

Applied Physics 1

Chapter: Physical World, Units and Measurements

Numerical Problems Set 3: Problems with Step-by-Step Solutions

For Engineering Students (Numericals Only, Exam Oriented)

Sections: A. Conversion of units, B. Dimensional analysis, C. Measuring instruments and least count, D. Errors and error propagation, E. Significant figures, F. Mixed problems. All problems are new and different from Sets 1 and 2.

Key formulae used

- Unit conversion: $n_2 = n_1 [M_1/M_2]^a [L_1/L_2]^b [T_1/T_2]^c$
- Vernier LC = 1 MSD / N (N VSD = N – 1 MSD); reading = MSR + VSR × LC
- Screw gauge LC = pitch / circular divisions; reading = LSR + CSR × LC; true = observed – zero error
- Sum or difference: $\Delta Z = \Delta A + \Delta B$. Product, quotient, power: $\Delta Z / Z = \Sigma (\text{power} \times \Delta A / A)$
- Percentage error = (mean absolute error / mean value) × 100

A. Conversion of Units

Q1. Express 1 kilowatt-hour in joule.

- 1 kW = 1000 W and 1 h = 3600 s.
- 1 kWh = 1000 W × 3600 s = 3.6×10^6 W s.

Answer: 1 kWh = 3.6×10^6 J.

Q2. A car moves at 20 m/s. Express this in km/h.

- 1 m = 10^{-3} km and 1 s = (1/3600) h.
- 1 m/s = $10^{-3} \times 3600 = 3.6$ km/h.
- 20 m/s = $20 \times 3.6 = 72$ km/h.

Answer: 72 km/h.

Q3. Using dimensions, convert 1 erg into joule.

- Energy = $[M L^2 T^{-2}]$, so a = 1, b = 2, c = -2.
- $n_2 = 1 \times (1 \text{ g} / 1 \text{ kg})^1 \times (1 \text{ cm} / 1 \text{ m})^2 \times (1 \text{ s} / 1 \text{ s})^{-2} = 10^{-3} \times 10^{-4} = 10^{-7}$.

Answer: 1 erg = 10^{-7} J.

Q4. Young's modulus of steel is 2×10^{11} N/m². Express it in dyne/cm².

- Young's modulus = $[M L^{-1} T^{-2}]$.
- $n_2 = 2 \times 10^{11} \times (1000)^1 \times (100)^{-1} \times (1)^{-2} = 2 \times 10^{11} \times 10$.

Answer: 2×10^{12} dyne/cm².

Q5. Convert g = 9.8 m/s² into km/h².

- Acceleration = $[L T^{-2}]$. 1 m = 10^{-3} km and 1 s = 1/3600 h.
- $n_2 = 9.8 \times (10^{-3}) \times (1/3600)^{-2} = 9.8 \times 10^{-3} \times 1.296 \times 10^7 = 9.8 \times 12\,960 = 127\,008$.

Answer: g = 1.27×10^5 km/h².

Q6. The density of iron is 7.8 g/cm^3 . Express it in kg/m^3 .

- Density = $[M L^{-3}]$.
- $n_2 = 7.8 \times (10^{-3})^1 \times (10^{-2})^{-3} = 7.8 \times 10^{-3} \times 10^6$.

Answer: 7800 kg/m^3 .

Q7. A body has momentum 5 kg m/s . Find its value in g cm/s .

- Momentum = $[M L T^{-1}]$.
- $n_2 = 5 \times (1000)^1 \times (100)^1 \times (1)^{-1} = 5 \times 10^5$.

Answer: $5 \times 10^5 \text{ g cm/s}$.

Q8. Convert an angular momentum of $1 \text{ kg m}^2/\text{s}$ into CGS units.

- Angular momentum = $[M L^2 T^{-1}]$.
- $n_2 = 1 \times (1000)^1 \times (100)^2 \times (1)^{-1} = 10^3 \times 10^4$.

Answer: $10^7 \text{ g cm}^2/\text{s}$.

Q9. A new system uses units of mass 10 g , length 5 cm and time 2 s . Find the value of 1 dyne in this system.

- Force = $[M L T^{-2}]$. Old units: g, cm, s (CGS).
- $n_2 = 1 \times (1/10)^1 \times (1/5)^1 \times (1/2)^{-2} = 0.1 \times 0.2 \times 4 = 0.08$.

Answer: $1 \text{ dyne} = 0.08 \text{ new units}$.

Q10. A system has units of length 1 km , mass $1 \text{ quintal (100 kg)}$ and time 1 minute . Find 1 newton in this system.

- Force = $[M L T^{-2}]$. Old units: kg, m, s.
- $n_2 = 1 \times (1/100)^1 \times (1/1000)^1 \times (1/60)^{-2} = 10^{-5} \times 3600 = 0.036$.

Answer: $1 \text{ N} = 0.036 \text{ new units}$.

Q11. The viscosity of water is 0.001 Pa s . Express it in poise.

- Viscosity = $[M L^{-1} T^{-1}]$. From earlier work, $1 \text{ Pa s} = 10 \text{ poise}$.
- Check: $n_2 = 0.001 \times (1000)^1 \times (100)^{-1} = 0.001 \times 10 = 0.01$.

Answer: 0.01 poise .

Q12. A vehicle runs at 54 km/h . Express its speed in m/s and cm/s .

- $54 \text{ km/h} = 54 \times 1000 / 3600 = 15 \text{ m/s}$.
- $15 \text{ m/s} = 15 \times 100 = 1500 \text{ cm/s}$.

Answer: $15 \text{ m/s} = 1500 \text{ cm/s}$.

B. Dimensional Analysis

Q13. The displacement is $s = a + b t + c t^2$, where t is time. Find the dimensions of a , b and c .

- By homogeneity, each term has the dimensions of $s = [L]$.
- $[a] = [L]$.
- $[b t] = [L]$, so $[b] = [L] / [T] = [L T^{-1}]$.
- $[c t^2] = [L]$, so $[c] = [L] / [T^2] = [L T^{-2}]$.

Answer: $[a] = [L]$, $[b] = [L T^{-1}]$, $[c] = [L T^{-2}]$.

Q14. In $F = a v + b v^2$, F is force and v is velocity. Find the dimensions of a and b .

- $[a v] = [F]$, so $[a] = [M L T^{-2}] / [L T^{-1}] = [M T^{-1}]$.
- $[b v^2] = [F]$, so $[b] = [M L T^{-2}] / [L^2 T^{-2}] = [M L^{-1}]$.

Answer: $[a] = [M T^{-1}]$, $[b] = [M L^{-1}]$.

Q15. In $y = a \sin(b t)$, y is in metre and t is in second. Find the dimensions of a and b .

- The argument of sine must be dimensionless, so $[b t] = 1$ and $[b] = [T^{-1}]$.
- $\sin$ is dimensionless, so $[a] = [y] = [L]$.

Answer: $[a] = [L]$, $[b] = [T^{-1}]$.

Q16. Find the dimensions of the permittivity of free space ϵ_0 from $F = q_1 q_2 / (4\pi \epsilon_0 r^2)$.

- $\epsilon_0 = q_1 q_2 / (4\pi F r^2)$. Charge $q = [A T]$.
- $[\epsilon_0] = [A T]^2 / ([M L T^{-2}][L^2]) = [A^2 T^2] / [M L^3 T^{-2}]$.

Answer: $[\epsilon_0] = [M^{-1} L^{-3} T^4 A^2]$.

Q17. Check the de Broglie relation $\lambda = h / (m v)$ dimensionally.

- $[h] = [M L^2 T^{-1}]$ and $[m v] = [M][L T^{-1}] = [M L T^{-1}]$.
- $[h / m v] = [M L^2 T^{-1}] / [M L T^{-1}] = [L]$, the same as $[\lambda] = [L]$.

Answer: The relation is dimensionally correct.

Q18. Check whether the speed of a transverse wave on a string, $v = \sqrt{F / \mu}$, is dimensionally correct (F = tension, μ = mass per unit length).

- $[F / \mu] = [M L T^{-2}] / [M L^{-1}] = [L^2 T^{-2}]$.
- $\sqrt{[F / \mu]} = [L T^{-1}] = [v]$.

Answer: Dimensionally correct.

Q19. Check the relation $E = p^2 / (2m)$, where p is momentum.

- $[p^2 / m] = [M L T^{-1}]^2 / [M] = [M L^2 T^{-2}]$.
- $[E] = [M L^2 T^{-2}]$. The factor 2 is dimensionless.

Answer: Dimensionally correct.

Q20. Check the relation $f = (1 / 2\pi)\sqrt{g / l}$ for the frequency of a pendulum.

- $[g / l] = [L T^{-2}] / [L] = [T^{-2}]$, so $\sqrt{[g / l]} = [T^{-1}]$.
- $[f] = [T^{-1}]$.

Answer: Dimensionally correct (it is the correct formula).

Q21. Using dimensions, derive the time of fall t of a body dropped from height h , assuming $t = k h^a g^b$.

- $[T] = [L]^a [L T^{-2}]^b = [L^{a+b} T^{-2b}]$.
- Time: $-2b = 1$, so $b = -1/2$. Length: $a + b = 0$, so $a = 1/2$.

Answer: $t = k\sqrt{h / g}$; experiment gives $k = \sqrt{2}$, so $t = \sqrt{2h / g}$.

Q22. The range R of a projectile depends on the speed u and g (for a fixed angle). Derive $R = k u^a g^b$.

- $[L] = [L T^{-1}]^a [L T^{-2}]^b = [L^{a+b} T^{-a-2b}]$.
- Length: $a + b = 1$. Time: $a + 2b = 0$.
- Solving: $b = -1$ and $a = 2$.

Answer: $R = k u^2 / g$ (the exact result is $u^2 \sin 2\theta / g$).

Q23. The speed v of deep-water waves depends on wavelength λ and g . Derive $v = k \lambda^a g^b$.

- $[L T^{-1}] = [L]^a [L T^{-2}]^b = [L^{a+b} T^{-2b}]$.
- Time: $-2b = -1$, so $b = 1/2$. Length: $a + b = 1$, so $a = 1/2$.

Answer: $v = k\sqrt{g \lambda}$. Density does not appear.

C. Measuring Instruments and Least Count

Q24. In a vernier callipers 1 MSD = 0.5 mm and 10 VSD = 9 MSD. The zero of the vernier lies after the 30th main scale division and the 7th vernier division coincides. Find the reading.

- $LC = 1 \text{ MSD} / 10 = 0.5 / 10 = 0.05 \text{ mm}$.
- $MSR = 30 \times 0.5 = 15.0 \text{ mm}$.
- $\text{Reading} = 15.0 + 7 \times 0.05 = 15.35 \text{ mm}$.

Answer: $15.35 \text{ mm} = 1.535 \text{ cm}$.

Q25. A vernier has 50 VSD equal to 49 MSD (1 MSD = 1 mm). With jaws closed, the 2nd vernier division coincides. For a block, MSR = 35 mm and VSR = 18. Find the true length.

- $LC = 1 / 50 = 0.02 \text{ mm}$.
- $\text{Observed} = 35 + 18 \times 0.02 = 35.36 \text{ mm}$.
- $\text{Zero error} = +2 \times 0.02 = +0.04 \text{ mm}$.
- $\text{True length} = 35.36 - 0.04 = 35.32 \text{ mm}$.

Answer: 35.32 mm .

Q26. A vernier has 25 VSD equal to 24 MSD and 1 MSD = 0.5 mm. Find its least count.

- $LC = 1 \text{ MSD} / 25 = 0.5 / 25 = 0.02 \text{ mm}$.

Answer: $0.02 \text{ mm} = 0.002 \text{ cm}$.

Q27. A screw gauge has pitch 1 mm and 50 circular divisions. The reading for a rod is LSR = 4 mm and CSR = 29, with zero error +0.06 mm. Find the true diameter.

- $LC = 1 / 50 = 0.02 \text{ mm}$.
- $\text{Observed} = 4 + 29 \times 0.02 = 4.58 \text{ mm}$.
- $\text{True} = 4.58 - 0.06 = 4.52 \text{ mm}$.

Answer: 4.52 mm .

Q28. A screw gauge has pitch 0.5 mm and 100 divisions. For a wire LSR = 1.5 mm and CSR = 36. The zero of the circular scale is 3 divisions above the reference line when the studs touch. Find the diameter.

- $LC = 0.5 / 100 = 0.005 \text{ mm}$.
- $\text{Observed} = 1.5 + 36 \times 0.005 = 1.68 \text{ mm}$.
- $\text{Zero error} = -3 \times 0.005 = -0.015 \text{ mm}$.
- $\text{True} = 1.68 + 0.015 = 1.695 \text{ mm}$.

Answer: 1.695 mm .

Q29. The screw of a screw gauge advances 3 mm in 6 complete rotations and the circular scale has 60 divisions. Find the pitch and the least count.

- Pitch = 3 mm / 6 = 0.5 mm.
- LC = 0.5 / 60 = 0.0083 mm.

Answer: Pitch = 0.5 mm, LC = 0.0083 mm (about 0.01 mm).

Q30. The time for 20 oscillations of a pendulum is 36.4 s, measured with a stopwatch of least count 0.1 s. Find the time period with its error.

- $T = 36.4 / 20 = 1.82$ s.
- Error in T = LC / 20 = 0.1 / 20 = 0.005 s.
- Percentage error = $0.005 / 1.82 \times 100 = 0.27$ %.

Answer: $T = (1.820 \pm 0.005)$ s, 0.27 % error. Timing many oscillations reduces the error.

Q31. The diameter of a wire is measured as 0.50 mm using a screw gauge of least count 0.01 mm. Find the area of cross-section with error.

- $A = \pi d^2 / 4 = 3.142 \times 0.25 / 4 = 0.196$ mm².
- Relative error in A = $2 \times (0.01 / 0.50) = 0.04$ (4 %).
- $\Delta A = 0.04 \times 0.196 = 0.008$ mm².

Answer: $A = (0.196 \pm 0.008)$ mm².

D. Errors and Error Propagation

Q32. Six readings of a length are 5.21, 5.23, 5.18, 5.22, 5.20 and 5.19 cm. Find the mean, mean absolute error, relative error and percentage error.

- Sum = 31.23, so mean = $31.23 / 6 = 5.205$ cm.
- Absolute errors: 0.005, 0.025, 0.025, 0.015, 0.005, 0.015 cm.
- Mean absolute error = $0.09 / 6 = 0.015$ cm.
- Relative error = $0.015 / 5.205 = 0.0029$; percentage error = 0.29 %.

Answer: $L = (5.205 \pm 0.015)$ cm, relative error 0.0029, percentage error 0.29 %.

Q33. The periods of a pendulum are 1.82, 1.85, 1.80, 1.83 and 1.85 s. Find the mean, mean absolute error and percentage error.

- Mean = $9.15 / 5 = 1.83$ s.
- Absolute errors: 0.01, 0.02, 0.03, 0.00, 0.02 s.
- Mean absolute error = $0.08 / 5 = 0.016$ s (about 0.02 s).
- Percentage error = $0.016 / 1.83 \times 100 = 0.87$ %.

Answer: $T = (1.83 \pm 0.02)$ s, 0.87 % error.

Q34. Two temperatures are (30.0 ± 0.5) °C and (80.0 ± 0.5) °C. Find their difference with error.

- Difference = $80.0 - 30.0 = 50.0$ °C.
- Error = $0.5 + 0.5 = 1.0$ °C (errors add even for a difference).

Answer: (50.0 ± 1.0) °C.

Q35. Two resistors $R_1 = (100 \pm 3) \Omega$ and $R_2 = (200 \pm 4) \Omega$ are connected (a) in series and (b) in parallel. Find the equivalent resistance with error.

- (a) $R_s = 300 \Omega$ and $\Delta R_s = 3 + 4 = 7 \Omega$.
- (b) $R_p = R_1 R_2 / (R_1 + R_2) = 20\,000 / 300 = 66.7 \Omega$.
- From $1/R_p = 1/R_1 + 1/R_2$: $\Delta R_p / R_p^2 = \Delta R_1 / R_1^2 + \Delta R_2 / R_2^2 = 3/10\,000 + 4/40\,000 = 0.0004$.
- $\Delta R_p = (66.7)^2 \times 0.0004 = 4444 \times 0.0004 = 1.8 \Omega$.

Answer: Series: $(300 \pm 7) \Omega$. Parallel: $(66.7 \pm 1.8) \Omega$.

Q36. A sphere has mass $(40.0 \pm 0.1) \text{ g}$ and radius $(2.00 \pm 0.02) \text{ cm}$. Find its density with error.

- $V = (4/3)\pi r^3 = 4.189 \times 8.00 = 33.51 \text{ cm}^3$, so $\rho = 40.0 / 33.51 = 1.194 \text{ g/cm}^3$.
- Relative error $= 0.1 / 40.0 + 3 \times (0.02 / 2.00) = 0.0025 + 0.030 = 0.0325$.
- $\Delta \rho = 0.0325 \times 1.194 = 0.039 \text{ g/cm}^3$.

Answer: $\rho = (1.19 \pm 0.04) \text{ g/cm}^3$, about 3.3 % error.

Q37. In the formula $Y = 4 F L / (\pi d^2 l)$, the percentage errors in F, L, d and l are 1 %, 0.5 %, 1 % and 2 %. Find the percentage error in Y.

- $\Delta Y / Y = \Delta F / F + \Delta L / L + 2 (\Delta d / d) + \Delta l / l$.
- $= 1 + 0.5 + 2 \times 1 + 2 = 5.5 \%$.

Answer: 5.5 %.

Q38. A ball is dropped from $h = (1.20 \pm 0.01) \text{ m}$ and takes $t = (0.50 \pm 0.01) \text{ s}$. Find $g = 2h / t^2$ with error.

- $g = 2 \times 1.20 / (0.50)^2 = 2.40 / 0.25 = 9.60 \text{ m/s}^2$.
- Relative error $= 0.01 / 1.20 + 2 \times (0.01 / 0.50) = 0.0083 + 0.0400 = 0.0483$.
- $\Delta g = 0.0483 \times 9.60 = 0.46 \text{ m/s}^2$.

Answer: $g = (9.60 \pm 0.46) \text{ m/s}^2$.

Q39. The power dissipated is $P = V^2 / R$. V is measured with 1 % error and R with 2 % error. Find the percentage error in P.

- $\Delta P / P = 2 (\Delta V / V) + \Delta R / R = 2 \times 1 + 2 = 4 \%$.

Answer: 4 %.

Q40. The moment of inertia of a disc is $I = \frac{1}{2} M R^2$. M has 1 % error and R has 0.5 % error. Find the percentage error in I.

- $\Delta I / I = \Delta M / M + 2 (\Delta R / R) = 1 + 2 \times 0.5 = 2 \%$.

Answer: 2 %.

Q41. $X = \sqrt{a} \times b^3 / (c^2 d)$. The percentage errors in a, b, c and d are 4 %, 1 %, 2 % and 3 %. Find the maximum percentage error in X.

- $\Delta X / X = \frac{1}{2} (4 \%) + 3 (1 \%) + 2 (2 \%) + 1 (3 \%) = 2 + 3 + 4 + 3 = 12 \%$.

Answer: 12 %.

Q42. The radius of a circle is $(5.0 \pm 0.1) \text{ cm}$. Find its area with error.

- $A = \pi r^2 = 3.142 \times 25.0 = 78.5 \text{ cm}^2$.
- Relative error $= 2 \times (0.1 / 5.0) = 0.04$.
- $\Delta A = 0.04 \times 78.5 = 3.1 \text{ cm}^2$.

Answer: $A = (78.5 \pm 3.1) \text{ cm}^2$.

Q43. A tuning fork has frequency $f = (500 \pm 5)$ Hz and the wavelength of its sound is $\lambda = (0.66 \pm 0.01)$ m. Find the speed of sound $v = f \lambda$ with error.

- $v = 500 \times 0.66 = 330$ m/s.
- Relative error = $5 / 500 + 0.01 / 0.66 = 0.0100 + 0.0152 = 0.0252$.
- $\Delta v = 0.0252 \times 330 = 8.3$ m/s.

Answer: $v = (330 \pm 8)$ m/s.

Q44. A 2.00 m table is measured with a metre rod of least count 1 mm. Find the percentage error.

- Absolute error = least count = 1 mm = 0.001 m.
- Percentage error = $0.001 / 2.00 \times 100 = 0.05 \%$.

Answer: 0.05 %.

E. Significant Figures

Q45. Add $1.234 + 12.3 + 0.12$ and give the answer with correct significant figures.

- Sum = 13.654.
- The least precise term (12.3) has 1 decimal place, so round to 1 decimal place.

Answer: 13.7.

Q46. Subtract 3.4 from 15.60.

- $15.60 - 3.4 = 12.20$.
- 3.4 has 1 decimal place, so the result has 1 decimal place.

Answer: 12.2.

Q47. Multiply 2.50 by 4.1.

- $2.50 \times 4.1 = 10.25$.
- 4.1 has 2 significant figures, so the result has 2 significant figures.

Answer: 10 (write as 1.0×10^1).

Q48. Divide 22.4 by 0.0160.

- $22.4 / 0.0160 = 1400$.
- Both numbers have 3 significant figures, so keep 3.

Answer: 1.40×10^3 .

Q49. A rectangle measures 12.5 cm by 3.2 cm. Find its area with correct significant figures.

- $A = 12.5 \times 3.2 = 40.00$ cm².
- 3.2 has 2 significant figures, so the result has 2.

Answer: 4.0×10^1 cm² (about 40 cm²).

Q50. The radius of a circle is 3.25 cm. Find the circumference with correct significant figures.

- $C = 2\pi r = 2 \times 3.1416 \times 3.25 = 20.42$ cm.
- The factor 2 is exact, so 3.25 (3 significant figures) decides.

Answer: 20.4 cm.

Q51. A metal block has mass 22.50 g and volume 8.0 cm³. Find its density.

- $\rho = 22.50 / 8.0 = 2.8125 \text{ g/cm}^3$.
- 8.0 has 2 significant figures.

Answer: 2.8 g/cm³.

Q52. Write 0.0001230 in scientific notation and state its number of significant figures.

- Leading zeros are not significant; the trailing zero after the decimal is significant.
- $0.0001230 = 1.230 \times 10^{-4}$.

Answer: 1.230×10^{-4} , with 4 significant figures.

F. Mixed Exam-Type Problems

Q53. A student wants to convert the viscosity 0.2 poise into SI and also check its dimensions as [M L⁻¹ T⁻¹]. Find the SI value.

- 1 poise = 0.1 Pa s.
- 0.2 poise = $0.2 \times 0.1 = 0.02 \text{ Pa s}$.

Answer: 0.02 Pa s (dimensions [M L⁻¹ T⁻¹]).

Q54. The period of a spring-mass system is $T = 2\pi\sqrt{m / k_s}$. With $m = (0.250 \pm 0.005) \text{ kg}$ and $T = (0.50 \pm 0.01) \text{ s}$, find $k_s = 4\pi^2 m / T^2$ with error.

- $k_s = 4 \times 9.870 \times 0.250 / 0.25 = 39.5 \text{ N/m}$.
- Relative error = $0.005 / 0.250 + 2 \times (0.01 / 0.50) = 0.020 + 0.040 = 0.060$.
- $\Delta k_s = 0.060 \times 39.5 = 2.4 \text{ N/m}$.

Answer: $k_s = (39.5 \pm 2.4) \text{ N/m}$, 6 % error.

Q55. Show that the quantity (mass $\times$ velocity² / length) has the dimensions of force, and convert 1 such unit in CGS to SI.

- $[m v^2 / l] = [M][L^2 T^{-2}] / [L] = [M L T^{-2}] = [\text{force}]$.
- So it converts like force: 1 dyne = 10^{-5} N .

Answer: It has the dimensions of force; 1 CGS unit = 10^{-5} SI unit.