

MATHEMATICS 1

Chapter: Quadratic Equation

Important Short and Broad Questions with Solutions

Section A: Important Formulas (Quick Revision)

- **Standard form:** $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$. It has exactly two roots (real or complex).
- **Quadratic formula:** $x = [-b \pm \sqrt{b^2 - 4ac}] / 2a$.
- **Discriminant:** $D = b^2 - 4ac$. $D > 0$: roots real and distinct. $D = 0$: roots real and equal, each $= -b/2a$. $D < 0$: roots complex (non-real) and conjugate.
- **Rational / irrational roots** (a, b, c rational): D a perfect square gives rational roots; D positive but not a perfect square gives irrational conjugate roots.
- **Sum and product of roots:** $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$.
- **Equation from given roots:** $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.
- **Useful identities:** $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$; $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$; $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$; $1/\alpha + 1/\beta = (\alpha + \beta)/\alpha\beta$; $\alpha/\beta + \beta/\alpha = (\alpha^2 + \beta^2)/\alpha\beta$.
- **Special conditions:** one root zero $\Rightarrow c = 0$; roots equal in magnitude but opposite in sign $\Rightarrow b = 0$; roots reciprocal of each other $\Rightarrow a = c$; one root twice the other $\Rightarrow 2b^2 = 9ac$; roots equal $\Rightarrow D = 0$.
- **Common root** of $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$: $(c_1a_2 - c_2a_1)^2 = (a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1)$. Both roots common $\Rightarrow a_1/a_2 = b_1/b_2 = c_1/c_2$.
- **Conjugate roots:** if the coefficients are real, complex roots occur in conjugate pairs ($p + iq$ and $p - iq$). If the coefficients are rational, irrational roots occur in conjugate pairs ($p + \sqrt{q}$ and $p - \sqrt{q}$).
- **Sign of the expression:** if $a > 0$ and $D < 0$, then $ax^2 + bx + c > 0$ for all real x . If $a < 0$ and $D < 0$, it is negative for all real x .
- **Equations reducible to quadratic form:** put $y = x^2$ for equations in x^4 and x^2 ; multiply by x for equations containing $x + 1/x$; use a substitution such as $y = x^2 + x$ for repeated expressions. For radical equations, square and always check the answers.

Section B: Short Answer Questions (2 Marks)

Part 1: Solving Quadratic Equations

Q1. Define a quadratic equation and write its standard form.

Solution: An equation in one variable whose highest power is 2 is called a quadratic equation. Its standard form is $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.

Ans. $ax^2 + bx + c = 0, a \neq 0$.

Q2. Solve $x^2 - 5x + 6 = 0$ by factorisation.

Solution:

1. Split the middle term: $x^2 - 2x - 3x + 6 = 0$
2. $x(x - 2) - 3(x - 2) = 0$, so $(x - 2)(x - 3) = 0$

Ans. $x = 2$ or $x = 3$.

Q3. Solve $2x^2 + x - 6 = 0$ by factorisation.

Solution:

1. Product = $2 \times (-6) = -12$ and sum = 1, so split as +4 and -3: $2x^2 + 4x - 3x - 6 = 0$
2. $2x(x + 2) - 3(x + 2) = 0$, so $(x + 2)(2x - 3) = 0$

Ans. $x = -2$ or $x = 3/2$.

Q4. Solve $x^2 + 4x - 1 = 0$ using the quadratic formula.

Solution:

1. Here $a = 1$, $b = 4$, $c = -1$. $D = 16 + 4 = 20$.
2. $x = (-4 \pm \sqrt{20})/2 = (-4 \pm 2\sqrt{5})/2$

Ans. $x = -2 + \sqrt{5}$ or $x = -2 - \sqrt{5}$.

Q5. Solve $x^2 + 6x + 4 = 0$ by completing the square.

Solution:

1. $x^2 + 6x = -4$. Add $(6/2)^2 = 9$ to both sides: $x^2 + 6x + 9 = 5$
2. $(x + 3)^2 = 5$, so $x + 3 = \pm\sqrt{5}$

Ans. $x = -3 + \sqrt{5}$ or $x = -3 - \sqrt{5}$.

Q6. Solve $x^4 - 5x^2 + 4 = 0$.

Solution:

1. Put $y = x^2$: $y^2 - 5y + 4 = 0$, so $(y - 1)(y - 4) = 0$, so $y = 1$ or $y = 4$.
2. $x^2 = 1$ gives $x = \pm 1$; $x^2 = 4$ gives $x = \pm 2$.

Ans. $x = \pm 1, \pm 2$.

Q7. Solve $x - 3/x = 2$.

Solution:

1. Multiply by x : $x^2 - 3 = 2x$, so $x^2 - 2x - 3 = 0$.
2. $(x - 3)(x + 1) = 0$. Check: $x = 3$ gives $3 - 1 = 2$ ✓; $x = -1$ gives $-1 + 3 = 2$ ✓

Ans. $x = 3$ or $x = -1$.

Q8. Solve $x = \sqrt{x + 2}$.

Solution:

1. Square both sides: $x^2 = x + 2$, so $x^2 - x - 2 = 0$, so $(x - 2)(x + 1) = 0$.
2. Check: $x = 2$ gives $\sqrt{4} = 2$ ✓. $x = -1$ gives $\sqrt{1} = 1 \neq -1$, so it is rejected.

Ans. $x = 2$ only.

Q9. Solve $x^2 + 2x + 5 = 0$.

Solution:

1. $D = 4 - 20 = -16 < 0$, so the roots are complex.

2. $x = (-2 \pm \sqrt{-16})/2 = (-2 \pm 4i)/2$

Ans. $x = -1 + 2i$ or $x = -1 - 2i$.

Part 2: Discriminant and Nature of Roots

Q10. Write the quadratic formula and the discriminant. State the nature of roots for $D > 0$, $D = 0$ and $D < 0$.

Solution:

1. Roots: $x = [-b \pm \sqrt{(b^2 - 4ac)}]/2a$. Discriminant $D = b^2 - 4ac$.

2. $D > 0$: two real and distinct roots. $D = 0$: two real and equal roots. $D < 0$: two complex conjugate roots.

Ans. As stated above.

Q11. Find the discriminant and nature of roots of $3x^2 - 2x + 5 = 0$.

Solution: $D = (-2)^2 - 4(3)(5) = 4 - 60 = -56$

Ans. $D = -56 < 0$, so the roots are complex conjugates (not real).

Q12. Find the nature of the roots of $2x^2 - 7x + 3 = 0$.

Solution: $D = 49 - 24 = 25$, which is positive and a perfect square.

Ans. Roots are real, distinct and rational (they are 3 and 1/2).

Q13. Find k if the roots of $x^2 + kx + 9 = 0$ are equal.

Solution: Equal roots need $D = 0$: $k^2 - 4(1)(9) = 0$, so $k^2 = 36$.

Ans. $k = \pm 6$.

Q14. Find k if the roots of $2x^2 - 4x + k = 0$ are equal.

Solution: $D = 16 - 8k = 0$

Ans. $k = 2$.

Q15. Find the values of k for which $x^2 + 4x + k = 0$ has real roots.

Solution: Real roots need $D \geq 0$: $16 - 4k \geq 0$, so $k \leq 4$.

Ans. $k \leq 4$.

Q16. Show that $x^2 + x + 1$ is positive for every real value of x.

Solution:

1. Here $a = 1 > 0$ and $D = 1 - 4 = -3 < 0$.

2. $D < 0$ means the graph never touches the x-axis, and $a > 0$ means it opens upwards, so it lies entirely above the axis.

Ans. $x^2 + x + 1 > 0$ for all real x. Hence shown.

Part 3: Sum and Product of Roots; Forming Equations

Q17. Find the sum and product of the roots of $3x^2 - 7x + 2 = 0$.

Solution: Sum = $-b/a = 7/3$; Product = $c/a = 2/3$

Ans. Sum = $7/3$, product = $2/3$.

Q18. Form the quadratic equation whose roots are 3 and -5.

Solution:

1. Sum = $3 + (-5) = -2$; Product = $3 \times (-5) = -15$.

2. $x^2 - (\text{sum})x + \text{product} = 0$

Ans. $x^2 + 2x - 15 = 0$.

Q19. Form the quadratic equation whose roots are $2 + \sqrt{3}$ and $2 - \sqrt{3}$.

Solution: Sum = 4; Product = $(2)^2 - (\sqrt{3})^2 = 4 - 3 = 1$.

Ans. $x^2 - 4x + 1 = 0$.

Q20. One root of a quadratic equation with real coefficients is $3 + 2i$. Form the equation.

Solution:

1. Complex roots occur in conjugate pairs, so the other root is $3 - 2i$.

2. Sum = 6; Product = $3^2 + 2^2 = 13$.

Ans. $x^2 - 6x + 13 = 0$.

Q21. If α and β are the roots of $x^2 - 5x + 6 = 0$, find $\alpha^2 + \beta^2$.

Solution:

1. $\alpha + \beta = 5$ and $\alpha\beta = 6$.

2. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 25 - 12$

Ans. $\alpha^2 + \beta^2 = 13$.

Q22. If α and β are the roots of $x^2 - 3x + 1 = 0$, find $1/\alpha + 1/\beta$.

Solution:

1. $\alpha + \beta = 3$ and $\alpha\beta = 1$.

2. $1/\alpha + 1/\beta = (\alpha + \beta)/\alpha\beta = 3/1$

Ans. 3.

Q23. If α and β are the roots of $x^2 - 7x + 12 = 0$, find $(\alpha - \beta)^2$.

Solution: $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = 49 - 48$

Ans. $(\alpha - \beta)^2 = 1$.

Q24. If α and β are the roots of $x^2 - 4x + 2 = 0$, find $\alpha^3 + \beta^3$.

Solution:

1. $\alpha + \beta = 4$ and $\alpha\beta = 2$.

2. $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = 64 - 3(2)(4) = 64 - 24$

Ans. $\alpha^3 + \beta^3 = 40$.

Q25. Given that 2 is a root of $x^2 - 3x + k = 0$, find k and the other root.

Solution:

1. Substitute $x = 2$: $4 - 6 + k = 0$, so $k = 2$.
2. The equation is $x^2 - 3x + 2 = 0$. Sum of roots = 3, so the other root = $3 - 2 = 1$.

Ans. $k = 2$ and the other root is 1.

Q26. Find k if the roots of $3x^2 - 10x + k = 0$ are reciprocals of each other. Also find the roots.

Solution:

1. Product of roots = $k/3 = 1$, so $k = 3$.
2. The equation is $3x^2 - 10x + 3 = 0$, so $(3x - 1)(x - 3) = 0$.

Ans. $k = 3$; the roots are 3 and $1/3$.

Q27. Find k if the roots of $3x^2 + (k - 1)x - 7 = 0$ are equal in magnitude but opposite in sign.

Solution: Roots α and $-\alpha$ give sum = 0, so $-(k - 1)/3 = 0$.

Ans. $k = 1$.

Part 4: Simple Word Problems

Q28. The product of two consecutive positive integers is 56. Find them.

Solution:

1. Let the numbers be x and $x + 1$. Then $x(x + 1) = 56$, so $x^2 + x - 56 = 0$.
2. $(x + 8)(x - 7) = 0$. Positive solution: $x = 7$.

Ans. The numbers are 7 and 8.

Q29. Find two numbers whose sum is 10 and product is 21.

Solution:

1. The numbers are the roots of $t^2 - (\text{sum})t + \text{product} = 0$, i.e. $t^2 - 10t + 21 = 0$.
2. $(t - 3)(t - 7) = 0$

Ans. The numbers are 3 and 7.

Q30. The length of a rectangle exceeds its breadth by 3 m and its area is 40 m^2 . Find its dimensions.

Solution:

1. Let the breadth = x m. Then $x(x + 3) = 40$, so $x^2 + 3x - 40 = 0$.
2. $(x + 8)(x - 5) = 0$. Length cannot be negative, so $x = 5$.

Ans. Breadth = 5 m and length = 8 m.

Section C: Broad Answer Questions (6 Marks)

Q31. Derive the quadratic formula for $ax^2 + bx + c = 0$ by completing the square. Hence show that the sum of the roots is $-b/a$ and the product is c/a .

Solution:

1. Start with $ax^2 + bx + c = 0$ ($a \neq 0$). Divide by a : $x^2 + (b/a)x + c/a = 0$.
2. Shift the constant: $x^2 + (b/a)x = -c/a$.

3. Add $(b/2a)^2$ to both sides: $x^2 + (b/a)x + b^2/4a^2 = b^2/4a^2 - c/a$.
4. Left side is a perfect square; simplify the right side: $(x + b/2a)^2 = (b^2 - 4ac)/4a^2$.
5. Take the square root: $x + b/2a = \pm \sqrt{(b^2 - 4ac)} / 2a$.
6. Therefore $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$.
7. Let $\alpha = [-b + \sqrt{D}]/2a$ and $\beta = [-b - \sqrt{D}]/2a$, where $D = b^2 - 4ac$.
8. Sum: $\alpha + \beta = -2b/2a = -b/a$.
9. Product: $\alpha\beta = (b^2 - D)/4a^2 = (b^2 - b^2 + 4ac)/4a^2 = c/a$.

Ans. $x = [-b \pm \sqrt{(b^2 - 4ac)}]/2a$; $\alpha + \beta = -b/a$; $\alpha\beta = c/a$.

Q32. Solve $2x^2 + 3x + 5 = 0$. Verify that the sum and product of the roots agree with the formulas $-b/a$ and c/a .

Solution:

1. Here $a = 2$, $b = 3$, $c = 5$.
2. $D = b^2 - 4ac = 9 - 40 = -31 < 0$, so the roots are complex conjugates.
3. $x = [-3 \pm \sqrt{-31}] / 4 = (-3 \pm i\sqrt{31})/4$.
4. Roots: $\alpha = (-3 + i\sqrt{31})/4$ and $\beta = (-3 - i\sqrt{31})/4$.
5. Sum: $\alpha + \beta = -6/4 = -3/2$, and $-b/a = -3/2$ ✓
6. Product: $\alpha\beta = [(-3)^2 - (i\sqrt{31})^2]/16 = (9 + 31)/16 = 40/16 = 5/2$, and $c/a = 5/2$ ✓

Ans. $x = (-3 \pm i\sqrt{31})/4$; sum = $-3/2$ and product = $5/2$ verified.

Q33. Find the value of k for which the equation $(k + 1)x^2 + 2(k + 3)x + (k + 8) = 0$ has equal roots. Find the equal root.

Solution:

1. Here $a = k + 1$, $b = 2(k + 3)$, $c = k + 8$.
2. For equal roots, $D = 0$: $[2(k + 3)]^2 - 4(k + 1)(k + 8) = 0$.
3. Divide by 4: $(k + 3)^2 - (k + 1)(k + 8) = 0$.
4. Expand: $k^2 + 6k + 9 - (k^2 + 9k + 8) = 0$.
5. Simplify: $-3k + 1 = 0$, so $k = 1/3$.
6. Substitute $k = 1/3$: $(4/3)x^2 + (20/3)x + 25/3 = 0$. Multiply by 3: $4x^2 + 20x + 25 = 0$.
7. $(2x + 5)^2 = 0$, so $x = -5/2$ (repeated).

Ans. $k = 1/3$, and the equal root is $x = -5/2$.

Q34. For what values of k are the roots of $(k - 2)x^2 + 2(2k - 3)x + (5k - 6) = 0$ (i) real and equal, (ii) real and distinct, (iii) imaginary?

Solution:

1. $a = k - 2$, $b = 2(2k - 3)$, $c = 5k - 6$, with $k \neq 2$ (so that the equation is quadratic).
2. $D = 4(2k - 3)^2 - 4(k - 2)(5k - 6) = 4[(4k^2 - 12k + 9) - (5k^2 - 16k + 12)]$.
3. $D = 4[-k^2 + 4k - 3] = -4(k^2 - 4k + 3) = -4(k - 1)(k - 3)$.

4. **(i) Real and equal:** $D = 0$, so $(k - 1)(k - 3) = 0$, so $k = 1$ or $k = 3$.
5. **(ii) Real and distinct:** $D > 0$, so $(k - 1)(k - 3) < 0$, so $1 < k < 3$. Excluding $k = 2$, we get $1 < k < 3, k \neq 2$.
6. **(iii) Imaginary:** $D < 0$, so $(k - 1)(k - 3) > 0$, so $k < 1$ or $k > 3$.
7. Check for $k = 1$: $-x^2 - 2x - 1 = -(x + 1)^2$, which has equal roots ✓

Ans. (i) $k = 1$ or 3 ; (ii) $1 < k < 3, k \neq 2$; (iii) $k < 1$ or $k > 3$.

Q35. Prove that the roots of $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$ are always real for real a, b, c .

Solution:

1. Expand each product and add: $3x^2 - 2(a + b + c)x + (ab + bc + ca) = 0$.
2. Here $A = 3, B = -2(a + b + c), C = ab + bc + ca$.
3. $D = B^2 - 4AC = 4(a + b + c)^2 - 12(ab + bc + ca)$.
4. Expand $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$. So $D = 4[a^2 + b^2 + c^2 - ab - bc - ca]$.
5. Use the identity $2(a^2 + b^2 + c^2 - ab - bc - ca) = (a - b)^2 + (b - c)^2 + (c - a)^2$.
6. So $D = 2[(a - b)^2 + (b - c)^2 + (c - a)^2]$.
7. Squares of real numbers are never negative, so $D \geq 0$.

Ans. $D \geq 0$, so the roots are always real. Hence proved.

Q36. If α and β are the roots of $2x^2 - 5x + 1 = 0$, find (i) $\alpha^2 + \beta^2$, (ii) $\alpha^3 + \beta^3$, (iii) $(\alpha - \beta)^2$, (iv) $\alpha/\beta + \beta/\alpha$.

Solution:

1. $\alpha + \beta = -b/a = 5/2$ and $\alpha\beta = c/a = 1/2$.
2. **(i)** $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 25/4 - 1 = 21/4$.
3. **(ii)** $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = 125/8 - 3(1/2)(5/2) = 125/8 - 15/4 = 125/8 - 30/8 = 95/8$.
4. **(iii)** $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = 25/4 - 2 = 17/4$.
5. **(iv)** $\alpha/\beta + \beta/\alpha = (\alpha^2 + \beta^2)/\alpha\beta = (21/4) \div (1/2) = 21/2$.

Ans. (i) $21/4$ (ii) $95/8$ (iii) $17/4$ (iv) $21/2$.

Q37. If α and β are the roots of $x^2 - 5x + 3 = 0$, form the quadratic equation whose roots are (i) α^2 and β^2 , (ii) $1/\alpha$ and $1/\beta$, (iii) $\alpha + 2$ and $\beta + 2$.

Solution:

1. $\alpha + \beta = 5$ and $\alpha\beta = 3$.
2. **(i)** Sum $= \alpha^2 + \beta^2 = 25 - 6 = 19$. Product $= \alpha^2\beta^2 = (\alpha\beta)^2 = 9$. Equation: $x^2 - 19x + 9 = 0$.

3. (ii) Sum = $1/\alpha + 1/\beta = (\alpha + \beta)/\alpha\beta = 5/3$. Product = $1/(\alpha\beta) = 1/3$. Equation: $x^2 - (5/3)x + 1/3 = 0$, i.e. $3x^2 - 5x + 1 = 0$.
4. (iii) Sum = $(\alpha + 2) + (\beta + 2) = 5 + 4 = 9$. Product = $\alpha\beta + 2(\alpha + \beta) + 4 = 3 + 10 + 4 = 17$. Equation: $x^2 - 9x + 17 = 0$.

Ans. (i) $x^2 - 19x + 9 = 0$ (ii) $3x^2 - 5x + 1 = 0$ (iii) $x^2 - 9x + 17 = 0$.

Q38. If α and β are the roots of $x^2 - px + q = 0$, form the equation whose roots are α/β and β/α .

Solution:

1. $\alpha + \beta = p$ and $\alpha\beta = q$.
2. Sum of new roots = $\alpha/\beta + \beta/\alpha = (\alpha^2 + \beta^2)/\alpha\beta = [(\alpha + \beta)^2 - 2\alpha\beta]/\alpha\beta = (p^2 - 2q)/q$.
3. Product of new roots = $(\alpha/\beta)(\beta/\alpha) = 1$.
4. Required equation: $x^2 - [(p^2 - 2q)/q]x + 1 = 0$.
5. Multiply by q : $qx^2 - (p^2 - 2q)x + q = 0$.

Ans. $qx^2 - (p^2 - 2q)x + q = 0$.

Q39. If one root of $x^2 + px + q = 0$ is twice the other, prove that $2p^2 = 9q$.

Solution:

1. Let the roots be α and 2α .
2. Sum of roots: $\alpha + 2\alpha = 3\alpha = -p$, so $\alpha = -p/3$.
3. Product of roots: $\alpha \times 2\alpha = 2\alpha^2 = q$.
4. Substitute $\alpha = -p/3$: $2(p^2/9) = q$.
5. So $2p^2/9 = q$, i.e. $2p^2 = 9q$.
6. Check with $x^2 - 3x + 2 = 0$ (roots 1 and 2): $2(9) = 18 = 9(2) \checkmark$

Ans. $2p^2 = 9q$. Hence proved.

Q40. If the equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ ($a \neq b$) have a common root, prove that $a + b + 1 = 0$.

Solution:

1. Let the common root be x . Then $x^2 + ax + b = 0 \dots$ (i) and $x^2 + bx + a = 0 \dots$ (ii)
2. Subtract (ii) from (i): $(a - b)x + (b - a) = 0$.
3. Take $(a - b)$ common: $(a - b)(x - 1) = 0$.
4. Since $a \neq b$, $x - 1 = 0$, so $x = 1$. The common root must be 1.
5. Substitute $x = 1$ in (i): $1 + a + b = 0$.
6. Check with $a = 2, b = -3$: $x^2 + 2x - 3 = 0$ has roots 1, -3; $x^2 - 3x + 2 = 0$ has roots 1, 2. The common root is 1 $\checkmark$

Ans. $a + b + 1 = 0$. Hence proved.

Q41. If one root of $ax^2 + bx + c = 0$ is the square of the other, prove that $b^3 + a^2c + ac^2 = 3abc$.

Solution:

1. Let the roots be α and α^2 .
2. Sum: $\alpha + \alpha^2 = -b/a \dots$ (i)
3. Product: $\alpha \cdot \alpha^2 = \alpha^3 = c/a \dots$ (ii)
4. Cube (i): $(\alpha + \alpha^2)^3 = \alpha^3 + 3\alpha^4 + 3\alpha^5 + \alpha^6 = \alpha^3 + \alpha^6 + 3\alpha^3(\alpha + \alpha^2) = -b^3/a^3$.
5. Substitute (i) and (ii): $c/a + c^2/a^2 + 3(c/a)(-b/a) = -b^3/a^3$.
6. Multiply by a^3 : $a^2c + ac^2 - 3abc = -b^3$.
7. Rearrange: $b^3 + a^2c + ac^2 = 3abc$.

Ans. $b^3 + a^2c + ac^2 = 3abc$. Hence proved.

Q42. Solve $(x^2 + x)^2 - 8(x^2 + x) + 12 = 0$.

Solution:

1. The expression $x^2 + x$ repeats, so put $y = x^2 + x$.
2. The equation becomes $y^2 - 8y + 12 = 0$, so $(y - 2)(y - 6) = 0$, so $y = 2$ or $y = 6$.
3. Case $y = 2$: $x^2 + x - 2 = 0$, so $(x + 2)(x - 1) = 0$, so $x = -2$ or $x = 1$.
4. Case $y = 6$: $x^2 + x - 6 = 0$, so $(x + 3)(x - 2) = 0$, so $x = -3$ or $x = 2$.
5. Check $x = 2$: $(4 + 2)^2 - 8(6) + 12 = 36 - 48 + 12 = 0 \checkmark$

Ans. $x = 1, -2, 2, -3$.

Q43. Solve $\sqrt{3x + 1} - \sqrt{x - 1} = 2$.

Solution:

1. Isolate one radical: $\sqrt{3x + 1} = 2 + \sqrt{x - 1}$.
2. Square both sides: $3x + 1 = 4 + 4\sqrt{x - 1} + (x - 1) = x + 3 + 4\sqrt{x - 1}$.
3. Simplify: $2x - 2 = 4\sqrt{x - 1}$, so $x - 1 = 2\sqrt{x - 1}$ (dividing by 2).
4. Put $t = \sqrt{x - 1}$: $t^2 = 2t$, so $t(t - 2) = 0$, so $t = 0$ or $t = 2$.
5. $t = 0$ gives $x = 1$; $t = 2$ gives $x - 1 = 4$, so $x = 5$.
6. Check $x = 1$: $\sqrt{4} - \sqrt{0} = 2 \checkmark$. Check $x = 5$: $\sqrt{16} - \sqrt{4} = 4 - 2 = 2 \checkmark$

Ans. $x = 1$ or $x = 5$.

Q44. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Solution:

1. Let the speed = x km/h. Time taken = $360/x$ hours.
2. With speed $(x + 5)$ km/h, time = $360/(x + 5)$ hours.
3. Given: $360/x - 360/(x + 5) = 1$.
4. Take common denominator: $360[(x + 5) - x] / [x(x + 5)] = 1$, so $1800 = x(x + 5)$.

5. $x^2 + 5x - 1800 = 0$.
6. $D = 25 + 7200 = 7225 = 85^2$, so $x = (-5 \pm 85)/2 = 40$ or -45 .
7. Speed cannot be negative, so $x = 40$.
8. Check: $360/40 = 9$ hours and $360/45 = 8$ hours, a difference of 1 hour ✓

Ans. Speed of the train = 40 km/h.

Q45. The hypotenuse of a right-angled triangle is 25 cm. One side is 5 cm longer than the other. Find the sides.

Solution:

1. Let the shorter side = x cm. Then the other side = $(x + 5)$ cm.
1. By Pythagoras' theorem: $x^2 + (x + 5)^2 = 25^2$.
2. $x^2 + x^2 + 10x + 25 = 625$, so $2x^2 + 10x - 600 = 0$, so $x^2 + 5x - 300 = 0$.
3. $D = 25 + 1200 = 1225 = 35^2$, so $x = (-5 \pm 35)/2 = 15$ or -20 .
4. Length cannot be negative, so $x = 15$ and the other side = 20.
5. Check: $15^2 + 20^2 = 225 + 400 = 625 = 25^2$ ✓

Ans. The sides are 15 cm and 20 cm.