

MATHEMATICS 1

Chapter: Quadratic Equation

Short Answer Questions (2 Marks) with Solutions: Set 3

Part 1: Solving Quadratic Equations

Q1. Solve $x^2 - 3x - 10 = 0$.

Solution: Split the middle term: $x^2 - 5x + 2x - 10 = 0$, so $x(x - 5) + 2(x - 5) = 0$, so $(x - 5)(x + 2) = 0$.

Ans. $x = 5$ or $x = -2$.

Q2. Solve $5x^2 - 6x + 1 = 0$.

Solution: Split $-6x$ as $-5x - x$: $5x^2 - 5x - x + 1 = 0$, so $5x(x - 1) - 1(x - 1) = 0$, so $(x - 1)(5x - 1) = 0$.

Ans. $x = 1$ or $x = 1/5$.

Q3. Solve $12x^2 - 7x - 10 = 0$.

Solution:

1. Product = $12 \times (-10) = -120$ and sum = -7 , so split as -15 and $+8$: $12x^2 - 15x + 8x - 10 = 0$.

2. $3x(4x - 5) + 2(4x - 5) = 0$, so $(4x - 5)(3x + 2) = 0$.

Ans. $x = 5/4$ or $x = -2/3$.

Q4. Solve $x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$.

Solution:

1. The sum of the numbers needed is $(\sqrt{2} + 1)$ and their product is $\sqrt{2}$, so they are $\sqrt{2}$ and 1 .

2. $(x - \sqrt{2})(x - 1) = 0$

Ans. $x = \sqrt{2}$ or $x = 1$.

Q5. Solve $x^2 - 6x + 7 = 0$ using the quadratic formula.

Solution:

1. $a = 1$, $b = -6$, $c = 7$. $D = 36 - 28 = 8$.

2. $x = (6 \pm \sqrt{8})/2 = (6 \pm 2\sqrt{2})/2$

Ans. $x = 3 + \sqrt{2}$ or $x = 3 - \sqrt{2}$.

Q6. Solve $2x^2 - 8x + 3 = 0$ by completing the square.

Solution:

1. Divide by 2: $x^2 - 4x + 3/2 = 0$, so $x^2 - 4x = -3/2$.

2. Add $(4/2)^2 = 4$: $(x - 2)^2 = 4 - 3/2 = 5/2$.

3. $x - 2 = \pm\sqrt{(5/2)} = \pm\sqrt{10}/2$

Ans. $x = (4 + \sqrt{10})/2$ or $x = (4 - \sqrt{10})/2$.

Q7. Solve $x^2 + 4x + 13 = 0$.

Solution:

1. $D = 16 - 52 = -36 < 0$, so the roots are complex.
2. $x = (-4 \pm 6i)/2$

Ans. $x = -2 + 3i$ or $x = -2 - 3i$.

Q8. Solve $3x^2 + 2 = 0$.

Solution: $x^2 = -2/3$, so $x = \pm\sqrt{-2/3} = \pm i\sqrt{2/3} = \pm i\sqrt{6}/3$

Ans. $x = i\sqrt{6}/3$ or $x = -i\sqrt{6}/3$ (imaginary roots).

Q9. Solve $(x - 2)^2 = 9$.

Solution: Take the square root of both sides: $x - 2 = \pm 3$.

Ans. $x = 5$ or $x = -1$.

Q10. Solve $(2x - 1)(x + 3) = 4$.

Solution:

1. The right side is not zero, so first expand: $2x^2 + 6x - x - 3 = 4$, so $2x^2 + 5x - 7 = 0$.
2. $(2x + 7)(x - 1) = 0$

Ans. $x = 1$ or $x = -7/2$.

Q11. Solve $x(x + 7) = 30$.

Solution: $x^2 + 7x - 30 = 0$, so $(x + 10)(x - 3) = 0$.

Ans. $x = 3$ or $x = -10$.

Q12. Solve $x^4 - 17x^2 + 16 = 0$.

Solution:

1. Put $y = x^2$: $y^2 - 17y + 16 = 0$, so $(y - 1)(y - 16) = 0$, so $y = 1$ or $y = 16$.
2. $x^2 = 1$ gives $x = \pm 1$; $x^2 = 16$ gives $x = \pm 4$.

Ans. $x = \pm 1, \pm 4$.

Q13. Solve $9^x - 12 \cdot 3^x + 27 = 0$.

Solution:

1. Put $y = 3^x$: $y^2 - 12y + 27 = 0$, so $(y - 3)(y - 9) = 0$, so $y = 3$ or $y = 9$.
2. $3^x = 3$ gives $x = 1$; $3^x = 9$ gives $x = 2$.

Ans. $x = 1$ or $x = 2$.

Q14. Solve $\sqrt{x} = 6 - x$.

Solution:

1. Square both sides: $x = (6 - x)^2 = 36 - 12x + x^2$, so $x^2 - 13x + 36 = 0$, so $(x - 4)(x - 9) = 0$.
2. Check: $x = 4$ gives $\sqrt{4} = 2$ and $6 - 4 = 2$ ✓. $x = 9$ gives $\sqrt{9} = 3$ but $6 - 9 = -3$, so it is rejected.

Ans. $x = 4$ only.

Q15. Solve $x/(x + 1) + (x + 1)/x = 5/2$.

Solution:

1. Put $y = x/(x + 1)$. Then $y + 1/y = 5/2$, so $2y^2 - 5y + 2 = 0$, so $(2y - 1)(y - 2) = 0$, so $y = 2$ or $y = 1/2$.
2. $y = 2$: $x = 2x + 2$, so $x = -2$. $y = 1/2$: $2x = x + 1$, so $x = 1$.
3. Check $x = 1$: $1/2 + 2 = 5/2$ ✓

Ans. $x = 1$ or $x = -2$.

Part 2: Discriminant and Nature of Roots

Q16. Find the nature of the roots of $9x^2 - 12x + 4 = 0$.

Solution: $D = 144 - 144 = 0$

Ans. Roots are real and equal ($x = 2/3, 2/3$).

Q17. Find the nature of the roots of $4x^2 + 4x + 5 = 0$.

Solution: $D = 16 - 80 = -64 < 0$

Ans. Roots are complex conjugates (not real).

Q18. Find the nature of the roots of $x^2 + x - 1 = 0$.

Solution: $D = 1 + 4 = 5$, which is positive but not a perfect square.

Ans. Roots are real, distinct and irrational.

Q19. Find k if the roots of $(k + 1)x^2 - 2(k - 1)x + 1 = 0$ are equal.

Solution:

1. $D = 4(k - 1)^2 - 4(k + 1) = 0$, so $(k - 1)^2 = k + 1$.
2. $k^2 - 2k + 1 = k + 1$, so $k^2 - 3k = 0$, so $k(k - 3) = 0$.

Ans. $k = 0$ or $k = 3$.

Q20. Find k if the roots of $x^2 - 2(5 + 2k)x + 3(7 + 10k) = 0$ are equal.

Solution:

1. $D/4 = (5 + 2k)^2 - 3(7 + 10k) = 0$.
2. $25 + 20k + 4k^2 - 21 - 30k = 0$, so $4k^2 - 10k + 4 = 0$, so $2k^2 - 5k + 2 = 0$.
3. $(2k - 1)(k - 2) = 0$

Ans. $k = 1/2$ or $k = 2$.

Q21. Find k if the roots of $2x^2 + kx + 3 = 0$ are equal.

Solution: $D = k^2 - 24 = 0$, so $k^2 = 24$.

Ans. $k = \pm 2\sqrt{6}$.

Q22. Find the values of k for which $x^2 + kx + 1 = 0$ has real and distinct roots.

Solution: $D = k^2 - 4 > 0$, so $(k - 2)(k + 2) > 0$.

Ans. $k < -2$ or $k > 2$.

Q23. Find the values of k for which $x^2 + kx + 4$ is positive for all real x .

Solution: Since $a = 1 > 0$, we need $D < 0$: $k^2 - 16 < 0$, so $(k - 4)(k + 4) < 0$.

Ans. $-4 < k < 4$.

Q24. If a and c have opposite signs, prove that the roots of $ax^2 + bx + c = 0$ are real and of opposite signs.

Solution:

1. a and c have opposite signs, so $ac < 0$ and hence $-4ac > 0$.
2. $D = b^2 - 4ac > 0$, so the roots are real and distinct.
3. Product of roots $= c/a < 0$, so one root is positive and the other negative.

Ans. Hence proved.

Part 3: Sum and Product of Roots

Q25. Find the sum and product of the roots of $7x^2 - 4x - 3 = 0$. Also find the roots.

Solution:

1. Sum $= -b/a = 4/7$; Product $= c/a = -3/7$.
2. Here $a + b + c = 7 - 4 - 3 = 0$, so $x = 1$ is a root. The other root $= \text{product} \div 1 = -3/7$.

Ans. Sum $= 4/7$, product $= -3/7$; roots are 1 and $-3/7$.

Q26. Form the quadratic equation whose roots are -2 and -7 .

Solution: Sum $= -9$; Product $= 14$.

Ans. $x^2 + 9x + 14 = 0$.

Q27. Form the quadratic equation whose roots are $3/4$ and $-2/3$.

Solution:

1. Sum $= 3/4 - 2/3 = 1/12$; Product $= (3/4)(-2/3) = -1/2$.
2. $x^2 - (1/12)x - 1/2 = 0$. Multiply by 12 to clear fractions.

Ans. $12x^2 - x - 6 = 0$.

Q28. Form the quadratic equation whose roots are $(-3 + \sqrt{5})/2$ and $(-3 - \sqrt{5})/2$.

Solution: Sum $= -3$; Product $= (9 - 5)/4 = 1$.

Ans. $x^2 + 3x + 1 = 0$.

Q29. Form the quadratic equation whose roots are $-1 + i\sqrt{2}$ and $-1 - i\sqrt{2}$.

Solution: Sum $= -2$; Product $= (-1)^2 + (\sqrt{2})^2 = 1 + 2 = 3$.

Ans. $x^2 + 2x + 3 = 0$.

Q30. If α and β are the roots of $x^2 - 3x + 2 = 0$, form the equation whose roots are 2α and 2β .

Solution:

1. $\alpha + \beta = 3$ and $\alpha\beta = 2$.
2. New sum $= 2(\alpha + \beta) = 6$; new product $= 4\alpha\beta = 8$.

Ans. $x^2 - 6x + 8 = 0$.

Q31. Form the equation whose roots are the reciprocals of the roots of $2x^2 - 7x + 3 = 0$.

Solution:

1. $\alpha + \beta = 7/2$ and $\alpha\beta = 3/2$.
2. New sum $= 1/\alpha + 1/\beta = (\alpha + \beta)/\alpha\beta = 7/3$; new product $= 1/(\alpha\beta) = 2/3$.
3. $x^2 - (7/3)x + 2/3 = 0$. Multiply by 3.

Ans. $3x^2 - 7x + 2 = 0$.

Q32. Form the equation whose roots are the negatives of the roots of $x^2 - 4x + 3 = 0$.

Solution: Replace x by $-x$: $(-x)^2 - 4(-x) + 3 = 0$.

Ans. $x^2 + 4x + 3 = 0$.

Q33. If α and β are the roots of $x^2 - 3x + 1 = 0$, find $\alpha^4 + \beta^4$.

Solution:

1. $\alpha + \beta = 3$ and $\alpha\beta = 1$, so $\alpha^2 + \beta^2 = 9 - 2 = 7$.
2. $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = 49 - 2$

Ans. $\alpha^4 + \beta^4 = 47$.

Q34. If $\alpha + \beta = 7$ and $\alpha\beta = 10$, find $\alpha^2 + \beta^2$ and $|\alpha - \beta|$.

Solution:

1. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 49 - 20 = 29$.
2. $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = 49 - 40 = 9$

Ans. $\alpha^2 + \beta^2 = 29$ and $|\alpha - \beta| = 3$.

Q35. If 2 is a root of $2x^2 + kx - 6 = 0$, find k and the other root.

Solution:

1. Substitute $x = 2$: $8 + 2k - 6 = 0$, so $k = -1$.
2. The equation is $2x^2 - x - 6 = 0$. Product of roots $= -3$, so the other root $= -3/2$.
3. Check: sum $= 2 - 3/2 = 1/2 = -b/a$ ✓

Ans. $k = -1$ and the other root is $-3/2$.

Q36. If the roots of $x^2 - px + 8 = 0$ differ by 2, find p .

Solution:

1. $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$, so $4 = p^2 - 32$, so $p^2 = 36$.
2. Check with $p = 6$: $x^2 - 6x + 8 = (x - 2)(x - 4)$, roots 2 and 4 ✓

Ans. $p = \pm 6$.

Q37. If the roots of $x^2 - 5x + k = 0$ are in the ratio 2 : 3, find k .

Solution:

1. Let the roots be $2m$ and $3m$. Sum: $5m = 5$, so $m = 1$.
2. Product: $k = 6m^2 = 6$. (Roots 2 and 3 ✓)

Ans. $k = 6$.

Part 4: Simple Word Problems

Q38. The product of two consecutive positive even numbers is 48. Find them.

Solution:

1. Let the numbers be x and $x + 2$. Then $x(x + 2) = 48$, so $x^2 + 2x - 48 = 0$, so $(x + 8)(x - 6) = 0$.
2. The positive solution is $x = 6$.

Ans. The numbers are 6 and 8.

Q39. The difference of two positive numbers is 4 and their product is 45. Find the numbers.

Solution:

1. Let the numbers be x and $x + 4$. Then $x(x + 4) = 45$, so $x^2 + 4x - 45 = 0$, so $(x + 9)(x - 5) = 0$.
2. The positive solution is $x = 5$.

Ans. The numbers are 5 and 9.

Q40. The base of a triangle is 4 cm more than its height. If the area is 30 cm^2 , find the base and height.

Solution:

1. Let the height = x cm and the base = $(x + 4)$ cm. Area = $(1/2) \times (x + 4) = 30$.
2. $x^2 + 4x - 60 = 0$, so $(x + 10)(x - 6) = 0$. Height cannot be negative, so $x = 6$.

Ans. Height = 6 cm and base = 10 cm.

Q41. A father is three times as old as his son, and the product of their ages is 1200. Find their ages.

Solution:

1. Let the son's age = x years, so the father's age = $3x$. Then $x \times 3x = 1200$, so $x^2 = 400$.
2. $x = 20$ (age is positive).

Ans. Son = 20 years and father = 60 years.

Q42. The perimeter of a rectangle is 28 cm and its area is 48 cm^2 . Find its length and breadth.

Solution:

1. $l + b = 14$ and $lb = 48$, so l and b are the roots of $t^2 - 14t + 48 = 0$.
2. $(t - 6)(t - 8) = 0$

Ans. Length = 8 cm and breadth = 6 cm.