MATHEMATICS 1

Chapter: Logarithm

Exercise Notes: Important Questions, Solutions and Answers

For Engineering and Polytechnic Students | Exam-Oriented

Note: In this document, "log" means $\log_{10}$ (base 10) unless another base is written. "ln" means $\log_e$ (natural logarithm). Questions are arranged from short answer to long answer, followed by applications and practice questions.

Part 1: Key Concepts and Formulas

A logarithm is the power to which a fixed number (the base) must be raised to get a given number. If $a^x = N$, where $a > 0$, $a \neq 1$ and $N > 0$, then x is called the logarithm of N to the base a and we write $x = \log_a N$. The two forms $a^x = N$ and $\log_a N = x$ are just two ways of writing the same relation, so always be ready to switch between them.

Basic points

- The base a must be positive and not equal to 1, and the number N must be positive. The logarithm of zero or of a negative number is not defined (in real numbers).
- Common logarithm has base 10 and is written log or $\log_{10}$. Natural logarithm has base $e \approx 2.718$ and is written ln.
- Relation between the two: $\ln N = 2.3026 \times \log_{10} N$, or $\log_{10} N = 0.4343 \times \ln N$.

Laws of logarithms

- $\log_a 1 = 0$ and $\log_a a = 1$
- Product law: $\log_a (MN) = \log_a M + \log_a N$
- Quotient law: $\log_a (M/N) = \log_a M - \log_a N$
- Power law: $\log_a (M^n) = n \log_a M$, and in particular $\log_a (1/M) = - \log_a M$
- Change of base: $\log_a M = \log_b M / \log_b a$ (use the same new base for every term)
- Reciprocal rule: $\log_a b = 1 / \log_b a$, so $\log_a b \times \log_b a = 1$
- Chain rule: $\log_a b \times \log_b c = \log_a c$, and $\log_a b \times \log_b c \times \log_c a = 1$
- Base as a power: the log of M to the base a^n equals $(1/n) \times \log_a M$
- Cancellation: a to the power ($\log_a N$) is equal to N; also $\ln e = 1$ and $\ln e^x = x$
- One-one property: if $\log_a M = \log_a N$, then $M = N$

Characteristic and mantissa (common logarithm)

Every common logarithm can be written as log N = characteristic + mantissa, where the characteristic is an integer and the mantissa is a decimal part that is always positive and lies between 0 and 1.

- If $N \geq 1$: characteristic = (number of digits before the decimal point) − 1.
- If $0 < N < 1$: characteristic = −(number of zeros just after the decimal point + 1). It is a negative number and is written with a bar on top.
- Number of digits in N (for $N \geq 1$) = characteristic of log N + 1.

Standard method to solve logarithmic equations

- Step 1: Write the condition that every argument of a log must be greater than zero.

- Step 2: Use the laws to combine the terms into a single log on each side.
- Step 3: Either equate the arguments, or convert the equation into exponential form.
- Step 4: Solve for x and reject any value that breaks the condition of Step 1.

Values frequently given in exams

$\log 2 = 0.3010$, $\log 3 = 0.4771$, $\log 5 = 0.6990$ (since $\log 5 = 1 - \log 2$), $\log 7 = 0.8451$, $\log 10 = 1$, $\ln 10 = 2.303$, $e \approx 2.718$.

Part 2: Short Answer Questions (2 Marks)

Q1. Convert into logarithmic form: (i) $2^5 = 32$ (ii) $10^{-2} = 0.01$ (iii) $9^{1/2} = 3$.

Solution:

- Use the rule $a^x = N \leftrightarrow \log_a N = x$. The base stays the base and the power becomes the logarithm.

Answer: (i) $\log_2 32 = 5$ (ii) $\log_{10} 0.01 = -2$ (iii) $\log_9 3 = 1/2$

Q2. Convert into exponential form: (i) $\log_3 81 = 4$ (ii) $\log_5 (1/25) = -2$ (iii) $\log_{16} 4 = 1/2$.

Solution:

- Use the rule $\log_a N = x \leftrightarrow a^x = N$.

Answer: (i) $3^4 = 81$ (ii) $5^{-2} = 1/25$ (iii) $16^{1/2} = 4$

Q3. Find the value of: (i) $\log_2 64$ (ii) $\log_{10} 0.001$ (iii) $\log_{\sqrt{3}} 81$.

Solution:

- (i) Let $\log_2 64 = x$. Then $2^x = 64 = 2^6$, so $x = 6$.
- (ii) Let $\log_{10} 0.001 = x$. Then $10^x = 0.001 = 10^{-3}$, so $x = -3$.
- (iii) Let $\log_{\sqrt{3}} 81 = x$. Then $(\sqrt{3})^x = 81$, i.e. $3^{x/2} = 3^4$. So $x/2 = 4$ and $x = 8$.

Answer: (i) 6 (ii) −3 (iii) 8

Q4. Evaluate $\log_2 8 + \log_3 27 - \log_5 25$.

Solution:

1. $\log_2 8 = 3$, because $2^3 = 8$.
2. $\log_3 27 = 3$, because $3^3 = 27$.
3. $\log_5 25 = 2$, because $5^2 = 25$.
4. Value $= 3 + 3 - 2 = 4$.

Answer: 4

Q5. Find x if: (i) $\log_x 64 = 3$ (ii) $\log_5 x = -2$ (iii) $\log_2 (x + 1) = 4$.

Solution:

- (i) Convert to exponential form: $x^3 = 64 = 4^3$, so $x = 4$.
- (ii) $x = 5^{-2} = 1/5^2 = 1/25$.
- (iii) $x + 1 = 2^4 = 16$, so $x = 15$.

Answer: (i) $x = 4$ (ii) $x = 1/25$ (iii) $x = 15$

Q6. Simplify (base 10): (i) log 2 + log 5   (ii) log 25 + log 4.

Solution:

- (i) By the product law, log 2 + log 5 = log (2 × 5) = log 10 = 1.
- (ii) log 25 + log 4 = log (25 × 4) = log 100 = log 10^2 = 2.

Answer: (i) 1   (ii) 2

Q7. Prove that $\log_a 1 = 0$ and $\log_a a = 1$.

Solution:

1. Let $\log_a 1 = x$. Then $a^x = 1 = a^0$, so x = 0.
2. Let $\log_a a = y$. Then $a^y = a = a^1$, so y = 1.
3. Hence proved.

Q8. If log 2 = 0.3010, find log 8 and log 5.

Solution:

- log 8 = log 2^3 = 3 log 2 = 3 × 0.3010 = 0.9030.
- log 5 = log (10/2) = log 10 − log 2 = 1 − 0.3010 = 0.6990.

Answer: log 8 = 0.9030 and log 5 = 0.6990

Q9. Write the characteristic of the logarithm of: (i) 4567.8   (ii) 45.6   (iii) 0.0345   (iv) 0.5.

Solution:

- For a number greater than 1, characteristic = (digits before the decimal point) − 1.
- (i) 4567.8 has 4 digits before the point, so the characteristic is 3.
- (ii) 45.6 has 2 digits before the point, so the characteristic is 1.
- For a number between 0 and 1, characteristic = −(zeros after the decimal point + 1).
- (iii) 0.0345 has one zero after the point, so the characteristic is −2 (bar 2).
- (iv) 0.5 has no zero after the point, so the characteristic is −1 (bar 1).

Answer: (i) 3   (ii) 1   (iii) −2   (iv) −1

Q10. Find the value of $\log_3 5 \times \log_5 9$.

Solution:

1. Use the chain rule $\log_a b \times \log_b c = \log_a c$.
2. So $\log_3 5 \times \log_5 9 = \log_3 9 = 2$, because $3^2 = 9$.
3. Check by change of base: (log 5 / log 3) × (log 9 / log 5) = log 9 / log 3 = 2 log 3 / log 3 = 2.

Answer: 2

Q11. Evaluate $\ln e^5 + e^{\ln 3}$.

Solution:

1. $\ln e^5 = 5 \ln e = 5 \times 1 = 5$.
2. $e^{\ln 3} = 3$, because e raised to ln N is N.
3. Sum = 5 + 3 = 8.

Answer: 8

Q12. Expand log (x^2 y / z^3).

Solution:

1. Quotient law: log (x^2 y / z^3) = log (x^2 y) − log z^3.
2. Product law: = log x^2 + log y − log z^3.
3. Power law: = 2 log x + log y − 3 log z.

Answer: 2 log x + log y − 3 log z

Q13. Write as a single logarithm: 2 log 3 + 3 log 2 − log 6.

Solution:

1. Power law: 2 log 3 + 3 log 2 − log 6 = log 9 + log 8 − log 6.
2. Product and quotient laws: = log (9 × 8 / 6) = log 12.

Answer: log 12

Q14. Evaluate: (i) 3 raised to the power $\log_3 7$ (ii) 10 raised to the power 2 log 5.

Solution:

- Use the result "a raised to the power $\log_a N$ equals N".
- (i) 3 raised to $\log_3 7$ = 7.
- (ii) 10 raised to 2 log 5 = 10 raised to log 25 = 25.

Answer: (i) 7 (ii) 25

Q15. Distinguish between common logarithm and natural logarithm.

Solution:

- Common logarithm: base is 10, written log or $\log_{10}$. Used in numerical calculation, characteristic–mantissa work and in engineering scales such as decibel.
- Natural logarithm: base is e ≈ 2.718, written ln or $\log_e$. Used in calculus, growth and decay, and circuit transients.
- Relation: ln N = 2.3026 × $\log_{10}$ N.

Part 3: Medium Answer Questions (3–4 Marks)

Q16. Simplify: log (75/16) − 2 log (5/9) + log (32/243).

Solution:

1. Use the power law on the middle term: 2 log (5/9) = log $(5/9)^2$ = log (25/81).
2. Expression = log (75/16) − log (25/81) + log (32/243).
3. Combine (subtraction means multiplying by the reciprocal): log [(75/16) × (81/25) × (32/243)].
4. Cancel: 75/25 = 3, 32/16 = 2 and 81/243 = 1/3. So the product is 3 × 2 × (1/3) = 2.

Answer: log 2

Q17. Given log 2 = 0.3010 and log 3 = 0.4771, find: (i) log 6 (ii) log 12 (iii) log 1.5 (iv) log 45.

Solution:

- (i) log 6 = log (2 × 3) = 0.3010 + 0.4771 = 0.7781.
- (ii) log 12 = log (2^2 × 3) = 2(0.3010) + 0.4771 = 1.0791.
- (iii) log 1.5 = log (3/2) = 0.4771 − 0.3010 = 0.1761.

- (iv) $\log 45 = \log (9 \times 5) = 2 \log 3 + \log 5$. Here $\log 5 = 1 - \log 2 = 0.6990$, so $\log 45 = 0.9542 + 0.6990 = 1.6532$.

Answer: (i) 0.7781 (ii) 1.0791 (iii) 0.1761 (iv) 1.6532

Q18. Solve: $\log (x + 2) + \log (x - 2) = \log 5$.

Solution:

1. Condition: $x + 2 > 0$ and $x - 2 > 0$, so $x > 2$.
2. Product law: $\log [(x + 2)(x - 2)] = \log 5$.
3. Equate the arguments: $x^2 - 4 = 5$, so $x^2 = 9$ and $x = \pm 3$.
4. $x = -3$ does not satisfy $x > 2$, so it is rejected.

Answer: $x = 3$

Q19. Solve: $\log_2 x + \log_2 (x - 2) = 3$.

Solution:

1. Condition: $x > 0$ and $x - 2 > 0$, so $x > 2$.
2. Combine: $\log_2 [x(x - 2)] = 3$, so $x(x - 2) = 2^3 = 8$.
3. $x^2 - 2x - 8 = 0$, i.e. $(x - 4)(x + 2) = 0$, so $x = 4$ or $x = -2$.
4. $x = -2$ violates $x > 2$, so it is rejected.

Answer: $x = 4$

Q20. Solve: $\log_5 (x + 3) + \log_5 (x - 1) = 1$.

Solution:

1. Condition: $x + 3 > 0$ and $x - 1 > 0$, so $x > 1$.
2. Combine: $\log_5 [(x + 3)(x - 1)] = 1$, so $(x + 3)(x - 1) = 5^1 = 5$.
3. $x^2 + 2x - 3 = 5$, so $x^2 + 2x - 8 = 0$, i.e. $(x + 4)(x - 2) = 0$.
4. $x = -4$ is rejected because it is not greater than 1.

Answer: $x = 2$

Q21. Find x if $\log_x (1/8) = -3/2$.

Solution:

1. Exponential form: $x^{-3/2} = 1/8$.
2. Take the reciprocal of both sides: $x^{3/2} = 8$.
3. Raise both sides to the power 2/3: $x = 8^{2/3} = (2^3)^{2/3} = 2^2 = 4$.

Answer: $x = 4$

Q22. Evaluate $\log_2 (\log_3 81)$.

Solution:

1. Work from the inside: $\log_3 81 = 4$, because $3^4 = 81$.
2. Now $\log_2 4 = 2$, because $2^2 = 4$.

Answer: 2

Q23. Prove that $\log_a b = 1 / \log_b a$. Hence evaluate $1/\log_2 36 + 1/\log_3 36$.

Solution:

1. Let $\log_a b = x$. Then $a^x = b$.
2. Raise both sides to the power $1/x$: $a = b^{1/x}$, so $\log_b a = 1/x = 1 / \log_a b$. Hence proved.
3. Using the result, $1/\log_2 36 = \log_{36} 2$ and $1/\log_3 36 = \log_{36} 3$.
4. Sum = $\log_{36} 2 + \log_{36} 3 = \log_{36} 6 = 1/2$, because $36^{1/2} = 6$.

Answer: 1/2

Q24. Show that $1/\log_2 N + 1/\log_3 N + 1/\log_5 N = 1/\log_{30} N$.

Solution:

1. Use $1/\log_a N = \log_N a$ for each term.
2. LHS = $\log_N 2 + \log_N 3 + \log_N 5 = \log_N (2 \times 3 \times 5) = \log_N 30$.
3. Again by the reciprocal rule, $\log_N 30 = 1/\log_{30} N$ = RHS. Hence proved.

Q25. Given log 2 = 0.3010 and log 3 = 0.4771, find the number of digits in: (i) 2^{50} (ii) 3^{20}.

Solution:

- (i) $\log 2^{50} = 50 \times 0.3010 = 15.05$. The characteristic is 15, so the number of digits = 15 + 1 = 16.
- (ii) $\log 3^{20} = 20 \times 0.4771 = 9.542$. The characteristic is 9, so the number of digits = 9 + 1 = 10.

Answer: (i) 16 digits (ii) 10 digits

Q26. Given log 2 = 0.3010 and log 3 = 0.4771, find log 0.0036 and state its characteristic and mantissa.

Solution:

1. Write $0.0036 = 36 / 10^4$, so $\log 0.0036 = \log 36 - 4$.
2. $\log 36 = 2 \log 6 = 2 \times 0.7781 = 1.5562$.
3. $\log 0.0036 = 1.5562 - 4 = -2.4438$.
4. The mantissa must be positive, so write $-2.4438 = -3 + 0.5562$ (bar 3 point 5562).

Answer: $\log 0.0036 = -3 + 0.5562$. Characteristic = −3, mantissa = 0.5562.

Q27. If $\log [(x + y)/3] = \frac{1}{2} (\log x + \log y)$, prove that $x/y + y/x = 7$.

Solution:

1. RHS = $\frac{1}{2} \log (xy) = \log \sqrt{(xy)}$.
2. Equate the arguments: $(x + y)/3 = \sqrt{(xy)}$.
3. Square both sides: $(x + y)^2 = 9xy$, so $x^2 + 2xy + y^2 = 9xy$.
4. Hence $x^2 + y^2 = 7xy$.
5. Divide by xy: $x/y + y/x = 7$. Hence proved.

Q28. Solve using logarithms (log 2 = 0.3010, log 3 = 0.4771, log 5 = 0.6990): (i) $3^x = 5$ (ii) $2^{x+1} = 3^{x-1}$.

Solution:

- (i) Take log on both sides: $x \log 3 = \log 5$, so $x = 0.6990 / 0.4771 = 1.465$.
- (ii) Take log on both sides: $(x + 1) \log 2 = (x - 1) \log 3$.
- Expand: $x \log 2 + \log 2 = x \log 3 - \log 3$, so $x (\log 3 - \log 2) = \log 3 + \log 2$.
- $x = \log 6 / \log 1.5 = 0.7781 / 0.1761 = 4.42$ (approximately).

Answer: (i) $x \approx 1.465$ (ii) $x \approx 4.42$

Part 4: Long Answer Questions (5–6 Marks)

Q29. Prove the product law, quotient law and power law of logarithms.

Solution:

1. Let $\log_a M = p$ and $\log_a N = q$. In exponential form, $M = a^p$ and $N = a^q$.
2. Product law: $MN = a^p \times a^q = a^{p+q}$. So $\log_a (MN) = p + q = \log_a M + \log_a N$.
3. Quotient law: $M/N = a^p / a^q = a^{p-q}$. So $\log_a (M/N) = p - q = \log_a M - \log_a N$.
4. Power law: $M^n = (a^p)^n = a^{np}$. So $\log_a (M^n) = np = n \log_a M$.
5. Hence all three laws are proved.

Q30. Prove that $\log_a M = \log_b M / \log_b a$. Hence show that $\log_a b \times \log_b c \times \log_c a = 1$.

Solution:

1. Let $\log_a M = x$, so $a^x = M$.
2. Take log to the base b on both sides: $x \log_b a = \log_b M$, so $x = \log_b M / \log_b a$. Hence the change of base formula is proved.
3. Now change each term to base 10: $\log_a b = \log b / \log a$, $\log_b c = \log c / \log b$, $\log_c a = \log a / \log c$.
4. Product = $(\log b / \log a) \times (\log c / \log b) \times (\log a / \log c) = 1$, since every factor cancels. Hence proved.

Q31. Prove that $7 \log (16/15) + 5 \log (25/24) + 3 \log (81/80) = \log 2$.

Solution:

1. Write each number in primes: $16 = 2^4$, $15 = 3 \times 5$, $25 = 5^2$, $24 = 2^3 \times 3$, $81 = 3^4$, $80 = 2^4 \times 5$.
2. Let $p = \log 2$, $q = \log 3$ and $r = \log 5$.
3. $7 \log (16/15) = 7 (4p - q - r) = 28p - 7q - 7r$.
4. $5 \log (25/24) = 5 (2r - 3p - q) = 10r - 15p - 5q$.
5. $3 \log (81/80) = 3 (4q - 4p - r) = 12q - 12p - 3r$.
6. Add: coefficient of $p = 28 - 15 - 12 = 1$; coefficient of $q = -7 - 5 + 12 = 0$; coefficient of $r = -7 + 10 - 3 = 0$.
7. So LHS = $p = \log 2$ = RHS. Hence proved.

Q32. If $2^x = 3^y = 6^{-z}$, prove that $1/x + 1/y + 1/z = 0$.

Solution:

1. Let $2^x = 3^y = 6^{-z} = k$ ($k \neq 1$).
2. Then $2 = k^{1/x}$, $3 = k^{1/y}$ and $6 = k^{-1/z}$.
3. Since $2 \times 3 = 6$, we get $k^{1/x} \times k^{1/y} = k^{-1/z}$, i.e. $k^{1/x + 1/y} = k^{-1/z}$.
4. Equate the powers: $1/x + 1/y = -1/z$.
5. Therefore $1/x + 1/y + 1/z = 0$. Hence proved.

Q33. Solve: $\log_2 x + \log_4 x + \log_{16} x = 7$.

Solution:

1. Change every logarithm to base 2.
2. $\log_4 x = \log_2 x / \log_2 4 = (1/2) \log_2 x$, and $\log_{16} x = \log_2 x / \log_2 16 = (1/4) \log_2 x$.
3. Equation becomes $(1 + 1/2 + 1/4) \log_2 x = 7$, i.e. $(7/4) \log_2 x = 7$.

4. So $\log_2 x = 4$ and $x = 2^4 = 16$.

Answer: $x = 16$

Q34. If $a = \log_{12} 18$ and $b = \log_{24} 54$, prove that $ab + 5(a - b) = 1$.

Solution:

1. Let $p = \log 2$ and $q = \log 3$ (same base throughout). Then $\log 18 = p + 2q$, $\log 12 = 2p + q$, $\log 54 = p + 3q$ and $\log 24 = 3p + q$.
2. So $a = (p + 2q)/(2p + q)$ and $b = (p + 3q)/(3p + q)$. Let $D = (2p + q)(3p + q) = 6p^2 + 5pq + q^2$.
3. $ab = (p + 2q)(p + 3q) / D = (p^2 + 5pq + 6q^2) / D$.
4. $a - b = [(p + 2q)(3p + q) - (p + 3q)(2p + q)] / D = [(3p^2 + 7pq + 2q^2) - (2p^2 + 7pq + 3q^2)] / D = (p^2 - q^2) / D$.
5. $ab + 5(a - b) = [p^2 + 5pq + 6q^2 + 5p^2 - 5q^2] / D = (6p^2 + 5pq + q^2) / D = D / D = 1$. Hence proved.

Q35. If $x = \log_{2a} a$, $y = \log_{3a} 2a$ and $z = \log_{4a} 3a$, prove that $xyz + 1 = 2yz$.

Solution:

1. Change to base 10: $x = \log a / \log 2a$, $y = \log 2a / \log 3a$, $z = \log 3a / \log 4a$.
2. $xyz = (\log a / \log 2a) \times (\log 2a / \log 3a) \times (\log 3a / \log 4a) = \log a / \log 4a$.
3. $yz = (\log 2a / \log 3a) \times (\log 3a / \log 4a) = \log 2a / \log 4a$.
4. LHS $= xyz + 1 = (\log a + \log 4a) / \log 4a = \log (4a^2) / \log 4a$.
5. RHS $= 2yz = 2 \log 2a / \log 4a = \log (2a)^2 / \log 4a = \log (4a^2) / \log 4a$.
6. LHS = RHS. Hence proved.

Part 5: Engineering Applications

Q36. The sound intensity level is given by $L = 10 \log_{10} (I / I_0)$ decibel. (i) Find L when $I = 10^{-5}$ W/m² and $I_0 = 10^{-12}$ W/m². (ii) By how many decibels does the level rise when the intensity is doubled?

Solution:

- (i) $I / I_0 = 10^{-5} / 10^{-12} = 10^7$. So $L = 10 \log 10^7 = 10 \times 7 = 70$ dB.
- (ii) New level − old level $= 10 \log (2I / I_0) - 10 \log (I / I_0) = 10 \log 2 = 10 \times 0.3010 \approx 3$ dB.

Answer: (i) 70 dB (ii) about 3 dB

Q37. A sum of money is invested at 8% compound interest per year. In how many years will it double? (Given $\log 2 = 0.3010$ and $\log 1.08 = 0.0334$.)

Solution:

1. Amount $A = P (1 + r)^n$. For doubling, $2P = P (1.08)^n$, so $(1.08)^n = 2$.
2. Take log on both sides: $n \log 1.08 = \log 2$.
3. $n = 0.3010 / 0.0334 = 9.01$.

Answer: About 9 years

Q38. In a charging RC circuit the capacitor voltage is $v = V (1 - e^{-t/RC})$. Find the time taken for the voltage to reach 90% of V. (Given $\ln 10 = 2.303$.)

Solution:

1. Put $v = 0.9$ V: $0.9 V = V (1 - e^{-t/RC})$, so $e^{-t/RC} = 0.1$.
2. Take natural log on both sides: $-t/RC = \ln 0.1 = -\ln 10$.

3. So t = RC ln 10 = 2.303 RC.

Answer: t ≈ 2.3 RC

Part 6: Practice Questions

Try these without looking at the solutions above. Check your work with the answer key given below.

Q39. Find the value of $\log_4 32$ and $\log_{27} 9$.

Q40. Find x if (i) $\log_x 125 = 3$ (ii) $\log_2 (3x - 1) = 5$ (iii) $\log_{\sqrt{2}} x = 6$.

Q41. Simplify: (i) 2 log 5 + log 4 (ii) log 4 + log 25 − log 10.

Q42. Evaluate $\log_2 (1/8) + \log_3 81$.

Q43. Solve: log (x + 1) + log (x − 1) = log 3.

Q44. Given log 2 = 0.3010, find (i) log 16 (ii) log 20 (iii) log 0.5.

Q45. Find the number of digits in 5^{30}, given log 5 = 0.6990.

Q46. Solve: $\log_3 x + \log_9 x = 3$.

Q47. Solve $4^x = 7$, given log 2 = 0.3010 and log 7 = 0.8451.

Q48. Evaluate $(\log_2 3)(\log_3 4)(\log_4 5)(\log_5 8)$.

Q49. If $\log_a x = 2$ and $\log_a y = 3$, find (i) $\log_a (x^2 y^3)$ (ii) $\log_a (x / y^2)$.

Q50. If log (x + y) = log x + log y, show that y = x / (x − 1).

Q51. Prove that $\log_b a \times \log_c b \times \log_a c = 1$.

Q52. Solve: $\log (x^2 - 9) - \log (x + 3) = 1$.

Answer Key for Practice Questions

Q39. $\log_4 32 = 5/2$ and $\log_{27} 9 = 2/3$

Q40. (i) 5 (ii) 11 (iii) 8

Q41. (i) 2 (ii) 1

Q42. 1

Q43. x = 2 (reject x = −2)

Q44. (i) 1.2040 (ii) 1.3010 (iii) −0.3010

Q45. 21 digits

Q46. x = 9

Q47. x ≈ 1.404

Q48. 3

Q49. (i) 13 (ii) −4

Q50. x + y = xy, so xy − y = x, i.e. y (x − 1) = x, hence y = x / (x − 1).

Q51. Change every term to base 10: (log a / log b)(log b / log c)(log c / log a) = 1.

Q52. x − 3 = 10, so x = 13

Part 7: Exam Tips and Common Mistakes

- Always write the condition first when solving a log equation: every argument must be positive. Reject any root that breaks it.

- log (A + B) is not log A + log B, and log A / log B is not log (A / B). The laws work only for a product, a quotient or a power inside the log.
- $(\log A)^n$ is not the same as n log A. The power law applies to the number inside the log, that is $\log A^n = n \log A$.
- The base must be positive and not equal to 1.
- The mantissa is always positive. If the log comes out negative, split it as (negative whole number) + (positive decimal).
- In "prove that" questions, start from the more complicated side, use one common base (base 10 is safest) and end with "Hence proved".
- In numerical questions with given values such as log 2 = 0.3010, break the number into factors of 2, 3, 5 or 10 before applying the laws.
- Number of digits in N = characteristic of log N + 1.
- Write the final answer on a separate line and, for equations, verify it by substituting back.