

APPLIED PHYSICS 1

Chapter: Work, Power and Energy

Proofs, Derivations and Numerical Problems with Solution Steps

(Exam-oriented practice for Engineering / Diploma students)

Contents

1. **Part A: Proofs and Derivations** (14 proofs): work formula, $\mu = \tan\phi$, angle of repose, work on rough incline, KE, PE, work-energy theorem, conservation of energy, $P = Fv$, kWh and more.
2. **Part B: Numerical Problems** (44 problems): Work, Friction, Rough Inclined Plane, Energy, Power. Each is solved as Given, Formula, Steps, Answer.

Tips

- Unless stated otherwise take $g = 9.8 \text{ m/s}^2$.
- Write the formula first, substitute with units, and underline the final answer.
- For proofs: draw a neat labelled diagram, state what is given, and write each step on a new line.

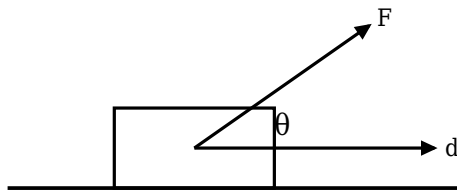
Formulae at a Glance

Quantity	Formula	SI Unit
Work	$W = F d \cos\theta$	joule (J)
Limiting friction; coefficient	$F = \mu R$; $\mu = F / R = \tan\phi = \tan\alpha$	N; no unit
Force up a rough incline	$F = mg \sin\theta + \mu mg \cos\theta$	newton (N)
Force down a rough incline	$F = mg \sin\theta - \mu mg \cos\theta$	newton (N)
Kinetic energy	$KE = \frac{1}{2} mv^2 = p^2 / 2m$	joule (J)
Potential energy	$PE = mgh$	joule (J)
Work-energy theorem	$W = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$	joule (J)
Speed of a freely falling body	$v = \sqrt{2gh}$	m/s
Power	$P = W / t = F v$	watt (W)
Efficiency	$\eta = (\text{output} / \text{input}) \times 100 \%$	percent
Conversions	$1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$; $1 \text{ HP} = 746 \text{ W}$; $1 \text{ J} = 10^7 \text{ erg}$	

PART A: PROOFS AND DERIVATIONS

Proof 1. Derive the expression for work done, $W = F d \cos\theta$. (4 marks)

$W = F d \cos\theta$ ($F \cos\theta$ is the component of force along displacement)



$\theta < 90^\circ \rightarrow W$ positive

$\theta = 90^\circ \rightarrow W$ zero

$\theta > 90^\circ$ (up to 180°) $\rightarrow W$ negative

Given: A constant force F acts on a body at an angle θ to the direction of displacement d .

1. Resolve F into two perpendicular components: $F \cos\theta$ along the displacement and $F \sin\theta$ perpendicular to it.
2. The perpendicular component $F \sin\theta$ causes no displacement in its own direction, so it does no work.
3. Only $F \cos\theta$ moves the body through d , so work done = (component of force along displacement) $\times$ (displacement).
4. Therefore **$W = (F \cos\theta) \times d = F d \cos\theta$** .

Special cases: $\theta = 0^\circ$ gives $W = F d$ (maximum, positive); $\theta = 90^\circ$ gives $W = 0$; $\theta = 180^\circ$ gives $W = -F d$ (negative).

Proof 2. Show that the coefficient of friction is equal to the tangent of the angle of friction, $\mu = \tan\phi$. (5 marks)

Given: A body of mass m rests on a rough horizontal surface and is just about to slide under an applied force P .

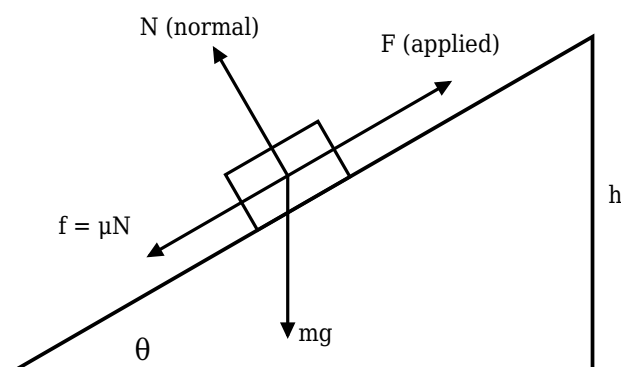
1. Forces at the contact: normal reaction R (perpendicular to the surface) and limiting friction F (along the surface, opposing motion).
2. The resultant of R and F is S . The angle between S and R is defined as the **angle of friction ϕ** .
3. In the right-angled triangle formed by R , F and S : $\tan\phi = F / R$ (opposite side / adjacent side).
4. By definition of the coefficient of limiting friction, $\mu = F / R$.
5. Comparing steps 3 and 4: **$\mu = \tan\phi$** . Also $S = \sqrt{R^2 + F^2} = R\sqrt{1 + \mu^2}$.

Proof 3. Define angle of repose and show that it is equal to the angle of friction. (5 marks)

Angle of repose (α) is the minimum angle of inclination of a plane at which a body placed on it just begins to slide down.

1. Let a body of mass m rest on a plane inclined at angle α , on the verge of sliding.
2. Resolve the weight mg : **$mg \sin\alpha$** along the plane (downward) and **$mg \cos\alpha$** perpendicular to the plane.
3. Normal reaction $R = mg \cos\alpha$. At the verge of sliding, limiting friction F (up the plane) balances the downward component: $F = mg \sin\alpha$.
4. $\mu = F / R = (mg \sin\alpha) / (mg \cos\alpha) = \mathbf{\tan\alpha}$.
5. Since $\mu = \tan\phi$ (Proof 2), $\tan\alpha = \tan\phi$, hence **$\alpha = \phi$** . Angle of repose equals angle of friction.

Proof 4. Derive an expression for the work done in pulling a body up a rough inclined plane. (7 marks)



(block of mass m pulled up a rough incline of angle θ)

Given: mass m , incline angle θ , coefficient of friction μ , distance moved along the plane d , force F applied parallel to the plane, constant speed.

1. Resolve weight mg : **$mg \sin\theta$** down the plane and **$mg \cos\theta$** perpendicular to the plane.
2. No motion perpendicular to the plane, so normal reaction **$N = mg \cos\theta$** .
3. Friction acts down the plane: **$f = \mu N = \mu mg \cos\theta$** .
4. Constant speed means zero net force along the plane: **$F = mg \sin\theta + \mu mg \cos\theta$** .
5. Work done by F through distance d : **$W = F d = mg (\sin\theta + \mu \cos\theta) d$** .

$$\mathbf{W = mg (\sin\theta + \mu \cos\theta) d}$$

Work against gravity = $mg \sin\theta \times d = mgh$; Work against friction = $\mu mg \cos\theta \times d$

Proof 5. Find the force required to move a body down a rough inclined plane at constant speed. (4 marks)

1. While moving down, friction acts up the plane: $f = \mu mg \cos\theta$. The weight component $mg \sin\theta$ acts down the plane.
2. Net unbalanced force down the plane = **$mg \sin\theta - \mu mg \cos\theta$** . For constant speed the net force must be zero.
3. If $\tan\theta = \mu$, the net force is zero and the body slides at constant speed with no external force (angle of repose).
4. If $\tan\theta > \mu$, the body accelerates down; a holding force **$F = mg \sin\theta - \mu mg \cos\theta$** must be applied up the plane to keep constant speed.
5. If $\tan\theta < \mu$, a push **$F = \mu mg \cos\theta - mg \sin\theta$** must be applied down the plane to keep the body moving.

Proof 6. Show that the work done against gravity in moving a body up a smooth incline depends only on the vertical height. (3 marks)

1. For a smooth plane $\mu = 0$, so $F = mg \sin\theta$ (from Proof 4).
2. Work $W = F d = mg \sin\theta \times d$.
3. The vertical height gained is $h = d \sin\theta$.
4. Hence **$W = mgh$** , which is independent of the length or angle of the incline. It is the same as lifting the body vertically through h .

Proof 7. Derive the expression for kinetic energy, $KE = \frac{1}{2} mv^2$. (5 marks)

Given: A body of mass m , initially at rest ($u = 0$), is acted on by a constant force F , gains acceleration a , and reaches velocity v after distance s .

1. Equation of motion: $v^2 = u^2 + 2as = 2as$, so $s = v^2 / 2a$.
2. Newton's second law: $F = ma$.
3. Work done: $W = F s = ma \times (v^2 / 2a) = \frac{1}{2} mv^2$.
4. This work is stored as the energy of motion. Hence **$KE = \frac{1}{2} mv^2$** .

Proof 8. Derive the expression for gravitational potential energy, $PE = mgh$. (4 marks)

1. A body of mass m is raised slowly (no acceleration) from the ground to a height h .
2. The lifting force equals the weight: $F = mg$, acting in the direction of displacement ($\theta = 0^\circ$).
3. Work done: $W = F h \cos 0^\circ = mg \times h$.
4. This work is stored in the body as potential energy. Hence **$PE = mgh$** (taking $PE = 0$ at ground).

Proof 9. State and prove the work-energy theorem. (5 marks)

Statement: The net work done on a body by all forces is equal to the change in its kinetic energy.

1. Let a net constant force F act on a body of mass m , changing its velocity from u to v over a distance s with acceleration a .
2. From $v^2 = u^2 + 2as$, we get $as = (v^2 - u^2) / 2$.
3. Work done: $W = F s = m a s = m (v^2 - u^2) / 2$.
4. Hence **$W = \frac{1}{2} mv^2 - \frac{1}{2} mu^2 = KE_{\text{final}} - KE_{\text{initial}}$** .

Proof 10. Prove the law of conservation of mechanical energy for a freely falling body. (7 marks)

Statement: For a body moving only under gravity, the sum of kinetic and potential energy is constant at every point of its path.

Given: mass m dropped ($u = 0$) from point A at height h . B is a point at distance x below A. C is the ground.

1. **At A:** $v = 0$, $KE = 0$, $PE = mgh$. Total $E_A = mgh$.
2. **At B:** $v^2 = 2gx$, so $KE = \frac{1}{2} m (2gx) = mgx$. Height = $h - x$, so $PE = mg(h - x)$. Total $E_B = mgx + mg(h - x) = mgh$.
3. **At C:** $x = h$, $v^2 = 2gh$, $KE = \frac{1}{2} m (2gh) = mgh$, $PE = 0$. Total $E_C = mgh$.
4. Since $E_A = E_B = E_C$, total mechanical energy is conserved. Equating KE at C with PE at A: $mgh = \frac{1}{2} mv^2$, giving **$v = \sqrt{2gh}$** , independent of mass.

Position	Height	KE	PE	Total
A (start)	h	0	mgh	mgh
B (fallen x)	$h - x$	mgx	$mg(h - x)$	mgh
Midpoint	$h / 2$	$mgh / 2$	$mgh / 2$	mgh
C (ground)	0	mgh	0	mgh

Proof 11. Show that a body thrown vertically upward with speed u reaches a maximum height $h = u^2 / 2g$. (3 marks)

1. At the point of projection (ground): $KE = \frac{1}{2} mu^2$, $PE = 0$.
2. At the highest point: $v = 0$, so $KE = 0$ and $PE = mgh$.
3. By conservation of mechanical energy: $\frac{1}{2} mu^2 = mgh$.
4. Therefore **$h = u^2 / 2g$** .

Proof 12. Show that $KE = p^2 / 2m$, where p is momentum. (3 marks)

1. $KE = \frac{1}{2} mv^2$. Multiply and divide by m : $KE = m^2 v^2 / 2m$.
2. Momentum $p = mv$, so $p^2 = m^2 v^2$.
3. Hence **$KE = p^2 / 2m$** . For equal momentum, the lighter body has greater KE.

Proof 13. Establish the relation between power and velocity, $P = F v$. (3 marks)

1. Power $P = W / t$.
2. For a constant force F acting along the displacement s , $W = F s$.
3. $P = F s / t = F (s / t)$.
4. Since $s / t = v$ (velocity), **$P = F v$** . At constant power, larger velocity means smaller force (hence low gear for climbing a hill).

Proof 14. Show that 1 kilowatt-hour = 3.6×10^6 joule. (2 marks)

1. $1 \text{ kWh} = 1 \text{ kW} \times 1 \text{ h} = 1000 \text{ W} \times 3600 \text{ s}$.
2. $1 \text{ W} = 1 \text{ J/s}$, so energy = $1000 \times 3600 \text{ J} = 3.6 \times 10^6 \text{ J}$.

PART B: NUMERICAL PROBLEMS

B1. Work

W1. A force of 50 N acts on a body at 60° to the horizontal and moves it 10 m horizontally. Find the work done.

Given: $F = 50 \text{ N}$, $\theta = 60^\circ$, $d = 10 \text{ m}$

Solution:

$$W = F d \cos\theta = 50 \times 10 \times \cos 60^\circ$$

$$W = 500 \times 0.5 = 250 \text{ J}$$

Answer: 250 J

W2. A man lifts a 5 kg bag to a height of 2 m. Calculate the work done against gravity.

Given: $m = 5 \text{ kg}$, $h = 2 \text{ m}$

Solution:

$$\text{Force} = mg = 5 \times 9.8 = 49 \text{ N}; \theta = 0^\circ$$

$$W = mgh = 49 \times 2 = 98 \text{ J}$$

Answer: 98 J

W3. A porter carries a 20 kg load and walks 100 m on a level road. How much work is done on the load?

Given: $m = 20 \text{ kg}$, $d = 100 \text{ m}$

Solution:

Weight acts vertically, displacement is horizontal, so $\theta = 90^\circ$

$$W = F d \cos 90^\circ = 0$$

Answer: Zero

W4. A frictional force of 20 N acts on a block which slides 5 m. Find the work done by friction.

Given: $f = 20 \text{ N}$, $d = 5 \text{ m}$, $\theta = 180^\circ$

Solution:

$$W = f d \cos 180^\circ = 20 \times 5 \times (-1) = -100 \text{ J}$$

Answer: -100 J (negative work)

W5. A force of 100 N moves a body through 4 m and does 200 J of work. Find the angle between force and displacement.

Given: $F = 100 \text{ N}$, $d = 4 \text{ m}$, $W = 200 \text{ J}$

Solution:

$$\cos\theta = W / (F d) = 200 / 400 = 0.5$$

$$\theta = \cos^{-1}(0.5) = 60^\circ$$

Answer: 60°

W6. A force of 40 N, acting at 30° above the horizontal, drags a sledge 15 m along level ground. Find the work done.

Given: $F = 40 \text{ N}$, $\theta = 30^\circ$, $d = 15 \text{ m}$

Solution:

$$W = F d \cos\theta = 40 \times 15 \times \cos 30^\circ$$

$$W = 600 \times 0.866 = 519.6 \text{ J}$$

Answer: 519.6 J

W7. A 2 kg block is raised from the ground to a height of 10 m. Find the work done by the lifting force.

Given: $m = 2 \text{ kg}$, $h = 10 \text{ m}$

Solution:

$$W = mgh = 2 \times 9.8 \times 10 = 196 \text{ J}$$

Answer: 196 J

W8. A 3 kg ball falls 5 m, and later is thrown up through 5 m. Find the work done by gravity in each case.

Given: $m = 3 \text{ kg}$, $d = 5 \text{ m}$

Solution:

$$\text{Weight} = mg = 3 \times 9.8 = 29.4 \text{ N}$$

$$\text{Falling: force and displacement in same direction, } W = 29.4 \times 5 = +147 \text{ J}$$

$$\text{Rising: force opposite to displacement } (\theta = 180^\circ), W = -147 \text{ J}$$

Answer: Falling: +147 J; rising: -147 J

B2. Friction

F1. A force of 29.4 N just slides a 10 kg block on a horizontal table. Find the coefficient of friction.

Given: $F = 29.4 \text{ N}$, $m = 10 \text{ kg}$

Solution:

$$R = mg = 10 \times 9.8 = 98 \text{ N}$$

$$\mu = F / R = 29.4 / 98 = 0.3$$

Answer: $\mu = 0.3$

F2. A 20 kg body rests on a surface with $\mu = 0.25$. Find the minimum force needed to move it.

Given: $m = 20 \text{ kg}$, $\mu = 0.25$

Solution:

$$R = mg = 196 \text{ N}$$

$$F = \mu R = 0.25 \times 196 = 49 \text{ N}$$

Answer: 49 N

F3. Find the angle of friction if $\mu = 0.577$.

Given: $\mu = 0.577$

Solution:

$$\tan\phi = \mu = 0.577$$

$$\phi = \tan^{-1}(0.577) = 30^\circ$$

Answer: 30°

F4. A block starts sliding down a plank tilted at 25° . Find the coefficient of friction.

Given: Angle of repose $\alpha = 25^\circ$

Solution:

$$\mu = \tan\alpha = \tan 25^\circ = 0.466$$

Answer: $\mu \approx 0.47$

F5. A 5 kg block is pulled on a floor by a 20 N force at constant speed. Find μ .

Given: $m = 5$ kg, applied force = 20 N

Solution:

Constant speed, so friction = 20 N

$$R = 5 \times 9.8 = 49 \text{ N}$$

$$\mu = 20 / 49 = 0.41$$

Answer: $\mu \approx 0.41$

F6. The coefficient of friction between a block and a plane is 0.36. Find the angle of repose.

Given: $\mu = 0.36$

Solution:

$$\tan\alpha = \mu = 0.36$$

$$\alpha = \tan^{-1}(0.36) = 19.8^\circ$$

Answer: $\approx 19.8^\circ$

F7. A 15 kg box is dragged 6 m along a floor with $\mu = 0.4$ at constant speed. Find the force applied and the work done.

Given: $m = 15$ kg, $\mu = 0.4$, $d = 6$ m

Solution:

$$R = mg = 15 \times 9.8 = 147 \text{ N}$$

$$F = \mu R = 0.4 \times 147 = 58.8 \text{ N}$$

$$W = F d = 58.8 \times 6 = 352.8 \text{ J}$$

Answer: Force = 58.8 N; work = 352.8 J

F8. The limiting friction is 60 N when the normal reaction is 200 N. Find μ , and the limiting friction if the normal reaction becomes 300 N.

Given: $F_1 = 60 \text{ N}$, $R_1 = 200 \text{ N}$, $R_2 = 300 \text{ N}$

Solution:

$$\mu = 60 / 200 = 0.3$$

$$F_2 = \mu R_2 = 0.3 \times 300 = 90 \text{ N}$$

Answer: $\mu = 0.3$; $F_2 = 90 \text{ N}$

B3. Rough Inclined Plane

I1. A 10 kg body is pulled 5 m up a rough plane inclined at 30° , with $\mu = 0.2$. Find (i) force required, (ii) total work, (iii) work against gravity, (iv) work against friction.

Given: $m = 10 \text{ kg}$, $\theta = 30^\circ$, $d = 5 \text{ m}$, $\mu = 0.2$

Solution:

$$mg = 98 \text{ N}$$

$$(i) F = mg \sin \theta + \mu mg \cos \theta = 98 \times 0.5 + 0.2 \times 98 \times 0.866 = 49 + 16.97 = 65.97 \text{ N}$$

$$(ii) W = F d = 65.97 \times 5 = 329.9 \text{ J}$$

$$(iii) \text{ Against gravity} = 49 \times 5 = 245 \text{ J}$$

$$(iv) \text{ Against friction} = 16.97 \times 5 = 84.9 \text{ J (check: } 245 + 84.9 = 329.9 \text{ J)}$$

Answer: $F = 65.97 \text{ N}$; $W = 329.9 \text{ J}$; gravity **245 J**; friction **84.9 J**

I2. A 20 kg box is pushed up a 10 m long ramp rising 5 m. If $\mu = 0.25$, find the work done.

Given: $m = 20 \text{ kg}$, $d = 10 \text{ m}$, $h = 5 \text{ m}$, $\mu = 0.25$

Solution:

$$\sin \theta = 5/10 = 0.5, \theta = 30^\circ, \cos \theta = 0.866; mg = 196 \text{ N}$$

$$F = 196 \times 0.5 + 0.25 \times 196 \times 0.866 = 98 + 42.4 = 140.4 \text{ N}$$

$$W = 140.4 \times 10 = 1404 \text{ J}$$

Answer: $\approx 1404 \text{ J}$

I3. Find the work done in pulling a 50 kg body 4 m up a smooth plane inclined at 30° .

Given: $m = 50 \text{ kg}$, $d = 4 \text{ m}$, $\theta = 30^\circ$, $\mu = 0$

Solution:

$$W = mg \sin \theta \times d = 50 \times 9.8 \times 0.5 \times 4 = 980 \text{ J}$$

$$\text{Check: } h = 4 \times 0.5 = 2 \text{ m, } mgh = 50 \times 9.8 \times 2 = 980 \text{ J}$$

Answer: **980 J**

14. A 2 kg body slides 3 m down a rough plane inclined at 30° with $\mu = 0.2$. Find the work done by friction.

Given: $m = 2 \text{ kg}$, $d = 3 \text{ m}$, $\theta = 30^\circ$, $\mu = 0.2$

Solution:

$$f = \mu mg \cos\theta = 0.2 \times 2 \times 9.8 \times 0.866 = 3.39 \text{ N}$$

$$\text{Friction opposes motion: } W = -f d = -3.39 \times 3 = -10.2 \text{ J}$$

Answer: -10.2 J

15. An 8 kg block slides 6 m down a smooth plane inclined at 30° . Find the work done by gravity.

Given: $m = 8 \text{ kg}$, $d = 6 \text{ m}$, $\theta = 30^\circ$

Solution:

$$\text{Component along plane} = mg \sin\theta = 8 \times 9.8 \times 0.5 = 39.2 \text{ N}$$

$$W = 39.2 \times 6 = 235.2 \text{ J}$$

$$\text{Check: } h = 6 \times 0.5 = 3 \text{ m, } mgh = 8 \times 9.8 \times 3 = 235.2 \text{ J}$$

Answer: 235.2 J

16. A 100 kg drum is pushed up a 6 m ramp to a platform 2 m high. If $\mu = 0.1$, find the force required and the work done.

Given: $m = 100 \text{ kg}$, $d = 6 \text{ m}$, $h = 2 \text{ m}$, $\mu = 0.1$

Solution:

$$\sin\theta = 2/6 = 1/3, \cos\theta = \sqrt{1 - 1/9} = 0.943; mg = 980 \text{ N}$$

$$F = mg \sin\theta + \mu mg \cos\theta = 980/3 + 0.1 \times 980 \times 0.943 = 326.7 + 92.4 = 419.1 \text{ N}$$

$$W = F d = 419.1 \times 6 = 2514 \text{ J}$$

Answer: $F = 419.1 \text{ N}$; $W = \approx 2514 \text{ J}$

B4. Energy

E1. Find the kinetic energy of a 2 kg body moving at 10 m/s.

Given: $m = 2 \text{ kg}$, $v = 10 \text{ m/s}$

Solution:

$$KE = \frac{1}{2} mv^2 = \frac{1}{2} \times 2 \times 100 = 100 \text{ J}$$

Answer: 100 J

E2. Find the potential energy of a 5 kg mass at a height of 20 m.

Given: $m = 5 \text{ kg}$, $h = 20 \text{ m}$

Solution:

$$PE = mgh = 5 \times 9.8 \times 20 = 980 \text{ J}$$

Answer: 980 J

E3. A ball is dropped from 20 m. Find its velocity on reaching the ground.

Given: $h = 20 \text{ m}$, $u = 0$

Solution:

$$mgh = \frac{1}{2} mv^2, \text{ so } v = \sqrt{2gh}$$

$$v = \sqrt{2 \times 9.8 \times 20} = \sqrt{392} = 19.8 \text{ m/s}$$

Answer: 19.8 m/s

E4. A 2 kg body is dropped from 50 m. Find KE and PE at 30 m above the ground and verify conservation of energy.

Given: $m = 2 \text{ kg}$, $H = 50 \text{ m}$, $h = 30 \text{ m}$, fallen $x = 20 \text{ m}$

Solution:

$$\text{Total energy at top} = mgH = 2 \times 9.8 \times 50 = 980 \text{ J}$$

$$\text{PE at 30 m} = 2 \times 9.8 \times 30 = 588 \text{ J}$$

$$\text{KE} = mgx = 2 \times 9.8 \times 20 = 392 \text{ J}$$

$$\text{KE} + \text{PE} = 392 + 588 = 980 \text{ J. Verified.}$$

Answer: KE = 392 J; PE = 588 J; total 980 J

E5. The velocity of a 4 kg body increases from 5 m/s to 10 m/s. Find the increase in KE.

Given: $m = 4 \text{ kg}$, $u = 5 \text{ m/s}$, $v = 10 \text{ m/s}$

Solution:

$$\text{Initial KE} = \frac{1}{2} \times 4 \times 25 = 50 \text{ J}$$

$$\text{Final KE} = \frac{1}{2} \times 4 \times 100 = 200 \text{ J}$$

$$\text{Increase} = 150 \text{ J}$$

Answer: 150 J

E6. A 2 kg body has 400 J of kinetic energy. Find its velocity.

Given: $m = 2 \text{ kg}$, $\text{KE} = 400 \text{ J}$

Solution:

$$v^2 = 2 \text{ KE} / m = 800 / 2 = 400$$

$$v = 20 \text{ m/s}$$

Answer: 20 m/s

E7. A 0.5 kg stone is thrown upward at 20 m/s. Find the maximum height.

Given: $m = 0.5 \text{ kg}$, $u = 20 \text{ m/s}$

Solution:

$$\frac{1}{2} mu^2 = mgh, \text{ so } h = u^2 / 2g$$

$$h = 400 / 19.6 = 20.4 \text{ m}$$

Answer: 20.4 m

E8. Find the kinetic energy of a 1000 kg car moving at 72 km/h.

Given: $m = 1000 \text{ kg}$, $v = 72 \text{ km/h}$

Solution:

$$v = 72 \times 1000 / 3600 = 20 \text{ m/s}$$

$$\text{KE} = \frac{1}{2} \times 1000 \times 20^2 = 200000 \text{ J}$$

Answer: $2 \times 10^5 \text{ J}$

E9. A body is dropped from 80 m. Find its speed when it is 20 m above the ground.

Given: $H = 80 \text{ m}$, $h = 20 \text{ m}$, $u = 0$

Solution:

$$\text{Distance fallen } x = 80 - 20 = 60 \text{ m}$$

$$v = \sqrt{(2gx)} = \sqrt{(2 \times 9.8 \times 60)} = \sqrt{1176} = 34.3 \text{ m/s}$$

Answer: 34.3 m/s

E10. A 0.2 kg ball dropped from 10 m rebounds to 6 m. Find the energy lost in the impact.

Given: $m = 0.2 \text{ kg}$, $h_1 = 10 \text{ m}$, $h_2 = 6 \text{ m}$

Solution:

$$\text{Energy lost} = mg(h_1 - h_2) = 0.2 \times 9.8 \times 4 = 7.84 \text{ J}$$

Answer: 7.84 J

E11. A 0.02 kg bullet moving at 300 m/s penetrates 0.1 m into a wooden block and stops. Find the average resisting force.

Given: $m = 0.02 \text{ kg}$, $u = 300 \text{ m/s}$, $v = 0$, $s = 0.1 \text{ m}$

Solution:

Work-energy theorem: work done by resistance = change in KE

$$\text{KE lost} = \frac{1}{2} \times 0.02 \times 300^2 = 900 \text{ J}$$

$$F \times s = 900, \text{ so } F = 900 / 0.1 = 9000 \text{ N}$$

Answer: 9000 N

B5. Power

P1. A motor lifts a 100 kg load to 20 m in 10 s. Find its power.

Given: $m = 100 \text{ kg}$, $h = 20 \text{ m}$, $t = 10 \text{ s}$

Solution:

$$W = mgh = 100 \times 9.8 \times 20 = 19600 \text{ J}$$

$$P = W / t = 1960 \text{ W}$$

Answer: $1960 \text{ W} = 1.96 \text{ kW}$

P2. A 50 kg boy climbs a 10 m staircase in 20 s. Find the power developed.

Given: $m = 50 \text{ kg}$, $h = 10 \text{ m}$, $t = 20 \text{ s}$

Solution:

$$W = 50 \times 9.8 \times 10 = 4900 \text{ J}$$

$$P = 4900 / 20 = 245 \text{ W}$$

Answer: 245 W

P3. A 5 HP engine works for 1 minute. Find the work done in joule.

Given: $P = 5 \text{ HP}$, $t = 60 \text{ s}$

Solution:

$$P = 5 \times 746 = 3730 \text{ W}$$

$$W = P t = 3730 \times 60 = 223800 \text{ J}$$

Answer: $2.238 \times 10^5 \text{ J}$

P4. A pump raises 1000 kg of water per minute from a 15 m deep well. Find (i) its power and (ii) input power if efficiency is 70 %.

Given: $m = 1000 \text{ kg}$, $h = 15 \text{ m}$, $t = 60 \text{ s}$, $\eta = 70 \%$

Solution:

$$(i) W = 1000 \times 9.8 \times 15 = 147000 \text{ J}; P = 147000 / 60 = 2450 \text{ W}$$

$$(ii) \text{Input} = \text{output} / \eta = 2450 / 0.70 = 3500 \text{ W}$$

Answer: 2450 W; input 3500 W

P5. A car moves at constant 20 m/s against a resistive force of 2000 N. Find the engine power.

Given: $v = 20 \text{ m/s}$, $F = 2000 \text{ N}$

Solution:

$$P = F v = 2000 \times 20 = 40000 \text{ W}$$

Answer: 40 kW (about 53.6 HP)

P6. A 100 W bulb is lit for 10 hours. Find the energy used in kWh and joule.

Given: $P = 0.1 \text{ kW}$, $t = 10 \text{ h}$

Solution:

$$E = 0.1 \times 10 = 1 \text{ kWh}$$

$$1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$$

Answer: 1 kWh = $3.6 \times 10^6 \text{ J}$

P7. How long will a 2 kW motor take to do $3.6 \times 10^5 \text{ J}$ of work?

Given: $P = 2000 \text{ W}$, $W = 360000 \text{ J}$

Solution:

$$t = W / P = 360000 / 2000 = 180 \text{ s}$$

Answer: 180 s (3 minutes)

P8. A lift raises an 800 kg load through 25 m in 30 s. Find the power in W and HP.

Given: $m = 800 \text{ kg}$, $h = 25 \text{ m}$, $t = 30 \text{ s}$

Solution:

$$W = 800 \times 9.8 \times 25 = 196000 \text{ J}$$

$$P = 196000 / 30 = 6533 \text{ W}$$

$$\text{In HP: } 6533 / 746 = 8.76 \text{ HP}$$

Answer: 6533 W $\approx$ 8.76 HP

P9. A 2 HP motor lifts water from a 10 m deep well. How much water can it lift per minute?

Given: $P = 2 \text{ HP}$, $h = 10 \text{ m}$, $t = 60 \text{ s}$

Solution:

$$P = 2 \times 746 = 1492 \text{ W}$$

$$\text{Work per minute} = 1492 \times 60 = 89520 \text{ J}$$

$$mgh = 89520, \text{ so } m = 89520 / (9.8 \times 10) = 913.5 \text{ kg}$$

Answer: $\approx$ 913.5 kg per minute

P10. A locomotive of power 50 kW pulls a train at 36 km/h. Find the pulling force.

Given: $P = 50000 \text{ W}$, $v = 36 \text{ km/h} = 10 \text{ m/s}$

Solution:

$$P = Fv, \text{ so } F = P / v = 50000 / 10 = 5000 \text{ N}$$

Answer: 5000 N

P11. Five 60 W bulbs burn 4 hours daily. Find the energy used in 30 days and the bill at ₹6 per unit.

Given: Total power = $5 \times 60 = 300 \text{ W} = 0.3 \text{ kW}$, $t = 4 \text{ h/day}$, 30 days

Solution:

$$\text{Energy per day} = 0.3 \times 4 = 1.2 \text{ kWh}$$

$$\text{Energy in 30 days} = 1.2 \times 30 = 36 \text{ kWh}$$

$$\text{Cost} = 36 \times 6 = ₹216$$

Answer: 36 kWh; ₹216

Exam tip: Marks are given for the formula, the steps, units and the final answer. Always show every step.