MATHEMATICS 1

Chapter: Logarithm

Numerical Problems, Solutions and MCQs

For Engineering and Polytechnic Students | Exam-Oriented

Part 1: Quick Toolkit for Numerical Problems

Most numerical questions in this chapter ask you to find a value, simplify an expression or solve an equation. Keep the following in mind before you start.

- $a^x = N \leftrightarrow \log_a N = x$. $\log_a 1 = 0$, $\log_a a = 1$.
- $\log_a (MN) = \log_a M + \log_a N$, $\log_a (M/N) = \log_a M - \log_a N$, $\log_a M^n = n \log_a M$.
- Change of base: $\log_a M = \log M / \log a$. Reciprocal: $\log_a b = 1 / \log_b a$. Chain: $\log_a b \times \log_b c = \log_a c$.
- To find a value, set $\log_a N = x$ and solve $a^x = N$ by writing both sides as powers of the same number.
- To solve an equation, write the condition (arguments positive, base positive and not 1) first, and reject invalid roots at the end.
- When values such as log 2 = 0.3010 and log 3 = 0.4771 are given, split the number into factors of 2, 3, 5 (use log 5 = 1 − log 2), 7 and 10.
- Number of digits in N = characteristic of log N + 1.

Note: In this document "log" means $\log_{10}$ unless another base is written. Values used: log 2 = 0.3010, log 3 = 0.4771, log 5 = 0.6990, log 7 = 0.8451, ln 2 = 0.6931.

Part 2: Find the Value

Q1. Find the value of (i) $\log_{\sqrt{2}} 64$ (ii) $\log_{1/4} 8$.

Solution:

1. (i) Let $\log_{\sqrt{2}} 64 = x$. Then $(\sqrt{2})^x = 64$, i.e. $2^{x/2} = 2^6$. So $x/2 = 6$ and $x = 12$.
2. (ii) Let $\log_{1/4} 8 = x$. Then $(1/4)^x = 8$, i.e. $2^{-2x} = 2^3$. So $-2x = 3$ and $x = -3/2$.

Answer: (i) 12 (ii) −3/2

Q2. Evaluate $\log_2 16 - \log_3 27 + \log_{10} 0.01$.

Solution:

1. $\log_2 16 = 4$, because $2^4 = 16$.
2. $\log_3 27 = 3$, because $3^3 = 27$.
3. $\log_{10} 0.01 = -2$, because $10^{-2} = 0.01$.
4. Value = 4 − 3 + (−2) = −1.

Answer: −1

Q3. Evaluate $\log_2 (\log_2 (\log_3 81))$.

Solution:

1. Innermost: $\log_3 81 = 4$, because $3^4 = 81$.
2. Next: $\log_2 4 = 2$, because $2^2 = 4$.

3. Outermost: $\log_2 2 = 1$.

Answer: 1

Q4. Evaluate $\log_4 8 \times \log_8 16 \times \log_{16} 2$.

Solution:

1. Use the chain rule: $\log_4 8 \times \log_8 16 = \log_4 16 = 2$.
2. $\log_{16} 2 = 1/4$, because $16^{1/4} = 2$.
3. Product = $2 \times 1/4 = 1/2$.

Answer: 1/2

Q5. Evaluate (i) $\log_5 3 \times \log_3 25$ (ii) $\log_9 27 + \log_{27} 9$.

Solution:

1. (i) By the chain rule, $\log_5 3 \times \log_3 25 = \log_5 25 = 2$.
2. (ii) $\log_9 27$: let it be x. Then $9^x = 27$, i.e. $3^{2x} = 3^3$, so $x = 3/2$.
3. $\log_{27} 9$: let it be y. Then $27^y = 9$, i.e. $3^{3y} = 3^2$, so $y = 2/3$.
4. Sum = $3/2 + 2/3 = 9/6 + 4/6 = 13/6$.

Answer: (i) 2 (ii) 13/6

Q6. Evaluate $1/\log_3 12 + 1/\log_4 12$.

Solution:

1. By the reciprocal rule, $1/\log_3 12 = \log_{12} 3$ and $1/\log_4 12 = \log_{12} 4$.
2. Sum = $\log_{12} 3 + \log_{12} 4 = \log_{12} (3 \times 4) = \log_{12} 12 = 1$.

Answer: 1

Q7. Evaluate (i) $\log 5 + \log 20 + \log 0.1$ (ii) $3 \log 2 + 2 \log 5 - \log 2$.

Solution:

1. (i) By the product law, $\log 5 + \log 20 + \log 0.1 = \log (5 \times 20 \times 0.1) = \log 10 = 1$.
2. (ii) $3 \log 2 - \log 2 = 2 \log 2$. So the expression = $2 \log 2 + 2 \log 5 = 2 (\log 2 + \log 5) = 2 \log 10 = 2$.

Answer: (i) 1 (ii) 2

Part 3: Simplify and Express

Q8. Simplify $2 \log (15/18) - \log (25/162) + \log (4/9)$.

Solution:

1. Power law: $2 \log (15/18) = \log (15/18)^2 = \log (225/324) = \log (25/36)$.
2. Combine: expression = $\log [(25/36) \times (162/25) \times (4/9)]$.
3. Cancel 25. Then $162/36 = 9/2$, and $(9/2) \times (4/9) = 2$.

Answer: log 2

Q9. Simplify $\frac{1}{2} \log 25 - 2 \log 3 + \log 18$.

Solution:

1. $\frac{1}{2} \log 25 = \log 25^{1/2} = \log 5$.

2. $2 \log 3 = \log 3^2 = \log 9$.
3. Expression = $\log 5 - \log 9 + \log 18 = \log (5 \times 18 / 9) = \log 10 = 1$.

Answer: 1

Q10. Express $\log (\sqrt{(x^3y)} / z^2)$ in terms of log x, log y and log z.

Solution:

1. Write $\sqrt{(x^3y)} = (x^3y)^{1/2}$. Quotient law: log = ½ log (x^3y) − log z^2.
2. Product and power laws: ½ (3 log x + log y) − 2 log z.

Answer: (3/2) log x + ½ log y − 2 log z

Q11. Write as a single logarithm: 3 log 2 − 2 log 3 + ½ log 81.

Solution:

1. 3 log 2 = log 8, 2 log 3 = log 9 and ½ log 81 = log $81^{1/2}$ = log 9.
2. Expression = log 8 − log 9 + log 9 = log 8.

Answer: log 8

Q12. Simplify $\log_2 12 + \log_2 6 - \log_2 9$.

Solution:

1. Combine: $\log_2 (12 \times 6 / 9) = \log_2 8$.
2. $\log_2 8 = 3$, because $2^3 = 8$.

Answer: 3

Q13. Simplify (i) log 125 / log 5 (ii) log 81 / log 27.

Solution:

1. (i) log 125 = log 5^3 = 3 log 5. So the ratio = 3 log 5 / log 5 = 3.
2. (ii) log 81 = 4 log 3 and log 27 = 3 log 3. So the ratio = 4 log 3 / 3 log 3 = 4/3.

Answer: (i) 3 (ii) 4/3

Part 4: Problems Using Given Logarithm Values

Given: log 2 = 0.3010, log 3 = 0.4771, log 7 = 0.8451.

Q14. Find (i) log 14 (ii) log 21 (iii) log 98.

Solution:

1. (i) log 14 = log (2 × 7) = 0.3010 + 0.8451 = 1.1461.
2. (ii) log 21 = log (3 × 7) = 0.4771 + 0.8451 = 1.3222.
3. (iii) log 98 = log (2×7^2) = 0.3010 + 2 (0.8451) = 0.3010 + 1.6902 = 1.9912.

Answer: (i) 1.1461 (ii) 1.3222 (iii) 1.9912

Q15. Find (i) log √21 (ii) log of the cube root of 49 (iii) log (2/7).

Solution:

1. (i) log $\sqrt{21}$ = ½ log 21 = ½ (1.3222) = 0.6611.
2. (ii) log $\sqrt[3]{49}$ = (1/3) log 7^2 = (2/3)(0.8451) = 1.6902 / 3 = 0.5634.

3. (iii) log (2/7) = 0.3010 − 0.8451 = −0.5441 = −1 + 0.4559.

Answer: (i) 0.6611 (ii) 0.5634 (iii) −0.5441 (characteristic −1, mantissa 0.4559)

Q16. Find log 0.14 and log 0.0021. Write the characteristic and mantissa of each.

Solution:

1. 0.14 = 14 / 100, so log 0.14 = log 14 − 2 = 1.1461 − 2 = −0.8539 = −1 + 0.1461.
2. 0.0021 = 21 / 10^4, so log 0.0021 = log 21 − 4 = 1.3222 − 4 = −2.6778 = −3 + 0.3222.
3. Cross-check with the rule: 0.14 has no zero after the decimal point (characteristic −1); 0.0021 has two zeros (characteristic −3).

Answer: log 0.14: characteristic −1, mantissa 0.1461. log 0.0021: characteristic −3, mantissa 0.3222.

Q17. Find the number of digits in (i) 7^{20} (ii) 12^{10} (iii) 14^{15}.

Solution:

1. (i) log 7^{20} = 20 × 0.8451 = 16.902. Characteristic 16, so the number of digits = 17.
2. (ii) log 12 = 2 (0.3010) + 0.4771 = 1.0791. log 12^{10} = 10.791. Characteristic 10, so the number of digits = 11.
3. (iii) log 14 = 1.1461. log 14^{15} = 15 × 1.1461 = 17.1915. Characteristic 17, so the number of digits = 18.

Answer: (i) 17 digits (ii) 11 digits (iii) 18 digits

Q18. Solve (i) $7^x = 100$ (ii) $2^x = 3^{x-1}$.

Solution:

1. (i) Take log on both sides: x log 7 = log 100 = 2. So x = 2 / 0.8451 = 2.367.
2. (ii) Take log on both sides: x log 2 = (x − 1) log 3, i.e. x log 2 = x log 3 − log 3.
3. So log 3 = x (log 3 − log 2), which gives x = 0.4771 / (0.4771 − 0.3010) = 0.4771 / 0.1761 = 2.71.

Answer: (i) x ≈ 2.367 (ii) x ≈ 2.71

Q19. Solve $2^{x+1} \times 3^x = 72$.

Solution:

1. Take log on both sides: (x + 1) log 2 + x log 3 = log 72.
2. log 72 = log ($2^3 \times 3^2$) = 3 (0.3010) + 2 (0.4771) = 1.8572.
3. Left side = 0.3010x + 0.3010 + 0.4771x = 0.7781x + 0.3010.
4. So 0.7781x = 1.8572 − 0.3010 = 1.5562, which gives x = 2.
5. Check: $2^3 \times 3^2 = 8 \times 9 = 72$.

Answer: x = 2

Part 5: Solve for x (Logarithmic Equations)

Q20. Solve (i) $\log_5 (2x - 1) = 2$ (ii) $\log_2 (x^2 - 2x) = 3$.

Solution:

1. (i) Exponential form: 2x − 1 = 5^2 = 25, so 2x = 26 and x = 13.

2. (ii) Exponential form: $x^2 - 2x = 2^3 = 8$, so $x^2 - 2x - 8 = 0$, i.e. $(x - 4)(x + 2) = 0$.
3. $x = 4$ or $x = -2$. For both, the argument $x^2 - 2x = 8$ is positive, so both are valid.

Answer: (i) $x = 13$ (ii) $x = 4$ or $x = -2$

Q21. Solve $\log (3x + 1) = \log (x + 9)$.

Solution:

1. Condition: $3x + 1 > 0$ and $x + 9 > 0$.
2. Equate the arguments: $3x + 1 = x + 9$, so $2x = 8$ and $x = 4$.
3. Check: $3(4) + 1 = 13 > 0$ and $4 + 9 = 13 > 0$. Valid.

Answer: $x = 4$

Q22. Solve $\log_2 (x - 1) + \log_2 (x + 1) = 3$.

Solution:

1. Condition: $x - 1 > 0$, so $x > 1$.
2. Combine: $\log_2 (x^2 - 1) = 3$, so $x^2 - 1 = 8$ and $x^2 = 9$.
3. $x = 3$ or $x = -3$. Reject $x = -3$ because it is not greater than 1.

Answer: $x = 3$

Q23. Solve $\log_3 x - \log_3 (x - 2) = 1$.

Solution:

1. Condition: $x > 0$ and $x - 2 > 0$, so $x > 2$.
2. Combine: $\log_3 [x / (x - 2)] = 1$, so $x / (x - 2) = 3$.
3. $x = 3x - 6$, so $2x = 6$ and $x = 3$. It satisfies $x > 2$.

Answer: $x = 3$

Q24. Solve $2 \log x = \log (x + 6)$.

Solution:

1. Condition: $x > 0$ (needed for $\log x$) and $x + 6 > 0$.
2. Power law: $\log x^2 = \log (x + 6)$, so $x^2 = x + 6$.
3. $x^2 - x - 6 = 0$, i.e. $(x - 3)(x + 2) = 0$, so $x = 3$ or $x = -2$.
4. Reject $x = -2$ because $\log x$ is not defined for a negative number.
5. Check: $2 \log 3 = \log 9 = \log (3 + 6)$.

Answer: $x = 3$

Q25. Solve $\log x + \log (x - 3) = 1$.

Solution:

1. Condition: $x > 3$.
2. Combine: $\log [x (x - 3)] = 1$, so $x (x - 3) = 10$.
3. $x^2 - 3x - 10 = 0$, i.e. $(x - 5)(x + 2) = 0$, so $x = 5$ or $x = -2$.
4. Reject $x = -2$ because it is not greater than 3.

Answer: $x = 5$

Q26. Solve $\log_3 x + \log_9 x + \log_{27} x = 11$.

Solution:

1. Change to base 3: $\log_9 x = (1/2) \log_3 x$ and $\log_{27} x = (1/3) \log_3 x$.
2. The equation becomes $(1 + 1/2 + 1/3) \log_3 x = 11$, i.e. $(11/6) \log_3 x = 11$.
3. So $\log_3 x = 6$ and $x = 3^6 = 729$.

Answer: $x = 729$

Q27. Solve $(\log x)^2 = 2 \log x$.

Solution:

1. Condition: $x > 0$.
2. Bring all terms to one side: $(\log x)^2 - 2 \log x = 0$, so $\log x (\log x - 2) = 0$.
3. $\log x = 0$ gives $x = 1$. $\log x = 2$ gives $x = 100$.
4. Both are positive, so both are valid.

Answer: $x = 1$ or $x = 100$

Q28. Find x if (i) $\log_x 9 = 2$ (ii) $\log_x (1/16) = -2$ (iii) $\log_x 3 = 1/2$.

Solution:

1. (i) $x^2 = 9$, so $x = 3$. The value -3 is rejected because a base must be positive.
2. (ii) $x^{-2} = 1/16$, so $x^2 = 16$ and $x = 4$.
3. (iii) $x^{1/2} = 3$, so $x = 3^2 = 9$.

Answer: (i) 3 (ii) 4 (iii) 9

Part 6: Exponential and Simultaneous Equations

Q29. Solve $4^x - 5 \times 2^x + 4 = 0$.

Solution:

1. Put $t = 2^x$. Then $4^x = (2^x)^2 = t^2$.
2. The equation becomes $t^2 - 5t + 4 = 0$, i.e. $(t - 1)(t - 4) = 0$, so $t = 1$ or $t = 4$.
3. $2^x = 1$ gives $x = 0$. $2^x = 4$ gives $x = 2$.

Answer: $x = 0$ or $x = 2$

Q30. Solve $5^{x+1} = 3^x$, given $\log 3 = 0.4771$ and $\log 5 = 0.6990$.

Solution:

1. Take log on both sides: $(x + 1) \log 5 = x \log 3$.
2. Expand: $x \log 5 + \log 5 = x \log 3$, so $\log 5 = x (\log 3 - \log 5)$.
3. $x = 0.6990 / (0.4771 - 0.6990) = 0.6990 / (-0.2219) = -3.15$.

Answer: $x \approx -3.15$

Q31. Solve $\log_2 x + \log_2 y = 5$ and $x + y = 12$.

Solution:

1. Here $x > 0$ and $y > 0$. From the first equation, $\log_2 (xy) = 5$, so $xy = 2^5 = 32$.
2. From the second, $y = 12 - x$. So $x (12 - x) = 32$, i.e. $x^2 - 12x + 32 = 0$.
3. Factorise: $(x - 4)(x - 8) = 0$, so $x = 4$ or $x = 8$. The matching values of y are 8 and 4.

4. Check: $4 \times 8 = 32$ and $4 + 8 = 12$.

Answer: $(x, y) = (4, 8)$ or $(8, 4)$

Q32. Solve $\log_x y = 2$ and $x + y = 6$.

Solution:

1. Condition: $x > 0$, $x \neq 1$ and $y > 0$.
2. From $\log_x y = 2$ we get $y = x^2$.
3. Substitute in $x + y = 6$: $x + x^2 = 6$, so $x^2 + x - 6 = 0$, i.e. $(x + 3)(x - 2) = 0$.
4. $x = -3$ is rejected because a base must be positive. So $x = 2$ and $y = 4$.
5. Check: $\log_2 4 = 2$ and $2 + 4 = 6$.

Answer: $x = 2$, $y = 4$

Part 7: Word Problems from Engineering and Science

Q33. The sound level in decibels is $L = 10 \log (I / I_0)$. Sound A has 1000 times the intensity of sound B. Find the difference between their levels.

Solution:

1. $L_A - L_B = 10 \log (I_A / I_0) - 10 \log (I_B / I_0) = 10 \log (I_A / I_B)$.
2. Since $I_A / I_B = 1000$, the difference $= 10 \log 1000 = 10 \times 3 = 30$ dB.

Answer: 30 dB

Q34. The Richter magnitude is $M = \log (A / A_0)$, where A is the amplitude of the earthquake wave. How many times is the amplitude of a magnitude 7 earthquake greater than that of a magnitude 5 earthquake?

Solution:

1. $M_1 - M_2 = \log (A_1 / A_0) - \log (A_2 / A_0) = \log (A_1 / A_2)$.
2. So $\log (A_1 / A_2) = 7 - 5 = 2$.
3. Exponential form: $A_1 / A_2 = 10^2 = 100$.

Answer: 100 times

Q35. A population grows as $P = P_0 e^{0.02t}$, where t is in years. Find the time taken to double. (Given $\ln 2 = 0.6931$.)

Solution:

1. Doubling means $P = 2P_0$, so $e^{0.02t} = 2$.
2. Take natural log: $0.02t = \ln 2 = 0.6931$.
3. $t = 0.6931 / 0.02 = 34.66$.

Answer: About 34.7 years

Part 8: Multiple Choice Questions (Test Yourself)

Choose the correct option. The answer key is given at the end of this part.

MCQ 1. The value of $\log_3 81$ is

(a) 2 (b) 3 (c) 4 (d) 9

MCQ 2. $\log_{10} 0.001$ is equal to

(a) 3 (b) −3 (c) 1/3 (d) −1/3

MCQ 3. If $\log_x 16 = 4$, then x is

(a) 2 (b) 4 (c) 8 (d) 16

MCQ 4. $\log_a 1$ is equal to

(a) 1 (b) a (c) 0 (d) undefined

MCQ 5. $\log 2 + \log 5$ is equal to

(a) log 7 (b) 1 (c) 10 (d) log 3

MCQ 6. $\log_5 125 - \log_5 5$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4

MCQ 7. If $\log 2 = 0.3010$, then log 16 is

(a) 0.6020 (b) 0.9030 (c) 1.2040 (d) 1.5050

MCQ 8. The characteristic of log 0.0456 is

(a) 2 (b) −2 (c) 1 (d) −1

MCQ 9. If the characteristic of log N is 5, the number of digits in N is

(a) 4 (b) 5 (c) 6 (d) 7

MCQ 10. $\log_a b \times \log_b a$ is equal to

(a) 0 (b) 1 (c) a (d) b

MCQ 11. $\log_8 32$ is equal to

(a) 3/5 (b) 5/3 (c) 4 (d) 5/2

MCQ 12. If $\log_{10} x = -2$, then x is

(a) 100 (b) 0.01 (c) −100 (d) 0.001

MCQ 13. The value of $\ln e^3$ is

(a) e (b) 3e (c) 3 (d) 1/3

MCQ 14. log (A + B) is equal to

(a) log A + log B (b) log A × log B (c) log (AB) (d) none of these

MCQ 15. $\log_2 (1/8)$ is equal to

(a) −3 (b) 3 (c) −1/3 (d) 1/3

MCQ 16. If $\log_3 x = 4$, then x is

(a) 12 (b) 64 (c) 81 (d) 7

MCQ 17. The value of $\log_2 3 \times \log_3 4$ is

(a) 1 (b) 2 (c) 3 (d) 4

MCQ 18. If $\log 3 = 0.4771$, then log 9 is

(a) 0.4771 (b) 0.9542 (c) 1.4313 (d) 0.2386

MCQ 19. If $\log_a x = 2$ and $\log_a y = 3$, then $\log_a (xy)$ is

(a) 5 (b) 6 (c) 1 (d) −1

MCQ 20. The value of $10^{\log 5}$ is

(a) 5 (b) 10 (c) 50 (d) 1/5

MCQ 21. $\log_4 2$ is equal to

(a) 2 (b) 1/2 (c) 4 (d) 1/4

MCQ 22. The solution of $\log_2 (x + 1) = 3$ is

(a) 5 (b) 6 (c) 7 (d) 8

MCQ 23. If log x + log 4 = log 20, then x is

(a) 5 (b) 16 (c) 80 (d) 24

MCQ 24. 2 log 5 + log 4 is equal to

(a) log 14 (b) log 20 (c) log 100 (d) log 40

MCQ 25. The base of the natural logarithm is

(a) 10 (b) 2 (c) e ≈ 2.718 (d) π

Answer Key

1 − (c) 2 − (b) 3 − (a) 4 − (c) 5 − (b)

6 − (b) 7 − (c) 8 − (b) 9 − (c) 10 − (b)

11 − (b) 12 − (b) 13 − (c) 14 − (d) 15 − (a)

16 − (c) 17 − (b) 18 − (b) 19 − (a) 20 − (a)

21 − (b) 22 − (c) 23 − (a) 24 − (c) 25 − (c)