

Applied Physics 1

Chapter: Force and Motion

Exercise Notes: Important Questions, Solution Steps and Numerical Problems

For Engineering Students | Short and Broad Answers | Exam Oriented

Topics Covered

- Force, Momentum, Conservation of Linear Momentum, Recoil of Gun, Rocket (concept only), Impulse and Impulsive Force
- Circular Motion: Angular Displacement, Angular Velocity, Angular Acceleration, Frequency, Time Period, relation between linear and angular quantities
- Centripetal and Centrifugal Forces (formula only), Banking of Roads, Bending of a Cyclist

Contents

- Part A: Quick Revision Notes and Formula Sheet
- Part B: Short Answer Questions (2 to 3 marks)
- Part C: Broad Answer Questions (5 to 6 marks)
- Part D: Numerical Problems with Solution Steps
- Part E: Practice Numericals with Answers

Note: Take $g = 9.8 \text{ m/s}^2$ unless stated otherwise. Use $\pi = 3.14$ (or $22/7$) in calculations. Always write Given, Formula, Substitution and the Unit in the final answer.

PART A: QUICK REVISION NOTES

1. Force

Force is a push or a pull which changes, or tends to change, the state of rest or of uniform motion of a body, or changes its shape and size. It is a vector quantity. Its SI unit is the newton (N), where $1 \text{ N} = 1 \text{ kg m/s}^2$. Dimensional formula: $[M L T^{-2}]$.

- **Newton's first law:** A body continues in its state of rest or of uniform motion in a straight line unless an external force acts on it (law of inertia).
- **Newton's second law:** The rate of change of momentum is directly proportional to the applied force and takes place in the direction of the force. $F = dp/dt = ma$.
- **Newton's third law:** To every action there is an equal and opposite reaction. Action and reaction act on different bodies.

2. Momentum

Linear momentum of a body is the product of its mass and velocity: $\mathbf{p} = m\mathbf{v}$. It is a vector quantity and has the same direction as the velocity. SI unit: kg m/s (or N s). Dimensional formula: $[M L T^{-1}]$. Newton's second law in terms of momentum: $F = (\text{change in momentum})/(\text{time}) = (mv - mu)/t$.

3. Conservation of Linear Momentum

Statement: If no external force acts on a system of bodies, the total linear momentum of the system remains constant (before and after collision or interaction). $m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$

4. Applications: Recoil of a Gun and Rocket Propulsion

- Before firing, the gun and bullet are at rest, so the total momentum is zero. After firing, the bullet moves forward and the gun moves backward (recoils) so that the total momentum stays zero.
- **Recoil velocity:** $\mathbf{V} = -m\mathbf{v}/M$ (m = mass of bullet, v = velocity of bullet, M = mass of gun). The negative sign shows that the gun moves opposite to the bullet.
- **Rocket (concept only):** A rocket burns fuel and throws hot gases backward at high speed. The backward momentum of the gases is equal to the forward momentum gained by the rocket. It works on Newton's third law and conservation of momentum, and it can work in vacuum (space) because it does not need air.

5. Impulse and Impulsive Force

- **Impulse** is the product of force and the small time for which it acts: $\mathbf{J} = \mathbf{F} \times \Delta t$. SI unit: N s (same as kg m/s). It is a vector quantity.
- **Impulse-momentum theorem:** Impulse = change in momentum = $mv - mu$.
- **Impulsive force** is a very large force which acts on a body for a very short time, e.g. a bat hitting a ball, a hammer striking a nail. Average impulsive force = $\Delta p/\Delta t$.

6. Circular Motion: Angular Quantities

- **Angular displacement (θ):** the angle swept by the radius vector in a given time. $\theta = s/r$ (radian). 1 revolution = 2π rad = 360° .
- **Angular velocity (ω):** rate of change of angular displacement. $\omega = \theta/t = 2\pi/T = 2\pi f$. Unit: rad/s. (1 rpm = $2\pi/60$ rad/s)
- **Angular acceleration (α):** rate of change of angular velocity. $\alpha = (\omega_2 - \omega_1)/t$. Unit: rad/s^2 .
- **Time period (T):** time taken for one complete revolution. **Frequency (f):** number of revolutions per second. $f = 1/T$, unit Hz.
- **Linear and angular relations:** $v = r\omega$, $a_t = r\alpha$ (tangential acceleration).

7. Centripetal and Centrifugal Force

Centripetal force is the real force acting towards the centre of the circle which keeps a body in circular motion. $F = mv^2/r = m\omega^2 r$. Centripetal acceleration $a_c = v^2/r = \omega^2 r$. **Centrifugal force** is the apparent (pseudo) force felt in a rotating frame, directed away from the centre. It is equal in magnitude to the centripetal force, mv^2/r .

8. Banking of Roads and Bending of a Cyclist

- **Banking of roads:** raising the outer edge of a curved road above the inner edge so that a component of the normal reaction supplies the centripetal force. $\tan \theta = v^2/(rg)$, and safe speed $v = \sqrt{rg \tan \theta}$.
- **Bending of a cyclist:** a cyclist leans inward while turning by an angle θ to the vertical, given by $\tan \theta = v^2/(rg)$.

Formula Sheet

Quantity	Formula	Unit
Force	$F = ma = dp/dt$	N
Momentum	$p = mv$	kg m/s
Impulse	$J = F\Delta t = mv - mu$	N s
Conservation of momentum	$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$	—
Recoil velocity of gun	$V = -mv/M$	m/s
Rocket thrust (concept)	$F = u (dm/dt)$	N
Angular displacement	$\theta = s/r$	rad
Angular velocity	$\omega = \theta/t = 2\pi f = 2\pi/T$	rad/s
Angular acceleration	$\alpha = (\omega_2 - \omega_1)/t$	rad/s^2
Frequency and time period	$f = 1/T$	Hz, s
Linear and angular velocity	$v = r\omega$	m/s
Tangential acceleration	$a_t = r\alpha$	m/s^2
Rotational equations	$\omega = \omega_0 + \alpha t$; $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$; $\omega^2 = \omega_0^2 + 2\alpha\theta$	—
Centripetal acceleration	$a_c = v^2/r = \omega^2 r$	m/s^2
Centripetal force	$F = mv^2/r = m\omega^2 r$	N
Banking angle (no friction)	$\tan \theta = v^2/(rg)$, $v = \sqrt{rg \tan \theta}$	—
Max. speed on flat road	$v = \sqrt{\mu rg}$	m/s

PART B: SHORT ANSWER QUESTIONS (2 TO 3 MARKS)

Q1. Define force. Write its SI unit and dimensional formula.

Ans. Force is a push or pull that changes or tends to change the state of rest or motion of a body. SI unit: newton (N), $1 \text{ N} = 1 \text{ kg m/s}^2$. Dimensional formula: $[M L T^{-2}]$.

Q2. State Newton's three laws of motion.

Ans.

- First law: A body stays at rest or in uniform motion in a straight line unless acted on by an external force.
- Second law: Rate of change of momentum is proportional to the applied force ($F = ma$).
- Third law: For every action there is an equal and opposite reaction.

Q3. Define linear momentum. Give its unit and dimensions.

Ans. Momentum is the product of mass and velocity of a body, $p = mv$. Unit: kg m/s . Dimensions: $[M L T^{-1}]$. It is a vector quantity.

Q4. State the law of conservation of linear momentum.

Ans. If no external force acts on a system, the total linear momentum of the system remains constant. Mathematically, $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$.

Q5. Define impulse. Write its unit.

Ans. Impulse is the product of force and the time for which it acts, $J = F \times \Delta t$. It equals the change in momentum of the body. Unit: N s ($= \text{kg m/s}$).

Q6. What is an impulsive force? Give two examples.

Ans. A very large force acting for a very short time is called an impulsive force. Examples: a bat hitting a ball, a hammer hitting a nail, collision of two cars.

Q7. Why does a gun recoil when a bullet is fired?

Ans. Before firing, total momentum of gun + bullet is zero. By conservation of momentum, after firing the forward momentum of the bullet must be balanced by an equal backward momentum of the gun, so the gun recoils. $V = -mv/M$.

Q8. Why is the recoil of a heavy gun less than that of a light gun for the same bullet?

Ans. Since $V = mv/M$, the recoil velocity is inversely proportional to the mass of the gun M . A heavier gun gives a smaller recoil velocity.

Q9. On which principle does a rocket work?

Ans. A rocket works on Newton's third law and the law of conservation of linear momentum. The backward momentum of the ejected gases equals the forward momentum gained by the rocket.

Q10. A cricket player lowers his hands while catching a fast ball. Why?

Ans. Lowering the hands increases the time of contact Δt . Since $F = \Delta p/\Delta t$ and the change in momentum is fixed, a larger Δt gives a smaller force on the hands, so the player does not get hurt.

Q11. Why are vehicles provided with shock absorbers or cushions?

Ans. They increase the time of impact during a jerk, which reduces the average force ($F = \Delta p/\Delta t$) transmitted to the passengers.

Q12. Define angular displacement. Write its unit.

Ans. Angular displacement is the angle swept by the radius vector of a particle moving in a circle in a given time. $\theta = s/r$. SI unit: radian (rad).

Q13. Define angular velocity. Write its formula and unit.

Ans. Angular velocity is the rate of change of angular displacement. $\omega = \theta/t = 2\pi f = 2\pi/T$. Unit: rad/s .

Q14. Define angular acceleration. Write its unit.

Ans. It is the rate of change of angular velocity. $\alpha = (\omega_2 - \omega_1)/t$. Unit: rad/s^2 .

Q15. Define time period and frequency. How are they related?

Ans. Time period T is the time taken for one complete revolution; frequency f is the number of revolutions per second. $f = 1/T$. Units: second and hertz (Hz).

Q16. Derive the relation between linear velocity and angular velocity.

Ans. For one revolution, distance covered = $2\pi r$ in time T . So $v = 2\pi r/T$. Since $\omega = 2\pi/T$, we get $v = r\omega$.

Q17. What is centripetal force? Write its formula with two examples.

Ans. The force acting towards the centre of the circle which keeps a body in circular motion is called centripetal force. $F = mv^2/r = m\omega^2 r$. Examples: gravitational force on the Moon around the Earth; tension in a string when a stone is whirled.

Q18. What is centrifugal force? Give two examples.

Ans. It is the apparent force that seems to act away from the centre on a body in a rotating frame. Its magnitude is mv^2/r . Examples: clothes pressed against the drum in a spin dryer; passengers thrown outward when a bus takes a sharp turn.

Q19. Can a body moving with constant speed have acceleration? Explain.

Ans. Yes. In uniform circular motion the speed is constant but the direction of velocity changes continuously, so the body has a centripetal acceleration $a = v^2/r$ directed towards the centre.

Q20. Why are roads banked at curves?

Ans. Banking provides the required centripetal force through a component of the normal reaction, so the vehicle can take the turn safely without depending only on friction. It reduces skidding and tyre wear.

Q21. Write the expression for the angle of banking. On what factors does it depend?

Ans. $\tan \theta = v^2/(rg)$. It depends on the speed of the vehicle v and the radius of the curve r . It is independent of the mass of the vehicle.

Q22. Why does a cyclist lean inward while taking a turn?

Ans. Leaning inward provides the horizontal component of reaction needed as centripetal force and balances the torque due to weight, so that the cyclist does not topple. The angle of leaning is $\tan \theta = v^2/(rg)$.

Q23. Differentiate between centripetal and centrifugal force.

Centripetal force	Centrifugal force
Real force	Apparent (pseudo) force
Acts towards the centre	Acts away from the centre
Seen in an inertial (ground) frame	Seen in a rotating (non-inertial) frame
Required to keep the body in circular path	Arises due to inertia of the body
Example: tension in string	Example: clothes in a spin dryer

PART C: BROAD ANSWER QUESTIONS (5 TO 6 MARKS)**Q1. Define force and momentum. State Newton's second law and show that $F = ma$.**

Ans.

- **Force:** a push or pull that changes the state of rest or motion of a body. SI unit: newton.
- **Momentum:** $p = mv$, the product of mass and velocity.
- **Newton's second law:** The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction of the force.

Proof: Let a body of mass m change its velocity from u to v in time t under a force F .

Initial momentum = mu , final momentum = mv . Rate of change of momentum = $(mv - mu)/t = m(v - u)/t$.

By the law, $F \propto m(v - u)/t$. Since $(v - u)/t = a$, $F \propto ma$, i.e. $F = kma$. With $k = 1$ in SI units, **$F = ma$** .

1 newton is the force which produces an acceleration of 1 m/s^2 in a body of mass 1 kg.

Q2. State and prove the law of conservation of linear momentum.

Ans. Statement: If no external force acts on a system, its total linear momentum remains constant.

Proof: Consider two bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 ($u_1 > u_2$) along the same line. They collide and, after time Δt , their velocities become v_1 and v_2 .

Force on m_2 by $m_1 = F_{21}$ and force on m_1 by $m_2 = F_{12}$. From Newton's third law, $F_{12} = -F_{21}$.

Using $F = (\text{change in momentum})/\Delta t$: $m_1(v_1 - u_1)/\Delta t = -m_2(v_2 - u_2)/\Delta t$

$$\Rightarrow m_1 v_1 - m_1 u_1 = -m_2 v_2 + m_2 u_2$$

$$\Rightarrow m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Hence total momentum before collision = total momentum after collision. Hence proved.

Q3. Explain the recoil of a gun using conservation of momentum. Obtain the expression for recoil velocity.

Ans. Let M = mass of gun, m = mass of bullet, v = velocity of the bullet and V = recoil velocity of the gun.

Before firing both are at rest, so total initial momentum = 0.

After firing, total momentum = $mv + MV$.

By conservation of momentum: $0 = mv + MV \Rightarrow V = -mv/M$

The negative sign shows that the gun moves in the direction opposite to the bullet.

- Recoil velocity is directly proportional to the mass and velocity of the bullet.
- Recoil velocity is inversely proportional to the mass of the gun, so a heavy gun recoils less.
- Kinetic energy: KE of gun / KE of bullet = m/M . The bullet gets much more kinetic energy than the gun, since $M > m$.
- The recoil is felt as a backward push on the shoulder. A shooter holds the gun firmly against the shoulder so that the shooter's mass also adds to M and the recoil is reduced.

Q4. Explain the working principle of a rocket (concept only).

Ans. A rocket is based on Newton's third law of motion and the law of conservation of linear momentum.

- Fuel is burnt inside the combustion chamber and the hot gases are ejected backward through the nozzle at a very high speed.
- Before launch, the total momentum is zero. The backward momentum of the gases is balanced by an equal forward momentum of the rocket, so the rocket moves forward.
- The forward force on the rocket is called **thrust**: $F = u \times (dm/dt)$, where u is the exhaust velocity and dm/dt is the mass of fuel burnt per second.
- As fuel burns, the mass of the rocket decreases, so its velocity and acceleration keep increasing.
- At launch, acceleration $a = (\text{Thrust} - mg)/m$. Thrust must be greater than the weight of the rocket for lift-off.
- A rocket can work in vacuum because it does not push against air. Multi-stage rockets are used to attain the escape velocity.
- Uses: launching satellites, space missions, missiles, fireworks.

Q5. What is impulse? Prove that impulse equals change in momentum. Explain impulsive force with applications.

Ans. Impulse $J = F \times \Delta t$, unit N s.

Proof: Let a constant force F act on a body of mass m for a short time Δt , changing its velocity from u to v .

$F = ma = m(v - u)/\Delta t \Rightarrow F \Delta t = mv - mu$. So **Impulse = change in momentum**.

Impulsive force: a very large force acting for a very short time, e.g. hitting a ball with a bat. The average impulsive force $F = \Delta p/\Delta t$, which is large because Δt is very small.

Applications:

- A cricketer pulls his hands back while catching a ball: Δt increases, force decreases.

- Vehicles have bumpers, airbags and shock absorbers to increase the time of collision and reduce the force.
- A person jumping on sand or a cushion is not hurt, while jumping on concrete is dangerous, as time of stopping is smaller on hard ground.
- A hammer, axe and nail work by delivering a large impulsive force in a small time.

Q6. Define angular displacement, angular velocity, angular acceleration, time period and frequency. Establish the relations between linear and angular quantities.

Ans.

- **Angular displacement θ :** angle described by the radius vector. $\theta = s/r$, in radian.
- **Angular velocity ω :** $\omega = \theta/t$, in rad/s.
- **Angular acceleration α :** $\alpha = (\omega_2 - \omega_1)/t$, in rad/s².
- **Time period T:** time for one revolution, in seconds. **Frequency f:** number of revolutions per second, in hertz. $f = 1/T$.
- **Relation of ω with T and f:** In one revolution $\theta = 2\pi$ in time T, so $\omega = 2\pi/T = 2\pi f$.

Relation $v = r\omega$: Arc length $s = r\theta$. Dividing by t: $s/t = r(\theta/t)$, so **$v = r\omega$** .

Relation $a_t = r\alpha$: From $v = r\omega$, change in v per unit time = $r \times$ (change in ω per unit time), so **$a_t = r\alpha$** .

Equations of rotational motion (compare with linear motion):

Linear motion	Angular (rotational) motion
s, u, v, a	$\theta, \omega_0, \omega, \alpha$
$v = u + at$	$\omega = \omega_0 + \alpha t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = u^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$

Q7. Explain centripetal and centrifugal forces with examples. Write the formulas (no derivation).

Ans. Centripetal force: It is the real force that acts along the radius towards the centre of the circular path and is necessary to keep a body in uniform circular motion.

Formula: $F = mv^2/r = m\omega^2 r$ (and centripetal acceleration $a_c = v^2/r = \omega^2 r$).

Examples of centripetal force:

- Gravitational force of the Sun on planets / Earth on the Moon.
- Tension in the string when a stone is whirled in a horizontal circle.
- Friction between tyres and road when a car takes a turn.
- Electrostatic attraction between the nucleus and the electron in an atom.

Centrifugal force: It is the apparent outward force experienced by a body in a rotating frame of reference. It is equal in magnitude and opposite in direction to the centripetal force. Formula: $F = mv^2/r = m\omega^2 r$.

Examples of centrifugal force:

- Spin dryer of a washing machine: water is thrown out through the holes of the drum.
- Cream separator and centrifuge machines separate substances of different density.
- A person in a bus or car feels thrown outward during a sharp turn.
- Riders in a merry-go-round or circular ride feel pushed outward.

Q8. What is banking of roads? Why is it required? Write the formula for the angle of banking and explain the factors it depends on.

Ans. Banking of roads means raising the outer edge of a curved road above the inner edge, so that the road surface is tilted inward by an angle θ .

Need: When a vehicle takes a turn, it needs a centripetal force. On a flat road only friction provides it, which may not be enough at high speed and the vehicle may skid or overturn. On a banked road, a component of the normal reaction N provides the centripetal force.

Concept: The normal reaction N is perpendicular to the road. Its two components are:

- $N \cos \theta$ (vertical) balances the weight mg : $N \cos \theta = mg$
- $N \sin \theta$ (horizontal, towards centre) provides the centripetal force: $N \sin \theta = mv^2/r$

Dividing the two: **$\tan \theta = v^2/(rg)$** and the safe speed is **$v = \sqrt{rg \tan \theta}$** .

If h is the height of the outer edge above the inner edge and b is the width of the road, then $\tan \theta = h/b$.

Points to remember:

- Angle of banking increases with speed and decreases with radius of the curve.
- It does not depend on the mass of the vehicle.
- For a given θ and r , there is only one safe speed at which no friction is needed.
- Benefits: reduces skidding, tyre wear and chances of overturning.

Q9. Explain the bending of a cyclist while taking a turn. Write the formula for the angle of leaning.

Ans. While moving along a curved path, a cyclist needs a centripetal force towards the centre. The ground reaction (resultant of normal reaction and friction) provides it.

Explanation: If the cyclist remains upright, the reaction passes through the centre of gravity but there is no horizontal component and the cyclist would be thrown outward. So the cyclist leans inward by an angle θ with the vertical. Now:

- Vertical component of reaction balances the weight: $R \cos \theta = mg$
- Horizontal component provides the centripetal force: $R \sin \theta = mv^2/r$

Dividing: **$\tan \theta = v^2/(rg)$** .

Conclusions: The higher the speed, the more the cyclist must lean. A sharper turn (smaller r) needs more leaning. The angle is independent of the mass of the cyclist. The same logic applies to motorcyclists and to aeroplanes banking while turning.

PART D: NUMERICAL PROBLEMS WITH SOLUTION STEPS

D1. Momentum, Impulse and Conservation of Momentum

N1. Find the momentum of a body of mass 5 kg moving with a velocity of 12 m/s.

Given: $m = 5 \text{ kg}$, $v = 12 \text{ m/s}$

Formula: $p = mv$

Solution:

$$p = 5 \times 12 = 60$$

Answer: $p = 60 \text{ kg m/s}$

N2. A force of 20 N acts on a body of mass 4 kg, initially at rest, for 0.5 s. Find the impulse and the final velocity.

Given: $F = 20 \text{ N}$, $\Delta t = 0.5 \text{ s}$, $m = 4 \text{ kg}$, $u = 0$

Formula: $J = F\Delta t$; $J = mv - mu$

Solution:

$$J = 20 \times 0.5 = 10 \text{ N s}$$

$$10 = 4 \times v - 0 \Rightarrow v = 10/4 = 2.5 \text{ m/s}$$

Answer: Impulse = 10 N s, final velocity = 2.5 m/s

N3. A ball of mass 0.15 kg moving at 20 m/s is hit by a bat and returns with a speed of 30 m/s in the opposite direction. Find the impulse. If the contact time is 0.01 s, find the average force.

Given: $m = 0.15 \text{ kg}$, $u = 20 \text{ m/s}$, $v = -30 \text{ m/s}$ (opposite direction), $\Delta t = 0.01 \text{ s}$

Formula: $J = m(v - u)$; $F = J/\Delta t$

Solution:

$$J = 0.15 \times (-30 - 20) = 0.15 \times (-50) = -7.5 \text{ N s}$$

$$\text{Magnitude of impulse} = 7.5 \text{ N s}$$

$$F = 7.5/0.01 = 750 \text{ N}$$

Answer: Impulse = 7.5 N s (opposite to initial motion), average force = 750 N

N4. A cricketer catches a ball of mass 0.16 kg moving at 25 m/s. Find the average force on his hands if the ball is stopped in (a) 0.05 s and (b) 0.5 s (by pulling the hands back).

Given: $m = 0.16 \text{ kg}$, $u = 25 \text{ m/s}$, $v = 0$

Formula: $F = (mv - mu)/\Delta t$

Solution:

$$\text{Change in momentum} = 0 - 0.16 \times 25 = -4 \text{ kg m/s}$$

$$(a) F = 4/0.05 = 80 \text{ N}$$

$$(b) F = 4/0.5 = 8 \text{ N}$$

Answer: (a) 80 N, (b) 8 N. A longer stopping time reduces the force ten times.

N5. A body of mass 5 kg moving with a velocity of 4 m/s collides with a stationary body of mass 3 kg and they stick together. Find their common velocity.

Given: $m_1 = 5 \text{ kg}$, $u_1 = 4 \text{ m/s}$, $m_2 = 3 \text{ kg}$, $u_2 = 0$, common velocity v

Formula: $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

Solution:

$$5 \times 4 + 3 \times 0 = (5 + 3) v$$

$$20 = 8v \Rightarrow v = 2.5 \text{ m/s}$$

Answer: $v = 2.5 \text{ m/s}$ in the direction of the 5 kg body

N6. A bullet of mass 20 g is fired from a gun of mass 4 kg with a velocity of 400 m/s. Find the recoil velocity of the gun.

Given: $m = 20 \text{ g} = 0.02 \text{ kg}$, $M = 4 \text{ kg}$, $v = 400 \text{ m/s}$

Formula: $V = -mv/M$

Solution:

$$V = -(0.02 \times 400)/4$$

$$V = -8/4 = -2 \text{ m/s}$$

Answer: Recoil velocity = 2 m/s, opposite to the direction of the bullet

N7. A gun of mass 5 kg fires a bullet of mass 25 g with a speed of 500 m/s. Find the recoil velocity of the gun and the ratio of kinetic energy of the gun to that of the bullet.

Given: $M = 5 \text{ kg}$, $m = 25 \text{ g} = 0.025 \text{ kg}$, $v = 500 \text{ m/s}$

Formula: $V = -mv/M$; $KE_{\text{gun}}/KE_{\text{bullet}} = m/M$

Solution:

$$V = -(0.025 \times 500)/5 = -12.5/5 = -2.5 \text{ m/s}$$

$$\text{Ratio} = m/M = 0.025/5 = 0.005 = 1/200$$

Answer: Recoil velocity = 2.5 m/s; KE ratio = 1 : 200

N8. A stationary shell of mass 10 kg explodes into two fragments of masses 6 kg and 4 kg. The 6 kg fragment moves with a velocity of 8 m/s. Find the velocity of the 4 kg fragment.

Given: $M = 10 \text{ kg}$ (at rest), $m_1 = 6 \text{ kg}$, $v_1 = 8 \text{ m/s}$, $m_2 = 4 \text{ kg}$, $v_2 = ?$

Formula: Initial momentum = final momentum

Solution:

$$0 = 6 \times 8 + 4 \times v_2$$

$$4v_2 = -48 \Rightarrow v_2 = -12 \text{ m/s}$$

Answer: $v_2 = 12 \text{ m/s}$ in the direction opposite to the 6 kg fragment

N9. (Rocket, simple) A rocket ejects gas at 2000 m/s at a rate of 50 kg/s. Find the thrust. If the rocket has a mass of 5000 kg, find its initial acceleration at launch.

Given: $u = 2000 \text{ m/s}$, $dm/dt = 50 \text{ kg/s}$, $M = 5000 \text{ kg}$, $g = 9.8 \text{ m/s}^2$

Formula: Thrust $F = u (dm/dt)$; $a = (F - Mg)/M$

Solution:

$$F = 2000 \times 50 = 1,00,000 \text{ N}$$

$$\text{Weight} = Mg = 5000 \times 9.8 = 49,000 \text{ N}$$

$$a = (1,00,000 - 49,000)/5000 = 51,000/5000 = 10.2 \text{ m/s}^2$$

Answer: Thrust = $1 \times 10^5 \text{ N}$, initial acceleration = 10.2 m/s^2

D2. Circular Motion

N10. A wheel makes 120 revolutions per minute. Find its angular velocity, frequency and time period.

Given: $n = 120 \text{ rpm}$

Formula: $f = n/60$; $\omega = 2\pi f$; $T = 1/f$

Solution:

$$f = 120/60 = 2 \text{ Hz}$$

$$\omega = 2\pi \times 2 = 4\pi = 4 \times 3.14 = 12.56 \text{ rad/s}$$

$$T = 1/f = 1/2 = 0.5 \text{ s}$$

Answer: $f = 2 \text{ Hz}$, $\omega = 12.56 \text{ rad/s}$, $T = 0.5 \text{ s}$

N11. A particle moves in a circle of radius 0.5 m with an angular velocity of 10 rad/s. Find its linear velocity and centripetal acceleration.

Given: $r = 0.5 \text{ m}$, $\omega = 10 \text{ rad/s}$

Formula: $v = r\omega$; $a_c = \omega^2 r$

Solution:

$$v = 0.5 \times 10 = 5 \text{ m/s}$$

$$a_c = (10)^2 \times 0.5 = 100 \times 0.5 = 50 \text{ m/s}^2$$

Answer: $v = 5 \text{ m/s}$, $a_c = 50 \text{ m/s}^2$

N12. The second hand of a wall clock is 10 cm long. Find its angular velocity and the linear velocity of its tip.

Given: $r = 10 \text{ cm} = 0.1 \text{ m}$, $T = 60 \text{ s}$

Formula: $\omega = 2\pi/T$; $v = r\omega$

Solution:

$$\omega = 2 \times 3.14/60 = 0.1047 \text{ rad/s}$$

$$v = 0.1 \times 0.1047 = 0.01047 \text{ m/s}$$

Answer: $\omega = 0.105 \text{ rad/s}$, $v = 1.047 \text{ cm/s}$

N13. A fan starts from rest and reaches a speed of 300 rpm in 10 s. Find (a) angular acceleration and (b) number of revolutions made in this time.

Given: $\omega_0 = 0$, $n = 300$ rpm, $t = 10$ s

Formula: $\omega = 2\pi n/60$; $\alpha = (\omega - \omega_0)/t$; $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

Solution:

$$\omega = 2\pi \times 300/60 = 10\pi = 31.4 \text{ rad/s}$$

$$(a) \alpha = (31.4 - 0)/10 = 3.14 \text{ rad/s}^2$$

$$(b) \theta = 0 + \frac{1}{2} \times 3.14 \times (10)^2 = 157 \text{ rad}$$

$$\text{Number of revolutions} = \theta/2\pi = 157/6.28 = 25$$

Answer: $\alpha = 3.14 \text{ rad/s}^2$, revolutions = 25

N14. A flywheel of radius 0.4 m starts from rest with a constant angular acceleration of 5 rad/s². Find the tangential acceleration and the linear velocity of a point on its rim after 4 s.

Given: $r = 0.4$ m, $\omega_0 = 0$, $\alpha = 5 \text{ rad/s}^2$, $t = 4$ s

Formula: $a_t = r\alpha$; $\omega = \omega_0 + \alpha t$; $v = r\omega$

Solution:

$$a_t = 0.4 \times 5 = 2 \text{ m/s}^2$$

$$\omega = 0 + 5 \times 4 = 20 \text{ rad/s}$$

$$v = 0.4 \times 20 = 8 \text{ m/s}$$

Answer: $a_t = 2 \text{ m/s}^2$, $v = 8 \text{ m/s}$

N15. A stone of mass 0.5 kg tied to a string 1.2 m long is whirled in a horizontal circle with a speed of 6 m/s. Find the tension in the string.

Given: $m = 0.5$ kg, $r = 1.2$ m, $v = 6$ m/s

Formula: $T = F = mv^2/r$

Solution:

$$T = 0.5 \times (6)^2/1.2$$

$$T = 0.5 \times 36/1.2 = 18/1.2 = 15 \text{ N}$$

Answer: Tension = 15 N

N16. A car of mass 1000 kg takes a turn on a circular path of radius 50 m with a speed of 10 m/s. Find the centripetal force required.

Given: $m = 1000$ kg, $r = 50$ m, $v = 10$ m/s

Formula: $F = mv^2/r$

Solution:

$$F = 1000 \times (10)^2/50 = 1000 \times 100/50 = 2000 \text{ N}$$

Answer: $F = 2000 \text{ N}$, provided by friction between tyres and road

N17. Find the maximum speed with which a car can take a turn of radius 50 m on a flat road if the coefficient of friction is 0.4.

Given: $r = 50$ m, $\mu = 0.4$, $g = 9.8 \text{ m/s}^2$

Formula: $v = \sqrt{\mu r g}$

Solution:

$$v = \sqrt{0.4 \times 50 \times 9.8}$$

$$v = \sqrt{196} = 14 \text{ m/s}$$

Answer: $v = 14 \text{ m/s}$ (= 50.4 km/h)

N18. A road of radius 100 m is to be banked for vehicles moving at 72 km/h. Find the angle of banking. ($g = 9.8 \text{ m/s}^2$)

Given: $r = 100 \text{ m}$, $v = 72 \text{ km/h} = 72 \times 5/18 = 20 \text{ m/s}$

Formula: $\tan \theta = v^2/(rg)$

Solution:

$$\tan \theta = (20)^2/(100 \times 9.8) = 400/980 = 0.408$$

$$\theta = \tan^{-1}(0.408) = 22.2^\circ$$

Answer: $\theta \approx 22.2^\circ$

N19. A curved road of radius 50 m is banked at an angle of 15° . Find the safe speed of a vehicle on this road. ($\tan 15^\circ = 0.268$)

Given: $r = 50 \text{ m}$, $\theta = 15^\circ$, $g = 9.8 \text{ m/s}^2$

Formula: $v = \sqrt{rg \tan \theta}$

Solution:

$$v = \sqrt{50 \times 9.8 \times 0.268}$$

$$v = \sqrt{131.3} = 11.46 \text{ m/s}$$

$$\text{In km/h: } 11.46 \times 3.6 = 41.3 \text{ km/h}$$

Answer: $v \approx 11.5 \text{ m/s}$ (**41.3 km/h**)

N20. A cyclist takes a turn of radius 20 m with a speed of 36 km/h. At what angle should he lean from the vertical?

Given: $r = 20 \text{ m}$, $v = 36 \text{ km/h} = 10 \text{ m/s}$, $g = 9.8 \text{ m/s}^2$

Formula: $\tan \theta = v^2/(rg)$

Solution:

$$\tan \theta = (10)^2/(20 \times 9.8) = 100/196 = 0.51$$

$$\theta = \tan^{-1}(0.51) = 27^\circ$$

Answer: $\theta \approx 27^\circ$ from the vertical

PART E: PRACTICE NUMERICALS WITH ANSWERS

Solve these on your own using the steps shown in Part D, then check your answers.

No.	Problem	Answer
P1	The velocity of a 2 kg body changes from 3 m/s to 8 m/s in 0.5 s in the same direction. Find the impulse and the force.	Impulse = 10 N s, $F = 20 \text{ N}$
P2	A gun of mass 3 kg fires a 30 g bullet with a velocity of 300 m/s. Find the recoil velocity of the gun.	3 m/s
P3	A ball of mass 0.5 kg moving at 10 m/s strikes a wall and rebounds with 6 m/s. Find the impulse, and the average force if contact lasts 0.02 s.	Impulse = 8 N s, $F = 400 \text{ N}$
P4	A wheel rotates at 60 rpm. Find ω and the linear speed of a point 0.25 m from the axis.	$\omega = 6.28 \text{ rad/s}$, $v = 1.57 \text{ m/s}$
P5	A body starts from rest and has angular acceleration of 2 rad/s^2 for 5 s. Find the final angular velocity and the angle turned.	$\omega = 10 \text{ rad/s}$, $\theta = 25 \text{ rad}$
P6	A car moves at 20 m/s on a circular curve of radius 80 m. Find its centripetal acceleration.	5 m/s^2
P7	A stone of mass 0.2 kg is whirled in a circle of radius 0.5 m at 5 m/s. Find the tension in the string.	10 N
P8	Find the angle of banking for a curve of radius 60 m for a speed of 15 m/s. ($g = 9.8 \text{ m/s}^2$)	$\tan \theta = 0.383$, $\theta \approx 20.9^\circ$
P9	Find the maximum safe speed on a flat road of radius 40 m if $\mu = 0.5$. ($g = 9.8 \text{ m/s}^2$)	14 m/s (50.4 km/h)

Exam tips:

- Convert all quantities to SI units before substituting (g to kg, km/h to m/s, rpm to rad/s, cm to m).
- Write the formula first, then substitute, then give the answer with the correct unit.
- For recoil and collision problems, fix a positive direction and use signs carefully.
- In banking and cyclist problems, mass cancels out, so it is never needed for the angle.

Poly Notes Hub