$ lies on the positive imaginary axis: modulus 32 and argument $\pi/2$.

Answer: Modulus = 32, argument = $\pi/2$

Note: By polar form: $(\sqrt{2})^{10} \text{cis}(10\pi/4) = 32 \text{cis}(5\pi/2) = 32 \text{cis}(\pi/2)$, the same result.

Broad Questions (6 Marks)

B3. (i) Find the modulus and argument of $z = (1 + i)^5/(1 - i)^3$. (ii) Express $(1 + i\sqrt{3})^4$ in the form $a + ib$. [6 Marks]

Solution:

1. (i) $1 + i = \sqrt{2}(\cos\pi/4 + i \sin\pi/4)$ and $1 - i = \sqrt{2}(\cos\pi/4 - i \sin\pi/4) = \sqrt{2} \text{cis}(-\pi/4)$.
2. $(1 + i)^5 = (\sqrt{2})^5 \text{cis}(5\pi/4) = 4\sqrt{2} \text{cis}(5\pi/4)$.
3. $(1 - i)^3 = (\sqrt{2})^3 \text{cis}(-3\pi/4) = 2\sqrt{2} \text{cis}(-3\pi/4)$.
4. $z = (4\sqrt{2}/2\sqrt{2}) \text{cis}[5\pi/4 - (-3\pi/4)] = 2 \text{cis}(2\pi) = 2(1 + 0i) = 2$. So $|z| = 2$ and $\arg z = 0$.
5. Check directly: $(1 + i)^5 = (2i)^2(1 + i) = -4 - 4i$ and $(1 - i)^3 = (-2i)(1 - i) = -2 - 2i$, and $(-4 - 4i)/(-2 - 2i) = 2$.
6. (ii) $1 + i\sqrt{3} = 2 \text{cis}(\pi/3)$, so $(1 + i\sqrt{3})^4 = 2^4 \text{cis}(4\pi/3) = 16(\cos 4\pi/3 + i \sin 4\pi/3) = 16(-1/2 - i\sqrt{3}/2)$.

Answer: (i) $|z| = 2$, $\arg z = 0$ ($z = 2$) (ii) $-8 - 8\sqrt{3}i$

Topic 3: De Moivre's Theorem

Short Questions (2 Marks)

S8. Evaluate $[\cos(\pi/8) + i \sin(\pi/8)]^4$. [2 Marks]

Solution:

1. By De Moivre: $= \cos(4\pi/8) + i \sin(4\pi/8)$.
2. $= \cos(\pi/2) + i \sin(\pi/2) = 0 + i$.

Answer: i

S9. If $z = \cos(2\pi/3) + i \sin(2\pi/3)$, find z^{50} . [2 Marks]

Solution:

1. By De Moivre: $z^{50} = \cos(100\pi/3) + i \sin(100\pi/3)$.
2. $100\pi/3 = 32\pi + 4\pi/3$, and 32π is a multiple of 2π , so the angle is equivalent to $4\pi/3$.
3. $z^{50} = \cos(4\pi/3) + i \sin(4\pi/3) = -1/2 - i\sqrt{3}/2$.

Answer: $-1/2 - (\sqrt{3}/2)i$

Note: z is a cube root of unity (ω), and $\omega^{50} = \omega^2$, which agrees.

Broad Questions (6 Marks)

B4. If $x = \cos\theta + i \sin\theta$, prove that (i) $(x^{2n} - 1)/(x^{2n} + 1) = i \tan n\theta$ (ii) $(1 + i \tan\theta)^n + (1 - i \tan\theta)^n = 2\cos n\theta / \cos^n\theta$. [6 Marks]

Solution:

1. (i) Divide the numerator and the denominator by x^n : $(x^{2n} - 1)/(x^{2n} + 1) = (x^n - x^{-n})/(x^n + x^{-n})$.
2. By De Moivre, $x^n = \cos n\theta + i \sin n\theta$ and $x^{-n} = \cos n\theta - i \sin n\theta$.
3. So $x^n - x^{-n} = 2i \sin n\theta$ and $x^n + x^{-n} = 2\cos n\theta$, giving the ratio $2i \sin n\theta / (2\cos n\theta) = i \tan n\theta$.
4. (ii) $1 + i \tan\theta = (\cos\theta + i \sin\theta)/\cos\theta$ and $1 - i \tan\theta = (\cos\theta - i \sin\theta)/\cos\theta$.
5. Raise to the power n and use De Moivre: $(1 + i \tan\theta)^n = (\cos n\theta + i \sin n\theta)/\cos^n\theta$ and $(1 - i \tan\theta)^n = (\cos n\theta - i \sin n\theta)/\cos^n\theta$.
6. Add: the sum $= 2\cos n\theta / \cos^n\theta$.

Answer: Both results proved.

Topic 4: Multiple Angles and Powers of $\sin\theta$, $\cos\theta$

Short Questions (2 Marks)

S10. Using De Moivre's theorem, deduce that $\tan 3\theta = (3\tan\theta - \tan^3\theta)/(1 - 3\tan^2\theta)$. [2 Marks]

Solution:

1. Let $c = \cos\theta$, $s = \sin\theta$. $(c + is)^3 = c^3 + 3ic^2s - 3cs^2 - is^3$, so $\cos 3\theta = c^3 - 3cs^2$ and $\sin 3\theta = 3c^2s - s^3$.
2. $\tan 3\theta = \sin 3\theta / \cos 3\theta$. Divide numerator and denominator by c^3 and put $t = s/c$: $\tan 3\theta = (3t - t^3)/(1 - 3t^2)$.

Answer: $\tan 3\theta = (3\tan\theta - \tan^3\theta)/(1 - 3\tan^2\theta)$

Broad Questions (6 Marks)

B5. Prove that $\sin 5\theta = \sin\theta(16\cos^4\theta - 12\cos^2\theta + 1)$. Hence show that $\cos^2 36^\circ = (3 + \sqrt{5})/8$. [6 Marks]

Solution:

1. Let $c = \cos\theta$, $s = \sin\theta$. From $(c + is)^5$, the imaginary part is $\sin 5\theta = 5c^4s - 10c^2s^3 + s^5$.
2. Take s common: $\sin 5\theta = s(5c^4 - 10c^2s^2 + s^4)$.
3. Put $s^2 = 1 - c^2$: $5c^4 - 10c^2(1 - c^2) + (1 - c^2)^2 = 5c^4 - 10c^2 + 10c^4 + 1 - 2c^2 + c^4 = 16c^4 - 12c^2 + 1$.
4. Hence $\sin 5\theta = \sin\theta(16\cos^4\theta - 12\cos^2\theta + 1)$.
5. Put $\theta = 36^\circ$. Then $\sin 5\theta = \sin 180^\circ = 0$ and $\sin 36^\circ \neq 0$, so $16c^4 - 12c^2 + 1 = 0$ with $c = \cos 36^\circ$.
6. This is a quadratic in c^2 : $c^2 = [12 \pm \sqrt{(144 - 64)}]/32 = (12 \pm 4\sqrt{5})/32 = (3 \pm \sqrt{5})/8$.
7. $\theta = 72^\circ$ also gives $\sin 5\theta = \sin 360^\circ = 0$, so the two values belong to $\cos^2 36^\circ$ and $\cos^2 72^\circ$. Since $\cos 36^\circ > \cos 72^\circ$, $\cos^2 36^\circ$ is the larger value.

Answer: $\sin 5\theta = \sin\theta(16\cos^4\theta - 12\cos^2\theta + 1)$; $\cos^2 36^\circ = (3 + \sqrt{5})/8$ (so $\cos 36^\circ = (1 + \sqrt{5})/4$)

Note: The other root gives $\cos^2 72^\circ = (3 - \sqrt{5})/8$.

B6. Prove that (i) $\cos^3\theta = (1/4)(\cos 3\theta + 3\cos\theta)$ (ii) $\sin^3\theta = (1/4)(3\sin\theta - \sin 3\theta)$. [6 Marks]

Solution:

1. Let $z = \cos\theta + i \sin\theta$. Then $z + 1/z = 2\cos\theta$, $z - 1/z = 2i \sin\theta$, $z^n + 1/z^n = 2\cos n\theta$ and $z^n - 1/z^n = 2i \sin n\theta$.
2. (i) $(2\cos\theta)^3 = (z + 1/z)^3 = z^3 + 3z + 3/z + 1/z^3 = (z^3 + 1/z^3) + 3(z + 1/z) = 2\cos 3\theta + 6\cos\theta$.
3. So $8\cos^3\theta = 2\cos 3\theta + 6\cos\theta$, giving $\cos^3\theta = (1/4)(\cos 3\theta + 3\cos\theta)$.
4. (ii) $(2i \sin\theta)^3 = (z - 1/z)^3 = z^3 - 3z + 3/z - 1/z^3 = (z^3 - 1/z^3) - 3(z - 1/z) = 2i \sin 3\theta - 6i \sin\theta$.
5. LHS = $8i^3 \sin^3\theta = -8i \sin^3\theta$. So $-8i \sin^3\theta = 2i \sin 3\theta - 6i \sin\theta$.
6. Divide by $-8i$: $\sin^3\theta = (6\sin\theta - 2\sin 3\theta)/8 = (1/4)(3\sin\theta - \sin 3\theta)$.

Answer: Both results proved.

Note: Check with $\theta = \pi/2$: $\sin^3 \theta = 1$ and $(3 + 1)/4 = 1$.

Topic 5: Roots of Complex Numbers

Short Questions (2 Marks)

S11. Solve $x^3 = 8$ and write the roots in the form $a + ib$. [2 Marks]

Solution:

- $x^3 = 8 \times 1$, so $x = 2 \times (\text{cube roots of unity}) = 2, 2\omega, 2\omega^2$, where $\omega = (-1 + i\sqrt{3})/2$.
- $2\omega = -1 + i\sqrt{3}$ and $2\omega^2 = -1 - i\sqrt{3}$.

Answer: $x = 2, -1 + i\sqrt{3}, -1 - i\sqrt{3}$

S12. Write the fifth roots of unity in exponential form and state their sum. [2 Marks]

Solution:

- $x^5 = 1 = e^{2k\pi i}$, so $x = e^{2k\pi i/5}$, $k = 0, 1, 2, 3, 4$.
- The roots are $1, e^{2\pi i/5}, e^{4\pi i/5}, e^{6\pi i/5}, e^{8\pi i/5}$. The equation $x^5 - 1 = 0$ has no x^4 term, so the sum of the roots is 0.

Answer: Roots: $e^{2k\pi i/5}$ ($k = 0$ to 4); sum = 0

S13. Find the square roots of $-8i$. [2 Marks]

Solution:

- Let $\sqrt{-8i} = p + iq$. Then $p^2 - q^2 = 0$, $2pq = -8$ (so $pq = -4$) and $p^2 + q^2 = |-8i| = 8$.
- Adding: $2p^2 = 8$, so $p = \pm 2$; then $q^2 = 4$. Since $pq = -4$ is negative, p and q have opposite signs.

Answer: $\pm(2 - 2i)$, i.e. $2 - 2i$ and $-2 + 2i$

Note: Check: $(2 - 2i)^2 = 4 - 8i + 4i^2 = -8i$.

Broad Questions (6 Marks)

B7. Find all the values of $(-16)^{1/4}$ and express them in the form $a + ib$. [6 Marks]

Solution:

- $-16 = 16[\cos(\pi + 2k\pi) + i \sin(\pi + 2k\pi)]$ (modulus 16, argument π).
- $(-16)^{1/4} = 16^{1/4}[\cos((\pi + 2k\pi)/4) + i \sin((\pi + 2k\pi)/4)] = 2[\cos((2k + 1)\pi/4) + i \sin((2k + 1)\pi/4)]$, $k = 0, 1, 2, 3$.
- $k = 0$: $2(\cos\pi/4 + i \sin\pi/4) = 2(1/\sqrt{2} + i/\sqrt{2}) = \sqrt{2} + i\sqrt{2}$.
- $k = 1$: $2(\cos 3\pi/4 + i \sin 3\pi/4) = -\sqrt{2} + i\sqrt{2}$.
- $k = 2$: $2(\cos 5\pi/4 + i \sin 5\pi/4) = -\sqrt{2} - i\sqrt{2}$.
- $k = 3$: $2(\cos 7\pi/4 + i \sin 7\pi/4) = \sqrt{2} - i\sqrt{2}$.

Answer: $\pm\sqrt{2} \pm i\sqrt{2}$ (all four sign combinations)

Note: Check: $(\sqrt{2} + i\sqrt{2})^2 = 2 - 2 + 4i = 4i$, and $(4i)^2 = -16$.

B8. Solve $x^5 = 1$ and hence prove that $\cos 72^\circ + \cos 144^\circ = -1/2$. [6 Marks]

Solution:

1. $x^5 = 1 = \cos 2k\pi + i \sin 2k\pi$, so $x = \cos(2k\pi/5) + i \sin(2k\pi/5) = \cos(72^\circ k) + i \sin(72^\circ k)$, $k = 0, 1, 2, 3, 4$.
2. The roots are: $1, \cos 72^\circ, \cos 144^\circ, \cos 216^\circ, \cos 288^\circ$.
3. Note $\cos 216^\circ = \cos(-144^\circ)$ is the conjugate of $\cos 144^\circ$, and $\cos 288^\circ = \cos(-72^\circ)$ is the conjugate of $\cos 72^\circ$.
4. The equation $x^5 - 1 = 0$ has no x^4 term, so the sum of its roots is zero: $1 + [\cos 72^\circ + \cos(-72^\circ)] + [\cos 144^\circ + \cos(-144^\circ)] = 0$.
5. A number plus its conjugate is twice its real part, so $1 + 2\cos 72^\circ + 2\cos 144^\circ = 0$.
6. Therefore $\cos 72^\circ + \cos 144^\circ = -1/2$.

Answer: Roots: $\cos(72^\circ k) + i \sin(72^\circ k)$, $k = 0$ to 4 ; $\cos 72^\circ + \cos 144^\circ = -1/2$ (proved)

Note: Numerical check: $0.309 + (-0.809) = -0.5$.

Topic 6: Cube Roots of Unity

Short Questions (2 Marks)

S14. If ω is a cube root of unity, find $(1 + \omega^2)/(1 + \omega)$. [2 Marks]

Solution:

1. $1 + \omega^2 = -\omega$ and $1 + \omega = -\omega^2$.
2. Ratio = $(-\omega)/(-\omega^2) = 1/\omega = \omega^2$ (since $\omega^3 = 1$).

Answer: ω^2

S15. If ω is a cube root of unity, find $\omega^{100} + \omega^{-100}$. [2 Marks]

Solution:

1. $100 = 3 \times 33 + 1$, so $\omega^{100} = \omega$. $\omega^{-100} = 1/\omega^{100} = 1/\omega = \omega^2$.
2. Sum = $\omega + \omega^2 = -1$.

Answer: -1

S16. If ω is a cube root of unity, find $1 + \omega^7 + \omega^{14}$. [2 Marks]

Solution:

1. $7 = 3 \times 2 + 1$, so $\omega^7 = \omega$. $14 = 3 \times 4 + 2$, so $\omega^{14} = \omega^2$.
2. Sum = $1 + \omega + \omega^2 = 0$.

Answer: 0

Broad Questions (6 Marks)

B9. If ω is a complex cube root of unity, show that $(x - \omega)(x - \omega^2) = x^2 + x + 1$. Hence prove that $(2 - \omega)(2 - \omega^2)(2 - \omega^{10})(2 - \omega^{11}) = 49$. [6 Marks]

Solution:

1. $(x - \omega)(x - \omega^2) = x^2 - (\omega + \omega^2)x + \omega^3$.
2. Since $\omega + \omega^2 = -1$ and $\omega^3 = 1$, this equals $x^2 + x + 1$.
3. Reduce the powers: $10 = 3 \times 3 + 1$, so $\omega^{10} = \omega$; $11 = 3 \times 3 + 2$, so $\omega^{11} = \omega^2$.
4. The given product $= (2 - \omega)(2 - \omega^2)(2 - \omega)(2 - \omega^2) = [(2 - \omega)(2 - \omega^2)]^2$.
5. Put $x = 2$ in the identity: $(2 - \omega)(2 - \omega^2) = 2^2 + 2 + 1 = 7$.
6. Therefore the product $= 7^2 = 49$.

Answer: $(x - \omega)(x - \omega^2) = x^2 + x + 1$; the product = 49 (both proved)

Note: The identity also gives $x^3 - 1 = (x - 1)(x - \omega)(x - \omega^2)$.

Topic 7: Locus in the Argand Plane

Short Questions (2 Marks)

S17. Describe the locus of z if $|z + 2i| = 2$. [2 Marks]

Solution:

1. Put $z = x + iy$: $|z + 2i| = |x + i(y + 2)| = 2$.
2. So $x^2 + (y + 2)^2 = 4$.

Answer: Circle with centre $(0, -2)$ and radius 2.

S18. Describe the locus of z if $\text{Im}(z^2) = 2$. [2 Marks]

Solution:

1. $z^2 = (x + iy)^2 = (x^2 - y^2) + 2ixy$, so $\text{Im}(z^2) = 2xy$.
2. $2xy = 2$, i.e. $xy = 1$.

Answer: The rectangular hyperbola $xy = 1$.

S19. Find the locus of z if $|z - 1| = |z - i|$. [2 Marks]

Solution:

1. Squaring: $(x - 1)^2 + y^2 = x^2 + (y - 1)^2$.
2. Expand: $-2x + 1 = -2y + 1$, so $x = y$.

Answer: The straight line $y = x$, the perpendicular bisector of the points 1 and i.

Broad Questions (6 Marks)

B10. Find the locus of the point $z = x + iy$ if (i) $|z - 4i| = 2|z - i|$ (ii) $(z - i)/(z + 1)$ is a real number. [6 Marks]

Solution:

1. (i) Squaring: $|z - 4i|^2 = 4|z - i|^2$, i.e. $x^2 + (y - 4)^2 = 4[x^2 + (y - 1)^2]$.
2. Expand: $x^2 + y^2 - 8y + 16 = 4x^2 + 4y^2 - 8y + 4$.
3. Simplify: $3x^2 + 3y^2 = 12$, so $x^2 + y^2 = 4$.
4. (ii) $(z - i)/(z + 1) = [x + i(y - 1)]/[x + 1 + iy]$. Multiply numerator and denominator by $(x + 1) - iy$.
5. Numerator $= x(x + 1) - ixy + i(y - 1)(x + 1) + y(y - 1) = [x^2 + x + y^2 - y] + i[-xy + xy + y - x - 1] = [x^2 + x + y^2 - y] + i(y - x - 1)$.
6. The number is real when its imaginary part is zero: $y - x - 1 = 0$, i.e. $y = x + 1$, with $z \neq -1$.

Answer: (i) Circle $x^2 + y^2 = 4$ with centre at the origin and radius 2. (ii) The line $y = x + 1$, excluding the point $(-1, 0)$.

Note: In (ii) the line passes through the points i and -1 , which is expected because the ratio is real when z , i and -1 are collinear.

Exam Strategy for Mixed Papers

- Attempt the short questions first. They are quick, and every one is worth 2 marks for 3 to 5 lines of work.
- In broad questions, write the given data, the formula used, every step, and a clearly marked final answer.
- If a broad question has parts (i) and (ii), label them clearly, because partial marks are awarded for each part.
- Check answers with a simple test value (such as $\theta = 0$ or $\pi/2$) when time permits.