MODEL TEST PAPER

Mathematics 1 | Chapter: Logarithm

For Engineering and Polytechnic Students

Full Marks: 50 **Time: 1 hour 30 minutes**

Name: _____ Roll No.: _____ Date: _____

Instructions

- All questions are compulsory. The marks for each question are shown in square brackets on the right.
- "log" means $\log_{10}$ (base 10) unless another base is written, and "ln" means $\log_e$.
- Show all steps in Sections B, C and D. Where values are needed, use the ones given in the question.

Section A: Multiple Choice Questions (10 × 1 = 10 Marks)

Choose the correct option.

Q1. The value of $\log_5 625$ is **[1]**

(a) 3 (b) 4 (c) 5 (d) 25

Q2. If $\log_2 x = -3$, then x is **[1]**

(a) −8 (b) 1/8 (c) −6 (d) 1/6

Q3. $\log 4 + \log 25$ is equal to **[1]**

(a) 2 (b) 1 (c) log 29 (d) 100

Q4. The value of $\log_9 3$ is **[1]**

(a) 1/2 (b) 2 (c) 3 (d) 1/3

Q5. If $\log 3 = 0.4771$, then $\log 30$ is **[1]**

(a) 1.4771 (b) 0.4771 (c) 4.771 (d) 3.4771

Q6. The characteristic of log 725.4 is **[1]**

(a) 2 (b) 3 (c) 1 (d) 725

Q7. The value of $\log_2 5 \times \log_5 8$ is **[1]**

(a) 1 (b) 2 (c) 3 (d) 4

Q8. The value of $\ln e^2 + \ln 1$ is **[1]**

(a) 2 (b) 3 (c) 1 (d) 0

Q9. If $\log (x - 1) = \log 4$, then x is **[1]**

(a) 3 (b) 4 (c) 5 (d) 6

Q10. Which of the following is correct? **[1]**

(a) $\log (a + b) = \log a + \log b$ (b) $\log (a/b) = \log a - \log b$ (c) $\log a / \log b = \log (a/b)$
(d) $(\log a)^2 = 2 \log a$

Section B: Short Answer Questions (5 × 2 = 10 Marks)

Q11. Write $3^4 = 81$ in logarithmic form, and write $\log_{10} 0.01 = -2$ in exponential form. **[2 Marks]**

Q12. Find the value of $\log_{\sqrt{5}} 125$. **[2 Marks]**

Q13. Simplify $\log 8 + \log 125 - \log 10$. **[2 Marks]**

Q14. Given $\log 2 = 0.3010$, find $\log 0.25$ and state its characteristic and mantissa. **[2 Marks]**

Q15. Express $\log (x^3\sqrt{y} / z)$ in terms of log x, log y and log z. **[2 Marks]**

Section C: Medium Answer Questions (4 × 4 = 16 Marks)

Q16. Solve $\log_2 (x + 3) + \log_2 (x - 3) = 4$. **[4 Marks]**

Q17. If $a^2 + b^2 = 34ab$ $(a, b > 0)$, prove that $\log [(a + b)/6] = ½ (\log a + \log b)$. **[4 Marks]**

Q18. Given $\log 2 = 0.3010$ and $\log 3 = 0.4771$, find (i) log 18 (ii) log 0.75 (iii) the logarithm of the cube root of 6. **[4 Marks]**

Q19. Given $\log 2 = 0.3010$ and $\log 3 = 0.4771$, find the number of digits in (i) 3^{40} (ii) 2^{64}. **[4 Marks]**

Section D: Long Answer Questions (2 × 7 = 14 Marks)

Q20.

(a) Prove that $\log_a M = \log_b M / \log_b a$. **[3]**

(b) Hence prove that $\log_a b \times \log_b c \times \log_c d = \log_a d$, and evaluate $\log_2 3 \times \log_3 5 \times \log_5 16$. **[4]**

Q21.

(a) If $\log x / (2a - b - c) = \log y / (2b - c - a) = \log z / (2c - a - b)$, prove that $xyz = 1$. **[4]**

(b) Solve $3^{2x} - 4 \times 3^x + 3 = 0$. **[3]**

— End of Question Paper —

Answer Key and Marking Scheme

This part is for checking answers after you attempt the paper. Marks are shown against each step so that you can evaluate yourself.

Section A: Answer Key (1 mark each)

Q1 − (b) Q2 − (b) Q3 − (a) Q4 − (a) Q5 − (a)

Q6 − (a) Q7 − (c) Q8 − (a) Q9 − (c) Q10 − (b)

Reasons:

- **Q1.** $5^4 = 625$, so $\log_5 625 = 4$.
- **Q2.** $x = 2^{-3} = 1/8$.
- **Q3.** $\log 4 + \log 25 = \log 100 = 2$.
- **Q4.** $9^{1/2} = 3$, so $\log_9 3 = 1/2$.
- **Q5.** $\log 30 = \log 3 + \log 10 = 0.4771 + 1 = 1.4771$.
- **Q6.** There are 3 digits before the decimal point, so the characteristic is $3 − 1 = 2$.
- **Q7.** By the chain rule, $\log_2 5 \times \log_5 8 = \log_2 8 = 3$.
- **Q8.** $\ln e^2 = 2$ and $\ln 1 = 0$, so the sum is 2.
- **Q9.** Equate the arguments: $x − 1 = 4$, so $x = 5$.
- **Q10.** Only the quotient law is correct; the others are common mistakes.

Section B: Solutions (2 marks each)

Q11.

1. $3^4 = 81 \rightarrow \log_3 81 = 4$.
2. $\log_{10} 0.01 = −2 \rightarrow 10^{-2} = 0.01$.

Marking: 1 mark for each conversion.

Q12.

1. Let $\log_{\sqrt{5}} 125 = x$. Then $(\sqrt{5})^x = 125$, i.e. $5^{x/2} = 5^3$.
2. So $x/2 = 3$ and $x = 6$.

Answer: 6

Marking: 1 mark for the exponential form, 1 mark for the value.

Q13.

1. $\log 8 + \log 125 − \log 10 = \log (8 \times 125 / 10)$.
2. $= \log 100 = 2$.

Answer: 2

Marking: 1 mark for combining, 1 mark for the value.

Q14.

1. $\log 0.25 = \log (1/4) = − \log 4 = −2 \log 2 = −2 (0.3010) = −0.6020$.
2. Make the mantissa positive: $−0.6020 = −1 + 0.3980$.

Answer: log 0.25 = −0.6020. Characteristic = −1, mantissa = 0.3980.

Marking: 1 mark for the value of the log, 1 mark for the characteristic and mantissa.

Q15.

1. $\log (x^3\sqrt{y} / z) = \log x^3 + \log \sqrt{y} - \log z$.
2. $= 3 \log x + ½ \log y - \log z$.

Answer: $3 \log x + ½ \log y - \log z$

Marking: 1 mark for the product and quotient laws, 1 mark for the power law.

Section C: Solutions (4 marks each)

Q16.

1. Condition: $x + 3 > 0$ and $x - 3 > 0$, so $x > 3$.
2. Combine: $\log_2 [(x + 3)(x - 3)] = 4$, so $x^2 - 9 = 2^4 = 16$.
3. $x^2 = 25$, so $x = 5$ or $x = -5$.
4. $x = -5$ does not satisfy $x > 3$, so it is rejected.

Answer: $x = 5$

Marking: 1 mark for the condition, 1 mark for combining, 1 mark for solving, 1 mark for rejecting $x = -5$.

Q17.

1. $(a + b)^2 = a^2 + b^2 + 2ab = 34ab + 2ab = 36ab$.
2. Divide by 36: $[(a + b)/6]^2 = ab$.
3. Take the positive square root: $(a + b)/6 = \sqrt{(ab)}$.
4. Take log: $\log [(a + b)/6] = ½ \log (ab) = ½ (\log a + \log b)$. Hence proved.

Marking: 1 mark for each step.

Q18.

1. (i) $\log 18 = \log (2 \times 3^2) = 0.3010 + 2 (0.4771) = 1.2552$.
2. (ii) $\log 0.75 = \log (3/4) = 0.4771 - 2 (0.3010) = -0.1249$ $(= -1 + 0.8751)$.
3. (iii) log of the cube root of $6 = (1/3) \log 6 = (1/3)(0.3010 + 0.4771) = 0.7781 / 3 = 0.2594$.

Answer: (i) 1.2552 (ii) −0.1249 (iii) 0.2594

Marking: 1 mark for (i), 2 marks for (ii), 1 mark for (iii).

Q19.

1. (i) $\log 3^{40} = 40 \times 0.4771 = 19.084$. The characteristic is 19, so the number of digits = 19 + 1 = 20.
2. (ii) $\log 2^{64} = 64 \times 0.3010 = 19.264$. The characteristic is 19, so the number of digits = 19 + 1 = 20.

Answer: (i) 20 digits (ii) 20 digits

Marking: 2 marks for each part (1 for the log value, 1 for the number of digits).

Section D: Solutions (7 marks each)

Q20 (a).

1. Let $\log_a M = x$. Then $a^x = M$.
2. Take log to the base b on both sides: $x \log_b a = \log_b M$.

3. So $x = \log_b M / \log_b a$, that is $\log_a M = \log_b M / \log_b a$. Hence proved.

Marking: 1 mark for the exponential form, 1 mark for taking the log, 1 mark for the result.

Q20 (b).

1. Change every logarithm to base 10: $\log_a b = \log b / \log a$, $\log_b c = \log c / \log b$, $\log_c d = \log d / \log c$.
2. Product = $(\log b / \log a) \times (\log c / \log b) \times (\log d / \log c) = \log d / \log a = \log_a d$. Hence proved.
3. Using this result, $\log_2 3 \times \log_3 5 \times \log_5 16 = \log_2 16$.
4. $\log_2 16 = 4$, because $2^4 = 16$.

Answer: 4

Marking: 1 mark for changing the base, 2 marks for the cancellation and result, 1 mark for the value.

Q21 (a).

1. Let each ratio be k. Then $\log x = k(2a - b - c)$, $\log y = k(2b - c - a)$ and $\log z = k(2c - a - b)$.
2. Add the three equations: $\log x + \log y + \log z = k[(2a - b - c) + (2b - c - a) + (2c - a - b)]$.
3. The bracket = $(2a - a - a) + (2b - b - b) + (2c - c - c) = 0$.
4. So $\log(xyz) = 0 = \log 1$, which gives $xyz = 1$. Hence proved.

Marking: 1 mark for putting the ratio equal to k, 1 mark for adding, 1 mark for the bracket being 0, 1 mark for the conclusion.

Q21 (b).

1. Put $t = 3^x$. Then $3^{2x} = t^2$.
2. The equation becomes $t^2 - 4t + 3 = 0$, i.e. $(t - 1)(t - 3) = 0$, so $t = 1$ or $t = 3$.
3. $3^x = 1$ gives $x = 0$. $3^x = 3$ gives $x = 1$.

Answer: $x = 0$ or $x = 1$

Marking: 1 mark for the substitution, 1 mark for solving the quadratic, 1 mark for the values of x.

Total: Section A 10 + Section B 10 + Section C 16 + Section D 14 = 50 marks.