

Applied Physics 1

Chapter: Force and Motion

Exercise Notes (Set 4): Broad-Type Numerical Questions with Step-by-Step Solutions

For Engineering Students | Multi-Part Numericals (5 to 6 marks) | Exam Oriented

Contents

- Part A: How to Answer a Broad Numerical (marking approach)
- Part B: Momentum, Impulse, Collision, Recoil and Rocket (Q1 to Q9)
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Note: Take $g = 9.8 \text{ m/s}^2$ and $\pi = 3.14$ unless stated otherwise. Each question has several parts (a), (b), (c) and so on, as asked in long-answer exams.

PART A: HOW TO ANSWER A BROAD NUMERICAL

- **Step 1, Given:** write all data in SI units (g to kg, km/h to m/s, rpm to rad/s, cm to m, minutes to seconds).
- **Step 2, Formula:** write the formula for each part before substituting. Formula carries marks.
- **Step 3, Sign convention:** choose a positive direction and give opposite velocities a negative sign.
- **Step 4, Substitution:** show the substitution line and then the calculation. Do not skip steps.
- **Step 5, Unit and result:** every answer needs its unit. Finish with a one-line conclusion where asked.

Typical marking for a 6-mark numerical: formulas 1 to 2 marks, substitution and steps 3 marks, final answers with units 1 to 2 marks.

PART B: MOMENTUM, IMPULSE, COLLISION, RECOIL AND ROCKET

Q1. A cricket ball of mass 150 g moving at 30 m/s is hit back along the same line with a speed of 40 m/s. The bat is in contact with the ball for 0.005 s. Find (a) the change in momentum, (b) the impulse, (c) the average force, (d) the acceleration of the ball and (e) how many times this force is the weight of the ball. [6 marks]

Given: $m = 150 \text{ g} = 0.15 \text{ kg}$, $u = +30 \text{ m/s}$, $v = -40 \text{ m/s}$ (reversed), $\Delta t = 0.005 \text{ s}$

Formulas: $p = mv$; $J = \Delta p$; $F = J/\Delta t$; $a = F/m$; $W = mg$

(a) Change in momentum

Initial momentum $= 0.15 \times 30 = 4.5 \text{ kg m/s}$

Final momentum $= 0.15 \times (-40) = -6.0 \text{ kg m/s}$

$\Delta p = -6.0 - 4.5 = -10.5 \text{ kg m/s}$

(b) Impulse

$J = \Delta p = 10.5 \text{ N s}$, directed opposite to the original motion

(c) Average force

$F = J/\Delta t = 10.5/0.005 = 2100 \text{ N}$

(d) Acceleration

$a = F/m = 2100/0.15 = 14,000 \text{ m/s}^2$

Check: $\Delta v/\Delta t = 70/0.005 = 14,000 \text{ m/s}^2$

(e) Comparison with weight

$W = 0.15 \times 9.8 = 1.47 \text{ N}$

$F/W = 2100/1.47 \approx 1430$

The impulsive force is about 1430 times the weight, so weight can be ignored during contact.

Answer: $\Delta p = 10.5 \text{ kg m/s}$, $J = 10.5 \text{ N s}$, $F = 2100 \text{ N}$, $a = 14,000 \text{ m/s}^2$, $F \approx 1430 \text{ W}$

Q2. A car of mass 2000 kg moving at 25 m/s is brought to rest (i) by normal braking in 5 s and (ii) in a crash in 0.1 s. For each case find the average retarding force, the deceleration and the stopping distance. Compare the two forces. [6 marks]

Given: $m = 2000 \text{ kg}$, $u = 25 \text{ m/s}$, $v = 0$, $t_1 = 5 \text{ s}$, $t_2 = 0.1 \text{ s}$

Formulas: $F = \Delta p/t$; $a = (v - u)/t$; $s = (u + v)t/2$

Change in momentum

$$\Delta p = 0 - 2000 \times 25 = -50,000 \text{ kg m/s (same in both cases)}$$

(i) Normal braking, $t = 5 \text{ s}$

$$F = 50,000/5 = 10,000 \text{ N}$$

$$a = 25/5 = 5 \text{ m/s}^2$$

$$s = (25 + 0)/2 \times 5 = 62.5 \text{ m}$$

(ii) Crash, $t = 0.1 \text{ s}$

$$F = 50,000/0.1 = 5,00,000 \text{ N}$$

$$a = 25/0.1 = 250 \text{ m/s}^2 = 250/9.8 \approx 25.5 \text{ g}$$

$$s = 12.5 \times 0.1 = 1.25 \text{ m}$$

Comparison

$$F_2/F_1 = 5,00,000/10,000 = 50$$

The momentum change is the same, but the shorter time makes the force 50 times larger. This is why airbags and crumple zones increase the stopping time.

Answer: (i) 10,000 N, 5 m/s², 62.5 m; (ii) 5 × 10⁵ N, 250 m/s², 1.25 m; ratio 50

Q3. A 6 kg body moving at 4 m/s collides with a 4 kg body moving at 2 m/s in the same direction. After the collision the 6 kg body moves at 3 m/s in the same direction. Find (a) the velocity of the 4 kg body, (b) the loss of kinetic energy and state the type of collision, (c) the impulse on each body and (d) the average force if the contact time is 0.02 s. [6 marks]

Given: $m_1 = 6 \text{ kg}$, $u_1 = 4 \text{ m/s}$, $v_1 = 3 \text{ m/s}$; $m_2 = 4 \text{ kg}$, $u_2 = 2 \text{ m/s}$

Formulas: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$; $K = \frac{1}{2}mv^2$; $J = m(v - u)$; $F = J/\Delta t$

(a) Velocity of the 4 kg body

$$6 \times 4 + 4 \times 2 = 6 \times 3 + 4v_2$$

$$24 + 8 = 18 + 4v_2 \Rightarrow 4v_2 = 14 \Rightarrow v_2 = 3.5 \text{ m/s}$$

(Check: $v_2 > v_1$, so the rear body cannot pass through the front body.)

(b) Kinetic energy

$$\text{Initial } K = \frac{1}{2} \times 6 \times 4^2 + \frac{1}{2} \times 4 \times 2^2 = 48 + 8 = 56 \text{ J}$$

$$\text{Final } K = \frac{1}{2} \times 6 \times 3^2 + \frac{1}{2} \times 4 \times (3.5)^2 = 27 + 24.5 = 51.5 \text{ J}$$

Loss = $56 - 51.5 = 4.5 \text{ J}$. Kinetic energy is not conserved, so the collision is inelastic (momentum is still conserved).

(c) Impulse on each body

$$\text{On the 6 kg body: } J_1 = 6 \times (3 - 4) = -6 \text{ N s}$$

$$\text{On the 4 kg body: } J_2 = 4 \times (3.5 - 2) = +6 \text{ N s}$$

Equal and opposite, as Newton's third law requires.

(d) Average force

$$F = 6/0.02 = 300 \text{ N on each body (in opposite directions)}$$

Answer: $v_2 = 3.5 \text{ m/s}$, KE loss = 4.5 J (inelastic), $J = \pm 6 \text{ N s}$, $F = 300 \text{ N}$

Q4. A gun of mass 5 kg fires a bullet of mass 20 g horizontally with a muzzle velocity of 250 m/s. Find (a) the recoil velocity of the gun, (b) the kinetic energies of the bullet and the gun and their ratio, (c) the average force needed to stop the gun in 0.05 s, and (d) the average force on the bullet inside the barrel if it takes 0.004 s to leave. [6 marks]

Given: $M = 5 \text{ kg}$, $m = 20 \text{ g} = 0.02 \text{ kg}$, $v = 250 \text{ m/s}$

Formulas: $V = -mv/M$; $K = \frac{1}{2}mv^2$; $F = \Delta p/t$

(a) Recoil velocity

$$V = - (0.02 \times 250)/5 = - 5/5 = - 1 \text{ m/s (opposite to the bullet)}$$

(b) Kinetic energies

$$K_{\text{bullet}} = \frac{1}{2} \times 0.02 \times (250)^2 = 0.01 \times 62,500 = 625 \text{ J}$$

$$K_{\text{gun}} = \frac{1}{2} \times 5 \times 1^2 = 2.5 \text{ J}$$

$$\text{Ratio } K_{\text{gun}}/K_{\text{bullet}} = 2.5/625 = 0.004 = m/M = 0.02/5$$

(c) Force to stop the gun

$$\text{Momentum of the gun} = 5 \times 1 = 5 \text{ kg m/s}$$

$$F = 5/0.05 = 100 \text{ N}$$

(d) Force on the bullet in the barrel

$$F = (0.02 \times 250)/0.004 = 5/0.004 = 1250 \text{ N}$$

The gun feels an equal and opposite force of 1250 N for 0.004 s: $a = 1250/5 = 250 \text{ m/s}^2$, $V = 250 \times 0.004 = 1 \text{ m/s}$, which matches part (a).

Answer: $V = 1 \text{ m/s}$, $K = 625 \text{ J}$ and 2.5 J (ratio 0.004), $F = 100 \text{ N}$, $F_{\text{bullet}} = 1250 \text{ N}$

Q5. A bullet of mass 25 g moving horizontally at 400 m/s strikes and embeds in a wooden block of mass 3.975 kg resting on a smooth horizontal surface. Find (a) the common velocity, (b) the kinetic energy before and after, (c) the percentage of kinetic energy lost and (d) the total momentum before and after impact. [6 marks]

Given: $m = 0.025 \text{ kg}$, $u = 400 \text{ m/s}$, $M = 3.975 \text{ kg}$ (at rest)

Formulas: $mu = (m + M)v$; $K = \frac{1}{2}mv^2$

(a) Common velocity

$$\text{Total mass} = 0.025 + 3.975 = 4.0 \text{ kg}$$

$$0.025 \times 400 = 4.0 \times v \Rightarrow 10 = 4v \Rightarrow v = 2.5 \text{ m/s}$$

(b) Kinetic energy

$$\text{Before: } K_1 = \frac{1}{2} \times 0.025 \times (400)^2 = 0.0125 \times 160,000 = 2000 \text{ J}$$

$$\text{After: } K_2 = \frac{1}{2} \times 4.0 \times (2.5)^2 = 2 \times 6.25 = 12.5 \text{ J}$$

(c) Percentage loss

$$\text{Loss} = 2000 - 12.5 = 1987.5 \text{ J}$$

$$\text{Percentage} = 1987.5/2000 \times 100 = 99.4 \%$$

Almost all the energy becomes heat and work done in deforming the block.

(d) Momentum

$$\text{Before} = 0.025 \times 400 = 10 \text{ kg m/s; after} = 4.0 \times 2.5 = 10 \text{ kg m/s. Momentum is conserved.}$$

Answer: $v = 2.5 \text{ m/s}$, $K = 2000 \text{ J} \rightarrow 12.5 \text{ J}$, 99.4 % lost, $p = 10 \text{ kg m/s}$

Q6. Two skaters of masses 50 kg and 70 kg stand at rest on smooth ice and push each other apart. The 50 kg skater moves at 2.8 m/s. Find (a) the speed of the other skater, (b) the total momentum before and after, (c) the total kinetic energy gained and (d) the ratio of their speeds. [5 marks]

Given: $m_1 = 50 \text{ kg}$, $v_1 = 2.8 \text{ m/s}$, $m_2 = 70 \text{ kg}$, initial velocities = 0

Formulas: $0 = m_1v_1 + m_2v_2$; $K = \frac{1}{2}mv^2$

(a) Speed of the second skater

$$0 = 50 \times 2.8 + 70 \times v_2 \Rightarrow 70v_2 = -140 \Rightarrow v_2 = -2 \text{ m/s}$$

Speed = 2 m/s, opposite to the first skater.

(b) Total momentum

Before = 0. After = $50 \times 2.8 + 70 \times (-2) = 140 - 140 = 0$. Conserved.

(c) Kinetic energy gained

$$K_1 = \frac{1}{2} \times 50 \times (2.8)^2 = 25 \times 7.84 = 196 \text{ J}$$

$$K_2 = \frac{1}{2} \times 70 \times 2^2 = 140 \text{ J}$$

Total = $196 + 140 = 336 \text{ J}$ (from the muscular energy of the skaters)

(d) Ratio of speeds

$$v_1/v_2 = 2.8/2 = 1.4 = m_2/m_1 = 70/50. \text{ The lighter skater moves faster.}$$

Answer: $v_2 = 2 \text{ m/s}$, momentum = 0, K = 336 J, $v_1 : v_2 = 1.4$

Q7. A rocket of mass 40,000 kg at lift-off ejects exhaust gas at 2500 m/s at the rate of 200 kg/s. Find (a) the thrust, (b) the weight, (c) the initial acceleration, (d) the acceleration after 100 s (assume the thrust stays the same) and (e) the minimum rate of fuel burning needed just to lift the rocket. [6 marks]

Given: $M = 40,000 \text{ kg}$, $u = 2500 \text{ m/s}$, $dm/dt = 200 \text{ kg/s}$, $g = 9.8 \text{ m/s}^2$

Formulas: $F = u (dm/dt)$; $a = (F - Mg)/M$

(a) Thrust

$$F = 2500 \times 200 = 5,00,000 \text{ N}$$

(b) Weight at lift-off

$$W = 40,000 \times 9.8 = 3,92,000 \text{ N}$$

(c) Initial acceleration

$$\text{Net force} = 5,00,000 - 3,92,000 = 1,08,000 \text{ N}$$

$$a = 1,08,000/40,000 = 2.7 \text{ m/s}^2$$

(d) After 100 s

$$\text{Fuel burnt} = 200 \times 100 = 20,000 \text{ kg, so } M = 40,000 - 20,000 = 20,000 \text{ kg}$$

$$W = 20,000 \times 9.8 = 1,96,000 \text{ N}$$

$$a = (5,00,000 - 1,96,000)/20,000 = 3,04,000/20,000 = 15.2 \text{ m/s}^2$$

The acceleration rises because the mass falls while the thrust stays constant.

(e) Minimum fuel rate for lift-off

Condition: thrust = weight, so $u (dm/dt) = Mg$

$$dm/dt = 3,92,000/2500 = 156.8 \text{ kg/s}$$

Answer: $F = 5 \times 10^5 \text{ N}$, $W = 3.92 \times 10^5 \text{ N}$, $a = 2.7 \text{ m/s}^2$ then 15.2 m/s^2 , minimum rate = 156.8 kg/s

Q8. Water leaves a hose at 12 m/s at the rate of 3 kg/s and strikes a vertical wall normally. Find the force on the wall if the water (a) stops on hitting the wall and (b) rebounds with the same speed. (c) Compare the two forces and give the reason. [5 marks]

Given: $v = 12 \text{ m/s}$, $dm/dt = 3 \text{ kg/s}$

Formulas: $F = (dm/dt) \times \Delta v$ (momentum change per second)

(a) Water stops

$$\text{Change in velocity} = 12 - 0 = 12 \text{ m/s}$$

$$F = 3 \times 12 = 36 \text{ N}$$

(b) Water rebounds

$$\text{Change in velocity} = 12 - (-12) = 24 \text{ m/s}$$

$$F = 3 \times 24 = 72 \text{ N}$$

(c) Comparison

$F_2/F_1 = 72/36 = 2$. The rebound doubles the change in momentum per second, so the force doubles.

Answer: (a) 36 N, (b) 72 N, ratio 2

Q9. An astronaut of mass 60 kg (with suit) is at rest in space. He throws a tool of mass 2 kg at 6 m/s. Find (a) his recoil velocity, (b) the distance he drifts in 10 s, and (c) the average force on the tool and the astronaut if the throw lasts 0.5 s. [5 marks]

Given: $M = 60 \text{ kg}$, $m = 2 \text{ kg}$, $v = 6 \text{ m/s}$, $t = 10 \text{ s}$, $\Delta t = 0.5 \text{ s}$

Formulas: $0 = mv + MV$; $s = Vt$; $F = \Delta p / \Delta t$

(a) Recoil velocity

$$0 = 2 \times 6 + 60 V \Rightarrow V = -12/60 = -0.2 \text{ m/s (opposite to the tool)}$$

(b) Distance drifted

There is no external force in space, so the speed stays constant: $s = 0.2 \times 10 = 2 \text{ m}$

(c) Force during the throw

$$F = (2 \times 6) / 0.5 = 24 \text{ N}$$

Acceleration of the astronaut $= 24/60 = 0.4 \text{ m/s}^2$, so $V = 0.4 \times 0.5 = 0.2 \text{ m/s}$, which matches part (a).

Answer: $V = 0.2 \text{ m/s}$, $s = 2 \text{ m}$, $F = 24 \text{ N}$

PART C: CIRCULAR MOTION

Q10. A ceiling fan with blades of tip radius 0.6 m is switched on from rest and reaches 240 rpm uniformly in 8 s. Find (a) the final angular velocity and the angular acceleration, (b) the number of revolutions in 8 s, and (c) at $t = 8 \text{ s}$, the linear speed, tangential acceleration, centripetal acceleration and total acceleration of a blade tip. [6 marks]

Given: $\omega_0 = 0$, $n = 240 \text{ rpm}$, $t = 8 \text{ s}$, $r = 0.6 \text{ m}$

Formulas: $\omega = 2\pi n / 60$; $\alpha = (\omega - \omega_0) / t$; $\theta = \frac{1}{2} \alpha t^2$; $v = r\omega$; $a_t = r\alpha$; $a_c = \omega^2 r$; $a = \sqrt{(a_t^2 + a_c^2)}$

(a) Final ω and α

$$\omega = 2 \times 3.14 \times 240 / 60 = 25.12 \text{ rad/s}$$

$$\alpha = (25.12 - 0) / 8 = 3.14 \text{ rad/s}^2$$

(b) Revolutions

$$\theta = \frac{1}{2} \times 3.14 \times 8^2 = 100.5 \text{ rad}$$

$$\text{Revolutions} = 100.5 / 6.28 = 16$$

(Check: average speed $120 \text{ rpm} = 2 \text{ rev/s}$, $2 \times 8 = 16$)

(c) Blade tip at $t = 8 \text{ s}$

$$v = r\omega = 0.6 \times 25.12 = 15.07 \text{ m/s}$$

$$a_t = r\alpha = 0.6 \times 3.14 = 1.88 \text{ m/s}^2$$

$$a_c = \omega^2 r = (25.12)^2 \times 0.6 = 631 \times 0.6 = 378.6 \text{ m/s}^2$$

$$a = \sqrt{(1.88^2 + 378.6^2)} \approx 378.6 \text{ m/s}^2 \text{ (} a_t \text{ is negligible compared with } a_c \text{)}$$

Answer: $\omega = 25.12 \text{ rad/s}$, $\alpha = 3.14 \text{ rad/s}^2$, 16 revolutions, $v = 15.07 \text{ m/s}$, $a_t = 1.88 \text{ m/s}^2$, $a_c = 378.6 \text{ m/s}^2$

Q11. A car of mass 1200 kg moves at a uniform speed of 30 m/s on a flat circular track of radius 150 m. Find (a) the time period, (b) the angular velocity, (c) the centripetal acceleration, (d) the centripetal force and (e) the minimum coefficient of friction needed. (f) What is the maximum safe speed if $\mu = 0.5$? [6 marks]

Given: $m = 1200 \text{ kg}$, $v = 30 \text{ m/s}$, $r = 150 \text{ m}$

Formulas: $T = 2\pi r / v$; $\omega = v / r$; $a_c = v^2 / r$; $F = mv^2 / r$; $\mu_{\min} = v^2 / (rg)$; $v = \sqrt{(\mu rg)}$

(a) Time period

$$T = 2 \times 3.14 \times 150 / 30 = 31.4 \text{ s}$$

(b) Angular velocity

$$\omega = v / r = 30 / 150 = 0.2 \text{ rad/s}$$

(c) Centripetal acceleration

$$a_c = 30^2 / 150 = 900 / 150 = 6 \text{ m/s}^2$$

(d) Centripetal force

$$F = 1200 \times 6 = 7200 \text{ N (supplied by friction)}$$

(e) Minimum μ

Friction must supply F , so $\mu mg = mv^2/r$

$$\mu = v^2/(rg) = 900/(150 \times 9.8) = 900/1470 = 0.61$$

(f) Maximum speed for $\mu = 0.5$

$$v = \sqrt{(0.5 \times 150 \times 9.8)} = \sqrt{735} = 27.1 \text{ m/s} \approx 97.6 \text{ km/h}$$

This is less than 30 m/s, so the car would skid at 30 m/s on this surface.

Answer: $T = 31.4 \text{ s}$, $\omega = 0.2 \text{ rad/s}$, $a_c = 6 \text{ m/s}^2$, $F = 7200 \text{ N}$, $\mu_{\min} = 0.61$, $v_{\max} = 27.1 \text{ m/s}$

Q12. The drum of a washing machine has a radius of 0.25 m and spins at 900 rpm during the spin cycle. Find (a) its angular velocity, (b) the speed of the drum wall, (c) the centripetal acceleration, (d) this acceleration in terms of g , and (e) the force on a wet cloth of mass 0.2 kg pressed against the wall. [6 marks]

Given: $r = 0.25 \text{ m}$, $n = 900 \text{ rpm}$, $m = 0.2 \text{ kg}$

Formulas: $\omega = 2\pi n/60$; $v = r\omega$; $a_c = \omega^2 r$; $F = ma_c$

(a) Angular velocity

$$\omega = 2 \times 3.14 \times 900/60 = 94.2 \text{ rad/s}$$

(b) Speed of the wall

$$v = 0.25 \times 94.2 = 23.55 \text{ m/s}$$

(c) Centripetal acceleration

$$a_c = (94.2)^2 \times 0.25 = 8874 \times 0.25 = 2218 \text{ m/s}^2$$

(d) In terms of g

$$a_c/g = 2218/9.8 \approx 226, \text{ so } a_c \approx 226 g$$

(e) Force on the cloth

$$F = 0.2 \times 2218 = 443.6 \text{ N}$$

Water droplets cannot be held by the cloth with this force and move outward through the holes of the drum.

Answer: $\omega = 94.2 \text{ rad/s}$, $v = 23.55 \text{ m/s}$, $a_c = 2218 \text{ m/s}^2 \approx 226 g$, $F = 443.6 \text{ N}$

Q13. A stone of mass 0.5 kg is tied to a string of length 0.8 m and whirled in a horizontal circle. The string breaks if the tension exceeds 50 N. Find (a) the maximum speed, (b) the maximum angular velocity, (c) the maximum rpm, (d) the tension at a speed of 6 m/s, and (e) the path of the stone after the string breaks. [6 marks]

Given: $m = 0.5 \text{ kg}$, $r = 0.8 \text{ m}$, $T_{\max} = 50 \text{ N}$

Formulas: $T = mv^2/r$; $\omega = v/r$; $n = 60\omega/2\pi$

(a) Maximum speed

$$50 = 0.5 \times v^2/0.8 \Rightarrow v^2 = 50 \times 0.8/0.5 = 80$$

$$v = \sqrt{80} = 8.94 \text{ m/s}$$

(b) Maximum angular velocity

$$\omega = v/r = 8.94/0.8 = 11.18 \text{ rad/s}$$

(c) Maximum rpm

$$n = 11.18 \times 60/2\pi = 106.8 \text{ rpm}$$

(d) Tension at 6 m/s

$$T = 0.5 \times 6^2/0.8 = 18/0.8 = 22.5 \text{ N (safe, less than 50 N)}$$

(e) After the string breaks

The centripetal force vanishes, so by inertia the stone flies off along the tangent to the circle at the point of breaking, with speed 8.94 m/s.

Answer: $v = 8.94 \text{ m/s}$, $\omega = 11.18 \text{ rad/s}$, 106.8 rpm , $T = 22.5 \text{ N}$ at 6 m/s , tangential path

Q14. A satellite completes one orbit around the Earth in 100 minutes at an orbital radius of 7.0×10^6 m. Find (a) its angular velocity, (b) its orbital speed, (c) its centripetal acceleration and (d) the number of orbits it makes in one day. [5 marks]

Given: $T = 100 \text{ min} = 6000 \text{ s}$, $r = 7.0 \times 10^6 \text{ m}$

Formulas: $\omega = 2\pi/T$; $v = r\omega$; $a_c = \omega^2 r$

(a) Angular velocity

$$\omega = 2 \times 3.14/6000 = 1.047 \times 10^{-3} \text{ rad/s}$$

(b) Orbital speed

$$v = 7.0 \times 10^6 \times 1.047 \times 10^{-3} = 7.33 \times 10^3 \text{ m/s} \approx 7.33 \text{ km/s}$$

(c) Centripetal acceleration

$$a_c = (1.047 \times 10^{-3})^2 \times 7.0 \times 10^6 = 1.096 \times 10^{-6} \times 7.0 \times 10^6 = 7.67 \text{ m/s}^2$$

(d) Orbits per day

$$1 \text{ day} = 24 \times 60 = 1440 \text{ min}$$

$$\text{Number of orbits} = 1440/100 = 14.4$$

Answer: $\omega = 1.047 \times 10^{-3} \text{ rad/s}$, $v = 7.33 \text{ km/s}$, $a_c = 7.67 \text{ m/s}^2$, 14.4 orbits/day

PART D: BANKING OF ROADS AND CYCLIST

Q15. A highway curve of radius 120 m is banked at 12° . ($\tan 12^\circ = 0.2126$) Find (a) the design speed (no friction needed), (b) the banking angle needed for a design speed of 20 m/s on the same curve, and (c) the height of the outer edge above the inner edge for the 12° banking if the road is 12 m wide. [6 marks]

Given: $r = 120 \text{ m}$, $\theta_1 = 12^\circ$, $v_2 = 20 \text{ m/s}$, $b = 12 \text{ m}$

Formulas: $v = \sqrt{rg \tan \theta}$; $\tan \theta = v^2/(rg)$; $h = b \tan \theta$

(a) Design speed for 12°

$$v^2 = 120 \times 9.8 \times 0.2126 = 1176 \times 0.2126 = 250$$

$$v = \sqrt{250} = 15.8 \text{ m/s} = 15.8 \times 3.6 = 56.9 \text{ km/h}$$

(b) Angle for 20 m/s

$$\tan \theta = 20^2/(120 \times 9.8) = 400/1176 = 0.340$$

$$\theta = \tan^{-1}(0.340) = 18.8^\circ$$

(c) Height of the outer edge

$$h = b \tan \theta = 12 \times 0.2126 = 2.55 \text{ m}$$

Answer: $v = 15.8 \text{ m/s}$ (56.9 km/h), $\theta = 18.8^\circ$, $h = 2.55 \text{ m}$

Q16. A cyclist and cycle of total mass 85 kg take a curve of radius 30 m at 12 m/s. Find (a) the angle of lean from the vertical, (b) the centripetal force, (c) whether the angle changes if a heavier rider (100 kg total) takes the curve at the same speed, and (d) the angle of lean if the radius is reduced to 15 m at the same speed. [6 marks]

Given: $m = 85 \text{ kg}$, $r_1 = 30 \text{ m}$, $v = 12 \text{ m/s}$, $r_2 = 15 \text{ m}$

Formulas: $\tan \theta = v^2/(rg)$; $F = mv^2/r$

(a) Angle of lean

$$\tan \theta = 12^2/(30 \times 9.8) = 144/294 = 0.49$$

$$\theta = \tan^{-1}(0.49) = 26.1^\circ$$

(b) Centripetal force

$$F = 85 \times 144/30 = 408 \text{ N}$$

(c) Heavier rider

Mass does not appear in $\tan \theta = v^2/(rg)$, so the angle stays 26.1° . (The force would rise to $100 \times 144/30 = 480 \text{ N}$.)

(d) Smaller radius

$$\tan \theta = 144/(15 \times 9.8) = 144/147 = 0.98$$

$$\theta = \tan^{-1}(0.98) = 44.4^\circ$$

A sharper curve at the same speed needs a much larger lean.

Answer: $\theta = 26.1^\circ$, $F = 408$ N, no change with mass, $\theta = 44.4^\circ$ for $r = 15$ m

Q17. A car must take a curve of radius 100 m at 72 km/h. Find (a) the coefficient of friction needed on a flat road, (b) whether the car is safe if $\mu = 0.35$ and the maximum safe speed in that case, (c) the banking angle needed so that no friction is required, and (d) the height of the outer edge if the road is 10 m wide. [6 marks]

Given: $r = 100$ m, $v = 72$ km/h $= 72 \times 5/18 = 20$ m/s, $\mu = 0.35$, $b = 10$ m

Formulas: $\mu = v^2/(rg)$; $v = \sqrt{\mu rg}$; $\tan \theta = v^2/(rg)$; $h = b \tan \theta$

(a) Friction needed

$$\mu = 20^2/(100 \times 9.8) = 400/980 = 0.408$$

(b) Safety for $\mu = 0.35$

Needed $\mu = 0.408$ is more than the available 0.35, so the car is not safe and may skid.

$$v_{\max} = \sqrt{0.35 \times 100 \times 9.8} = \sqrt{343} = 18.5 \text{ m/s} = 66.7 \text{ km/h}$$

(c) Banking angle

$$\tan \theta = v^2/(rg) = 0.408$$

$$\theta = \tan^{-1}(0.408) = 22.2^\circ$$

(d) Height of the outer edge

$$h = b \tan \theta = 10 \times 0.408 = 4.08 \text{ m}$$

Answer: $\mu = 0.408$ (not safe for 0.35, $v_{\max} = 18.5$ m/s), $\theta = 22.2^\circ$, $h = 4.08$ m

PART E: ANSWER SUMMARY

Use this table to check your final answers after solving the questions yourself.

No.	Final answers
Q1	$\Delta p = 10.5$ kg m/s, $J = 10.5$ N s, $F = 2100$ N, $a = 14,000$ m/s ² , $F \approx 1430$ W
Q2	(i) 10,000 N, 5 m/s ² , 62.5 m; (ii) 5×10^5 N, 250 m/s ² , 1.25 m; ratio 50
Q3	$v_2 = 3.5$ m/s, KE loss = 4.5 J (inelastic), $J = \pm 6$ N s, $F = 300$ N
Q4	$V = 1$ m/s, $K = 625$ J and 2.5 J (ratio 0.004), $F = 100$ N, $F_{\text{bullet}} = 1250$ N
Q5	$v = 2.5$ m/s, $K = 2000$ J $\rightarrow$ 12.5 J, 99.4 % lost, $p = 10$ kg m/s
Q6	$v_2 = 2$ m/s, momentum = 0, $K = 336$ J, $v_1 : v_2 = 1.4$
Q7	$F = 5 \times 10^5$ N, $W = 3.92 \times 10^5$ N, $a = 2.7$ m/s ² then 15.2 m/s ² , minimum rate = 156.8 kg/s
Q8	(a) 36 N, (b) 72 N, ratio 2
Q9	$V = 0.2$ m/s, $s = 2$ m, $F = 24$ N
Q10	$\omega = 25.12$ rad/s, $\alpha = 3.14$ rad/s ² , 16 revolutions, $v = 15.07$ m/s, $a_t = 1.88$ m/s ² , $a_c = 378.6$ m/s ²
Q11	$T = 31.4$ s, $\omega = 0.2$ rad/s, $a_c = 6$ m/s ² , $F = 7200$ N, $\mu_{\min} = 0.61$, $v_{\max} = 27.1$ m/s
Q12	$\omega = 94.2$ rad/s, $v = 23.55$ m/s, $a_c = 2218$ m/s ² ≈ 226 g, $F = 443.6$ N
Q13	$v = 8.94$ m/s, $\omega = 11.18$ rad/s, 106.8 rpm, $T = 22.5$ N at 6 m/s, tangential path
Q14	$\omega = 1.047 \times 10^{-3}$ rad/s, $v = 7.33$ km/s, $a_c = 7.67$ m/s ² , 14.4 orbits/day
Q15	$v = 15.8$ m/s (56.9 km/h), $\theta = 18.8^\circ$, $h = 2.55$ m
Q16	$\theta = 26.1^\circ$, $F = 408$ N, no change with mass, $\theta = 44.4^\circ$ for $r = 15$ m
Q17	$\mu = 0.408$ (not safe for 0.35, $v_{\max} = 18.5$ m/s), $\theta = 22.2^\circ$, $h = 4.08$ m

Exam tips:

- Write each part label (a), (b), (c) clearly and keep the working under it. Examiners give marks part by part.

- In collision, recoil and rocket problems, check your answer by confirming that momentum before equals momentum after.
- In circular motion problems, convert rpm to rad/s first and then use $v = r\omega$, $a_c = \omega^2 r$.
- In banking and cyclist problems, mass cancels out, so the angle depends only on v and r .

Poly Notes Hub