

MATHEMATICS 1

Chapter: Quadratic Equation

Short Answer Questions (2 Marks) with Solutions: Set 2

Part 1: Solving Quadratic Equations

Q1. Solve $x^2 - 7x + 12 = 0$.

Solution: Split the middle term: $x^2 - 3x - 4x + 12 = 0$, so $x(x - 3) - 4(x - 3) = 0$, so $(x - 3)(x - 4) = 0$.

Ans. $x = 3$ or $x = 4$.

Q2. Solve $6x^2 - x - 2 = 0$.

Solution:

1. Product = $6 \times (-2) = -12$ and sum = -1 , so split as $+3$ and -4 : $6x^2 + 3x - 4x - 2 = 0$.

2. $3x(2x + 1) - 2(2x + 1) = 0$, so $(2x + 1)(3x - 2) = 0$.

Ans. $x = -1/2$ or $x = 2/3$.

Q3. Solve $3x^2 - 5x - 2 = 0$.

Solution:

1. Split $-5x$ as $-6x + x$: $3x^2 - 6x + x - 2 = 0$.

2. $3x(x - 2) + 1(x - 2) = 0$, so $(x - 2)(3x + 1) = 0$.

Ans. $x = 2$ or $x = -1/3$.

Q4. Solve $2x^2 = 3x$.

Solution:

1. Bring all terms to one side: $2x^2 - 3x = 0$, so $x(2x - 3) = 0$.

2. Note: do not divide by x , as that would lose the root $x = 0$.

Ans. $x = 0$ or $x = 3/2$.

Q5. Solve $4x^2 - 12x + 9 = 0$.

Solution: The left side is a perfect square: $(2x - 3)^2 = 0$.

Ans. $x = 3/2$ (repeated root).

Q6. Solve $x^2 - 8x + 5 = 0$ by completing the square.

Solution:

1. $x^2 - 8x = -5$. Add $(8/2)^2 = 16$ to both sides: $(x - 4)^2 = 11$.

2. $x - 4 = \pm\sqrt{11}$

Ans. $x = 4 + \sqrt{11}$ or $x = 4 - \sqrt{11}$.

Q7. Solve $2x^2 - 3x - 4 = 0$ using the quadratic formula.

Solution:

1. $a = 2, b = -3, c = -4. D = 9 + 32 = 41.$

2. $x = (3 \pm \sqrt{41})/4$

Ans. $x = (3 + \sqrt{41})/4$ or $x = (3 - \sqrt{41})/4.$

Q8. Solve $x^2 + 9 = 0$.

Solution: $x^2 = -9$, so $x = \pm\sqrt{-9}$

Ans. $x = 3i$ or $x = -3i.$

Q9. Solve $x^2 - 2x + 5 = 0$.

Solution:

1. $D = 4 - 20 = -16 < 0$, so the roots are complex.

2. $x = (2 \pm 4i)/2$

Ans. $x = 1 + 2i$ or $x = 1 - 2i.$

Q10. Solve $x + 1/x = 5/2$.

Solution:

1. Multiply by $2x$: $2x^2 + 2 = 5x$, so $2x^2 - 5x + 2 = 0.$

2. $(2x - 1)(x - 2) = 0$

Ans. $x = 1/2$ or $x = 2.$

Q11. Solve $x^4 - 13x^2 + 36 = 0$.

Solution:

1. Put $y = x^2$: $y^2 - 13y + 36 = 0$, so $(y - 4)(y - 9) = 0$, so $y = 4$ or $y = 9.$

2. $x^2 = 4$ gives $x = \pm 2$; $x^2 = 9$ gives $x = \pm 3.$

Ans. $x = \pm 2, \pm 3.$

Q12. Solve $2^{2x} - 5 \cdot 2^x + 4 = 0$.

Solution:

1. Put $y = 2^x$: $y^2 - 5y + 4 = 0$, so $(y - 1)(y - 4) = 0$, so $y = 1$ or $y = 4.$

2. $2^x = 1$ gives $x = 0$; $2^x = 4$ gives $x = 2.$

Ans. $x = 0$ or $x = 2.$

Q13. Solve $\sqrt{x + 7} = x - 5$.

Solution:

1. Square both sides: $x + 7 = x^2 - 10x + 25$, so $x^2 - 11x + 18 = 0$, so $(x - 2)(x - 9) = 0.$

2. Check: $x = 9$ gives $\sqrt{16} = 4$ and $9 - 5 = 4 \checkmark$. $x = 2$ gives $\sqrt{9} = 3$ but $2 - 5 = -3$, so it is rejected.

Ans. $x = 9$ only.

Q14. Solve $1/x + 1/(x + 1) = 5/6$.

Solution:

1. Combine the left side: $(2x + 1)/[x(x + 1)] = 5/6$, so $6(2x + 1) = 5x(x + 1)$.
2. $12x + 6 = 5x^2 + 5x$, so $5x^2 - 7x - 6 = 0$, so $(x - 2)(5x + 3) = 0$.
3. Check $x = 2$: $1/2 + 1/3 = 5/6$ ✓

Ans. $x = 2$ or $x = -3/5$.

Part 2: Discriminant and Nature of Roots

Q15. Find the nature of the roots of $x^2 - 6x + 9 = 0$.

Solution: $D = 36 - 36 = 0$

Ans. Roots are real and equal ($x = 3, 3$).

Q16. Find the nature of the roots of $2x^2 + 5x - 3 = 0$.

Solution: $D = 25 + 24 = 49$, which is positive and a perfect square.

Ans. Roots are real, distinct and rational (they are $1/2$ and -3).

Q17. Find the nature of the roots of $x^2 + 2x + 3 = 0$.

Solution: $D = 4 - 12 = -8 < 0$

Ans. Roots are complex conjugates (not real).

Q18. Find the nature of the roots of $x^2 - 4x + 1 = 0$.

Solution: $D = 16 - 4 = 12$, which is positive but not a perfect square.

Ans. Roots are real, distinct and irrational ($2 \pm \sqrt{3}$).

Q19. Find k if the roots of $kx^2 + 4x + 1 = 0$ are equal.

Solution: $D = 16 - 4k = 0$

Ans. $k = 4$.

Q20. Find the values of k for which $x^2 - kx + 4 = 0$ has real roots.

Solution: $D = k^2 - 16 \geq 0$, so $(k - 4)(k + 4) \geq 0$.

Ans. $k \leq -4$ or $k \geq 4$.

Q21. Find the values of k for which $3x^2 + kx + 3 = 0$ has no real roots.

Solution: No real roots need $D < 0$: $k^2 - 36 < 0$, so $(k - 6)(k + 6) < 0$.

Ans. $-6 < k < 6$.

Q22. If one root of $x^2 - 4x + (k - 2) = 0$ is zero, find k and the other root.

Solution:

1. Product of roots = $k - 2$. One root is 0, so the product is 0: $k - 2 = 0$, so $k = 2$.
2. The equation becomes $x^2 - 4x = 0$, so $x(x - 4) = 0$.

Ans. $k = 2$ and the other root is 4.

Q23. Show that $x^2 - 5x + 7$ is positive for all real values of x .

Solution:

1. Here $a = 1 > 0$ and $D = 25 - 28 = -3 < 0$.
2. Since $D < 0$ the graph never meets the x -axis, and since $a > 0$ it opens upwards, so the expression is always positive.

Ans. $x^2 - 5x + 7 > 0$ for all real x . Hence shown.

Part 3: Sum and Product of Roots

Q24. Find the sum and product of the roots of $2x^2 + 5x - 12 = 0$.

Solution: Sum $= -b/a = -5/2$; Product $= c/a = -12/2 = -6$

Ans. Sum $= -5/2$, product $= -6$.

Q25. Find the sum and product of the roots of $x^2 - 7 = 0$.

Solution: Here $b = 0$, so the sum $= 0$. Product $= c/a = -7$. (The roots are $+\sqrt{7}$ and $-\sqrt{7}$.)

Ans. Sum $= 0$, product $= -7$.

Q26. Form the quadratic equation whose roots are $1/2$ and -3 .

Solution:

1. Sum $= 1/2 - 3 = -5/2$; Product $= (1/2)(-3) = -3/2$.
2. $x^2 + (5/2)x - 3/2 = 0$. Multiply by 2 to clear fractions.

Ans. $2x^2 + 5x - 3 = 0$.

Q27. Form the quadratic equation whose roots are $1 + \sqrt{2}$ and $1 - \sqrt{2}$.

Solution: Sum $= 2$; Product $= 1 - 2 = -1$.

Ans. $x^2 - 2x - 1 = 0$.

Q28. Form the quadratic equation whose roots are $2 + i$ and $2 - i$.

Solution: Sum $= 4$; Product $= 2^2 + 1^2 = 5$.

Ans. $x^2 - 4x + 5 = 0$.

Q29. One root of a quadratic equation with rational coefficients is $2 + \sqrt{5}$. Find the equation.

Solution:

1. Irrational roots occur in conjugate pairs, so the other root is $2 - \sqrt{5}$.
2. Sum $= 4$; Product $= 4 - 5 = -1$.

Ans. $x^2 - 4x - 1 = 0$.

Q30. If α and β are the roots of $2x^2 - 3x - 2 = 0$, find $\alpha^2 + \beta^2$.

Solution:

1. $\alpha + \beta = 3/2$ and $\alpha\beta = -1$.
2. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 9/4 + 2$

Ans. $\alpha^2 + \beta^2 = 17/4$.

Q31. If α and β are the roots of $x^2 + 2x - 3 = 0$, find $|\alpha - \beta|$.

Solution:

1. $\alpha + \beta = -2$ and $\alpha\beta = -3$.
2. $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = 4 + 12 = 16$

Ans. $|\alpha - \beta| = 4$.

Q32. If α and β are the roots of $x^2 - 4x + 2 = 0$, find $\alpha/\beta + \beta/\alpha$.

Solution:

1. $\alpha + \beta = 4$ and $\alpha\beta = 2$, so $\alpha^2 + \beta^2 = 16 - 4 = 12$.
2. $\alpha/\beta + \beta/\alpha = (\alpha^2 + \beta^2)/\alpha\beta = 12/2$

Ans. 6.

Q33. If α and β are the roots of $x^2 - 3x + 1 = 0$, find $1/\alpha^2 + 1/\beta^2$.

Solution:

1. $\alpha + \beta = 3$ and $\alpha\beta = 1$, so $\alpha^2 + \beta^2 = 9 - 2 = 7$.
2. $1/\alpha^2 + 1/\beta^2 = (\alpha^2 + \beta^2)/(\alpha\beta)^2 = 7/1$

Ans. 7.

Q34. If α and β are the roots of $x^2 - 5x + 3 = 0$, find $\alpha^2\beta + \alpha\beta^2$.

Solution: $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta) = 3 \times 5$

Ans. 15.

Q35. Find k if the sum of the roots of $3x^2 + (2k + 1)x - (k + 5) = 0$ is equal to the product of the roots.

Solution:

1. Sum $= -(2k + 1)/3$ and product $= -(k + 5)/3$.
2. Equate: $2k + 1 = k + 5$

Ans. $k = 4$.

Q36. Find k if one root of $x^2 - 6x + k = 0$ is twice the other.

Solution:

1. Let the roots be α and 2α . Sum: $3\alpha = 6$, so $\alpha = 2$.
2. Product: $k = \alpha \times 2\alpha = 2 \times 4 = 8$. (Roots 2 and 4 ✓)

Ans. $k = 8$.

Part 4: Simple Word Problems

Q37. Find two numbers whose sum is 13 and product is 36.

Solution: The numbers are the roots of $t^2 - 13t + 36 = 0$, so $(t - 4)(t - 9) = 0$.

Ans. The numbers are 4 and 9.

Q38. A number added to its reciprocal gives $10/3$. Find the number.

Solution:

1. $x + 1/x = 10/3$, so $3x^2 + 3 = 10x$, so $3x^2 - 10x + 3 = 0$.
2. $(3x - 1)(x - 3) = 0$

Ans. The number is 3 (or $1/3$).

Q39. The sum of a positive number and its square is 42. Find the number.

Solution:

1. $x^2 + x = 42$, so $x^2 + x - 42 = 0$, so $(x + 7)(x - 6) = 0$.
2. The positive solution is $x = 6$.

Ans. The number is 6.

Q40. The length of a rectangle is twice its breadth and its area is 72 m^2 . Find its dimensions.

Solution: Let the breadth = $x \text{ m}$, so the length = $2x \text{ m}$. Area: $x(2x) = 72$, so $x^2 = 36$, so $x = 6$ (positive).

Ans. Breadth = 6 m and length = 12 m.