

APPLIED PHYSICS 1

Chapter: Work, Power and Energy

SET 2: Proofs and Numerical Problems with Solution Steps

(New questions, not repeated from Set 1)

Contents

1. **Part A:** 12 "Show that / Prove" problems
2. **Part B:** 39 numerical problems on Work, Friction, Rough Inclined Plane, Energy and Power

Reminders

- Take **$g = 9.8 \text{ m/s}^2$** unless stated otherwise.
- Use the standard formulae: $W = F d \cos\theta$, $F = \mu R$, $F = mg \sin\theta + \mu mg \cos\theta$, $KE = \frac{1}{2} mv^2$, $PE = mgh$, $P = W / t = F v$.
- Write formula, substitution with units, and the final answer underlined.

PART A: SHOW THAT / PROVE

A1. Show that the kinetic energy of a body becomes four times when its speed is doubled. (3 marks)

1. Initial $KE_1 = \frac{1}{2} mv^2$.
2. New speed $= 2v$, so $KE_2 = \frac{1}{2} m (2v)^2 = \frac{1}{2} m \times 4v^2 = 4 \times (\frac{1}{2} mv^2)$.
3. Hence **$KE_2 = 4 KE_1$** . In general $KE \propto v^2$, so tripling the speed gives 9 times the KE.

A2. Show that at the mid-point of its fall a freely falling body has equal KE and PE. (3 marks)

1. Let the body of mass m fall from height h . At the mid-point it has fallen $x = h/2$ and its height is $h/2$.
2. $PE = mg (h/2) = mgh / 2$.
3. $v^2 = 2gx = 2g (h/2) = gh$, so $KE = \frac{1}{2} m (gh) = mgh / 2$.
4. Hence **$KE = PE = mgh / 2$** and their sum is mgh .

A3. Show that the speed with which a freely falling body reaches the ground is independent of its mass. (3 marks)

1. PE at height $h = mgh$. KE on reaching the ground $= \frac{1}{2} mv^2$.
2. By conservation of energy: $mgh = \frac{1}{2} mv^2$.
3. Mass m cancels from both sides: **$v = \sqrt{2gh}$** .
4. Hence heavy and light bodies dropped from the same height (in the absence of air resistance) reach the ground with the same speed.

A4. Show that the total mechanical energy of a body thrown vertically upward with speed u is constant at every height x . (5 marks)

1. At the ground: $KE = \frac{1}{2} mu^2$, $PE = 0$, total $= \frac{1}{2} mu^2$.

- At height x , from $v^2 = u^2 - 2gx$, $KE = \frac{1}{2} m (u^2 - 2gx) = \frac{1}{2} mu^2 - mgx$.
- PE at height $x = mgx$.
- Total $= (\frac{1}{2} mu^2 - mgx) + mgx = \frac{1}{2} mu^2$, which is constant.
- At the top $v = 0$, so $mgh = \frac{1}{2} mu^2$ and $h = u^2 / 2g$.

A5. Show that a body passes through any given height with the same speed while going up and while coming down. (4 marks)

- Let the body be thrown up with speed u . At height x its speed is v (going up).
- By conservation of energy: $\frac{1}{2} mu^2 = \frac{1}{2} mv^2 + mgx$, so $v = \sqrt{u^2 - 2gx}$.
- While coming down, the body has the same total energy $\frac{1}{2} mu^2$ and the same PE mgx at height x , so its KE is the same.
- Hence the **speed** is the same at that height in both directions (only the direction of velocity differs). It returns to the ground with speed u .

A6. Show that a body sliding down a smooth inclined plane of height h reaches the bottom with speed $\sqrt{2gh}$, independent of the angle of incline. (4 marks)

- Let the length of the plane be d and the angle θ . Then $h = d \sin\theta$.
- Force along the plane $= mg \sin\theta$ (no friction). Work done by gravity $= mg \sin\theta \times d = mg (d \sin\theta) = mgh$.
- By the work-energy theorem (starting from rest): $mgh = \frac{1}{2} mv^2$.
- Therefore **$v = \sqrt{2gh}$** , which depends only on the height h and not on θ or m .

A7. Show that the acceleration of a body sliding down a rough inclined plane is $a = g (\sin\theta - \mu \cos\theta)$. (5 marks)

- Forces along the plane: $mg \sin\theta$ down the plane and friction f up the plane.
- Normal reaction $N = mg \cos\theta$, so friction $f = \mu N = \mu mg \cos\theta$.
- Net force down the plane $= mg \sin\theta - \mu mg \cos\theta$.
- Newton's second law: $ma = mg \sin\theta - \mu mg \cos\theta$.
- Dividing by m : **$a = g (\sin\theta - \mu \cos\theta)$** . Motion down occurs only if $\sin\theta > \mu \cos\theta$, i.e. $\tan\theta > \mu$.

A8. Show that a body placed on a rough incline will not slide as long as $\tan\theta \leq \mu$. (4 marks)

- Component of weight along the plane $= mg \sin\theta$ (tends to cause sliding).
- Maximum friction available (limiting) $= \mu N = \mu mg \cos\theta$.
- The body stays at rest if $mg \sin\theta \leq \mu mg \cos\theta$.
- Dividing by $mg \cos\theta$: **$\tan\theta \leq \mu$** . Since $\mu = \tan\alpha$ (angle of repose), the condition is $\theta \leq \alpha$.

A9. Show that the work done against friction in dragging a body of mass m through a distance d on a rough horizontal surface is μmgd . (3 marks)

- On a horizontal surface the normal reaction $R = mg$.
- Limiting (kinetic) friction $f = \mu R = \mu mg$.
- To drag the body at constant speed, applied force $= f$.
- Work done $= f \times d = \mu mgd$.

A10. Show that the power needed to lift a load m at constant speed v is $P = mgv$. (3 marks)

1. At constant speed the lifting force equals the weight: $F = mg$.
2. Power $P = F v$ (Proof of $P = Fv$: $P = W/t = Fs/t = Fv$).
3. Therefore **$P = mgv$** .

A11. Show that efficiency of a machine can be written as $\eta = P_{\text{out}} / P_{\text{in}}$ and is always less than 1. (4 marks)

1. Efficiency = useful work output / total work input = $W_{\text{out}} / W_{\text{in}}$.
2. Both works are done in the same time t , so $W_{\text{out}} / W_{\text{in}} = (P_{\text{out}} t) / (P_{\text{in}} t) = P_{\text{out}} / P_{\text{in}}$.
3. Some energy is always lost as heat due to friction, so $W_{\text{out}} < W_{\text{in}}$.
4. Hence **$\eta = P_{\text{out}} / P_{\text{in}} < 1$** (or $< 100\%$).

A12. Show that if two bodies have equal kinetic energy, the lighter body has the greater speed. (3 marks)

1. Let masses m_1 and m_2 have speeds v_1 and v_2 with equal KE: $\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2$.
2. So $v_1^2 / v_2^2 = m_2 / m_1$, i.e. **$v_1 / v_2 = \sqrt{(m_2 / m_1)}$** .
3. If $m_1 < m_2$, the ratio is greater than 1, so $v_1 > v_2$: the lighter body is faster.

PART B: NUMERICAL PROBLEMS

B1. Work

W1. A force of 25 N acts at 37° to the displacement and moves a body 12 m. Find the work done. ($\cos 37^\circ = 0.8$)

Given: $F = 25 \text{ N}$, $\theta = 37^\circ$, $d = 12 \text{ m}$

Solution:

$$W = F d \cos\theta = 25 \times 12 \times 0.8 = 240 \text{ J}$$

Answer: 240 J

W2. Find the work done in drawing a 12 kg bucket of water from an 8 m deep well.

Given: $m = 12 \text{ kg}$, $h = 8 \text{ m}$

Solution:

$$W = mgh = 12 \times 9.8 \times 8 = 940.8 \text{ J}$$

Answer: 940.8 J

W3. A force of 30 N does -150 J of work during a displacement of 10 m. Find the angle between force and displacement.

Given: $F = 30 \text{ N}$, $d = 10 \text{ m}$, $W = -150 \text{ J}$

Solution:

$$\cos\theta = W / (F d) = -150 / 300 = -0.5$$

$$\theta = 120^\circ$$

Answer: 120°

W4. A cyclist applies brakes giving a retarding force of 200 N over 25 m. Find the work done by the brakes.

Given: $f = 200 \text{ N}$, $d = 25 \text{ m}$, $\theta = 180^\circ$

Solution:

$$W = f d \cos 180^\circ = -200 \times 25 = -5000 \text{ J}$$

Answer: -5000 J

W5. A 60 kg person climbs 20 steps, each 0.2 m high. Find the work done against gravity.

Given: $m = 60 \text{ kg}$, $20 \text{ steps} \times 0.2 \text{ m}$

Solution:

$$\text{Total height } h = 20 \times 0.2 = 4 \text{ m}$$

$$W = mgh = 60 \times 9.8 \times 4 = 2352 \text{ J}$$

Answer: 2352 J

W6. A trolley is pulled 5 m along level ground by a rope inclined at 60° to the ground with a tension of 80 N. Find the work done.

Given: $T = 80 \text{ N}$, $\theta = 60^\circ$, $d = 5 \text{ m}$

Solution:

$$W = T d \cos\theta = 80 \times 5 \times 0.5 = 200 \text{ J}$$

Answer: 200 J

W7. A body is acted on by a 10 N force. It moves 5 m along the force, then 3 m perpendicular to the force, then 4 m opposite to the force. Find the total work done.

Given: $F = 10 \text{ N}$

Solution:

$$\text{Along: } W_1 = 10 \times 5 = +50 \text{ J}$$

$$\text{Perpendicular: } W_2 = 10 \times 3 \times \cos 90^\circ = 0$$

$$\text{Opposite: } W_3 = 10 \times 4 \times \cos 180^\circ = -40 \text{ J}$$

$$\text{Total} = 50 + 0 - 40 = 10 \text{ J}$$

Answer: 10 J

B2. Friction

F1. A 40 kg crate rests on a floor with $\mu = 0.3$. Find the minimum force required to start it moving.

Given: $m = 40 \text{ kg}$, $\mu = 0.3$

Solution:

$$R = mg = 40 \times 9.8 = 392 \text{ N}$$

$$F = \mu R = 0.3 \times 392 = 117.6 \text{ N}$$

Answer: 117.6 N

F2. A force of 150 N is just enough to move a 50 kg body on level ground. Find μ .

Given: $F = 150 \text{ N}$, $m = 50 \text{ kg}$

Solution:

$$R = 50 \times 9.8 = 490 \text{ N}$$

$$\mu = 150 / 490 = 0.306$$

Answer: $\mu \approx 0.31$

F3. A 10 kg block just begins to slide on a plane inclined at 30° . Find μ and the limiting friction.

Given: $m = 10 \text{ kg}$, angle of repose $\alpha = 30^\circ$

Solution:

$$\mu = \tan\alpha = \tan 30^\circ = 0.577$$

$$\text{Limiting friction} = mg \sin\alpha = 10 \times 9.8 \times 0.5 = 49 \text{ N}$$

$$\text{Check: } \mu mg \cos\alpha = 0.577 \times 98 \times 0.866 = 49 \text{ N}$$

Answer: $\mu = 0.577$; $F = 49 \text{ N}$

F4. If $\mu = 0.5$, find the angle of friction.

Given: $\mu = 0.5$

Solution:

$$\tan\phi = 0.5$$

$$\phi = \tan^{-1}(0.5) = 26.6^\circ$$

Answer: 26.6°

F5. A 1200 kg car on a level road has $\mu = 0.6$ between tyres and road. Find the maximum braking force, the retardation and the stopping distance from 20 m/s.

Given: $m = 1200 \text{ kg}$, $\mu = 0.6$, $u = 20 \text{ m/s}$, $v = 0$

Solution:

$$\text{Maximum braking force} = \mu mg = 0.6 \times 1200 \times 9.8 = 7056 \text{ N}$$

$$\text{Retardation } a = F / m = 7056 / 1200 = 5.88 \text{ m/s}^2$$

$$\text{Stopping distance } s = u^2 / 2a = 400 / 11.76 = 34.0 \text{ m}$$

Answer: $F = 7056 \text{ N}$; $a = 5.88 \text{ m/s}^2$; $s = 34 \text{ m}$

F6. A horizontal force of 30 N is applied to a 10 kg block on a floor with $\mu_s = 0.4$. Will the block move? What is the friction acting?

Given: $F = 30 \text{ N}$, $m = 10 \text{ kg}$, $\mu = 0.4$

Solution:

$$\text{Limiting friction} = \mu mg = 0.4 \times 10 \times 9.8 = 39.2 \text{ N}$$

Applied force $30 \text{ N} < 39.2 \text{ N}$, so the block does not move

Static friction adjusts itself and equals the applied force = 30 N

Answer: Block does not move; friction = 30 N

F7. Find the work done against friction in dragging a 25 kg box 8 m on a floor with $\mu = 0.2$.

Given: $m = 25 \text{ kg}$, $d = 8 \text{ m}$, $\mu = 0.2$

Solution:

$$f = \mu mg = 0.2 \times 25 \times 9.8 = 49 \text{ N}$$

$$W = f d = 49 \times 8 = 392 \text{ J}$$

Answer: 392 J

B3. Rough Inclined Plane

11. A 5 kg body is pulled 8 m up a plane inclined at 30° with $\mu = 0.3$. Find the force and the work done.

Given: $m = 5 \text{ kg}$, $\theta = 30^\circ$, $d = 8 \text{ m}$, $\mu = 0.3$

Solution:

$$mg = 49 \text{ N}; mg \sin\theta = 24.5 \text{ N}$$

$$\mu mg \cos\theta = 0.3 \times 49 \times 0.866 = 12.73 \text{ N}$$

$$F = 24.5 + 12.73 = 37.23 \text{ N}$$

$$W = 37.23 \times 8 = 297.8 \text{ J}$$

Answer: $F = 37.23 \text{ N}$; $W = 297.8 \text{ J}$

12. A 20 kg load is pulled 3 m up a plane inclined at 45° with $\mu = 0.1$. Find the work done.

Given: $m = 20 \text{ kg}$, $\theta = 45^\circ$, $d = 3 \text{ m}$, $\mu = 0.1$

Solution:

$$mg = 196 \text{ N}; \sin 45^\circ = \cos 45^\circ = 0.7071$$

$$F = 196 \times 0.7071 + 0.1 \times 196 \times 0.7071 = 138.6 + 13.9 = 152.5 \text{ N}$$

$$W = 152.5 \times 3 = 457.4 \text{ J}$$

Answer: $\approx 457.4 \text{ J}$

13. A block starts sliding down a plane when it is tilted to 20° . Find the coefficient of friction.

Given: $\alpha = 20^\circ$

Solution:

$$\mu = \tan\alpha = \tan 20^\circ = 0.364$$

Answer: $\mu = 0.364$

14. A 30 kg body is pushed up a smooth ramp 5 m long and 1.5 m high. Find the force and the work done.

Given: $m = 30 \text{ kg}$, $d = 5 \text{ m}$, $h = 1.5 \text{ m}$

Solution:

$$\sin\theta = h / d = 0.3$$

$$F = mg \sin\theta = 30 \times 9.8 \times 0.3 = 88.2 \text{ N}$$

$$W = F d = 88.2 \times 5 = 441 \text{ J} (= mgh = 30 \times 9.8 \times 1.5)$$

Answer: $F = 88.2 \text{ N}$; $W = 441 \text{ J}$

15. A body slides down a plane inclined at 30° with $\mu = 0.2$. Find its acceleration and the speed after sliding 4 m from rest.

Given: $\theta = 30^\circ$, $\mu = 0.2$, $s = 4 \text{ m}$, $u = 0$

Solution:

$$a = g (\sin\theta - \mu \cos\theta) = 9.8 (0.5 - 0.2 \times 0.866) = 9.8 \times 0.3268 = 3.20 \text{ m/s}^2$$

$$v^2 = 2as = 2 \times 3.20 \times 4 = 25.6$$

$$v = 5.06 \text{ m/s}$$

Answer: $a = 3.20 \text{ m/s}^2$; $v = 5.06 \text{ m/s}$

16. A 10 kg block rests on a plane inclined at 15° with $\mu = 0.3$. Will it slide? Find the friction acting on it.

Given: $m = 10 \text{ kg}$, $\theta = 15^\circ$, $\mu = 0.3$

Solution:

$\tan 15^\circ = 0.268$ and $\mu = 0.3$, so $\tan\theta < \mu$: the block does not slide

Friction needed to hold it $= mg \sin\theta = 98 \times 0.2588 = 25.4 \text{ N}$

Answer: Does not slide; friction = 25.4 N

B4. Energy

E1. Find the KE of a 0.5 kg ball moving at 12 m/s.

Given: $m = 0.5 \text{ kg}$, $v = 12 \text{ m/s}$

Solution:

$$KE = \frac{1}{2} \times 0.5 \times 144 = 36 \text{ J}$$

Answer: 36 J

E2. A tank holds 500 kg of water at a height of 12 m. Find its PE.

Given: $m = 500 \text{ kg}$, $h = 12 \text{ m}$

Solution:

$$PE = mgh = 500 \times 9.8 \times 12 = 58800 \text{ J}$$

Answer: 58800 J

E3. A stone is dropped from a 45 m high building. Find its speed on reaching the ground.

Given: $h = 45 \text{ m}$

Solution:

$$v = \sqrt{(2gh)} = \sqrt{(2 \times 9.8 \times 45)} = \sqrt{882} = 29.7 \text{ m/s}$$

Answer: 29.7 m/s

E4. A 3 kg body is dropped from 40 m. Find its KE and PE at 15 m above the ground.

Given: $m = 3 \text{ kg}$, $H = 40 \text{ m}$, $h = 15 \text{ m}$, fallen $x = 25 \text{ m}$

Solution:

$$PE = 3 \times 9.8 \times 15 = 441 \text{ J}$$

$$KE = mgx = 3 \times 9.8 \times 25 = 735 \text{ J}$$

$$\text{Total} = 1176 \text{ J} = mgH = 3 \times 9.8 \times 40 \text{ (conserved)}$$

Answer: PE = 441 J; KE = 735 J

E5. A body has 100 J of KE. What is its KE if the speed becomes three times?

Given: $KE_1 = 100 \text{ J}$, $v_2 = 3v_1$

Solution:

$$KE \propto v^2$$

$$KE_2 = 9 \times 100 = 900 \text{ J}$$

Answer: 900 J

E6. Find the work required to increase the speed of a 1500 kg car from 10 m/s to 20 m/s.

Given: $m = 1500 \text{ kg}$, $u = 10 \text{ m/s}$, $v = 20 \text{ m/s}$

Solution:

By the work-energy theorem, $W = \frac{1}{2} m (v^2 - u^2)$

$$W = \frac{1}{2} \times 1500 \times (400 - 100) = 750 \times 300 = 225000 \text{ J}$$

Answer: $2.25 \times 10^5 \text{ J}$

E7. 1000 kg of water falls from a height of 50 m. Find its KE and speed just before reaching the bottom.

Given: $m = 1000 \text{ kg}$, $h = 50 \text{ m}$

Solution:

$$\text{KE} = mgh = 1000 \times 9.8 \times 50 = 490000 \text{ J}$$

$$v = \sqrt{2gh} = \sqrt{980} = 31.3 \text{ m/s}$$

Answer: $\text{KE} = 4.9 \times 10^5 \text{ J}$; $v = 31.3 \text{ m/s}$

E8. A 0.5 kg pendulum bob is raised 0.2 m above its lowest point and released. Find its speed at the lowest point.

Given: $m = 0.5 \text{ kg}$, $h = 0.2 \text{ m}$

Solution:

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.2} = \sqrt{3.92} = 1.98 \text{ m/s}$$

$$\text{KE at bottom} = mgh = 0.5 \times 9.8 \times 0.2 = 0.98 \text{ J}$$

Answer: $v = 1.98 \text{ m/s}$

E9. A 400 kg roller-coaster car is released from rest at 30 m height. Find its speed at 5 m height (frictionless).

Given: $h_1 = 30 \text{ m}$, $h_2 = 5 \text{ m}$

Solution:

$$\text{Height lost} = 30 - 5 = 25 \text{ m}$$

$$v = \sqrt{2 \times 9.8 \times 25} = \sqrt{490} = 22.1 \text{ m/s}$$

Answer: 22.1 m/s

E10. A ball is thrown upward at 15 m/s. Find the maximum height and its speed at 5 m height.

Given: $u = 15 \text{ m/s}$

Solution:

$$\text{Max height } h = \frac{u^2}{2g} = \frac{225}{19.6} = 11.5 \text{ m}$$

$$\text{At } 5 \text{ m: } v^2 = u^2 - 2gx = 225 - 98 = 127$$

$$v = 11.3 \text{ m/s}$$

Answer: $h = 11.5 \text{ m}$; $v = 11.3 \text{ m/s}$

B5. Power

P1. A crane lifts a 2000 kg load at a constant speed of 0.5 m/s. Find the power.

Given: $m = 2000 \text{ kg}$, $v = 0.5 \text{ m/s}$

Solution:

$$P = F v = mgv = 2000 \times 9.8 \times 0.5 = 9800 \text{ W}$$

Answer: 9.8 kW

P2. A 1.5 kW pump lifts water to a height of 12 m for 1 hour. Find the mass of water lifted.

Given: $P = 1500 \text{ W}$, $h = 12 \text{ m}$, $t = 3600 \text{ s}$

Solution:

$$W = P t = 1500 \times 3600 = 5.4 \times 10^6 \text{ J}$$

$$m = W / gh = 5.4 \times 10^6 / (9.8 \times 12) = 45918 \text{ kg}$$

Answer: $\approx 4.59 \times 10^4 \text{ kg}$

P3. A motor draws 2 kW and has 80 % efficiency. Find the output power and the work done in 5 minutes.

Given: $P_{\text{in}} = 2000 \text{ W}$, $\eta = 0.8$, $t = 300 \text{ s}$

Solution:

$$P_{\text{out}} = \eta P_{\text{in}} = 0.8 \times 2000 = 1600 \text{ W}$$

$$W = 1600 \times 300 = 480000 \text{ J}$$

Answer: 1600 W; $4.8 \times 10^5 \text{ J}$

P4. Convert 10 HP into kW.

Given: $1 \text{ HP} = 746 \text{ W}$

Solution:

$$10 \times 746 = 7460 \text{ W} = 7.46 \text{ kW}$$

Answer: 7.46 kW

P5. A cyclist rides at 5 m/s against a resistive force of 40 N. Find the power developed.

Given: $v = 5 \text{ m/s}$, $F = 40 \text{ N}$

Solution:

$$P = F v = 40 \times 5 = 200 \text{ W}$$

Answer: 200 W

P6. A machine gives 4 kW output with 80 % efficiency. Find the input power and the power lost.

Given: $P_{\text{out}} = 4 \text{ kW}$, $\eta = 0.8$

Solution:

$$P_{\text{in}} = 4 / 0.8 = 5 \text{ kW}$$

$$\text{Power lost} = 5 - 4 = 1 \text{ kW}$$

Answer: Input = 5 kW; loss = 1 kW

P7. A 1500 W heater works 2 hours daily. Find the energy used in 30 days and the cost at ₹7 per unit.

Given: $P = 1.5 \text{ kW}$, 2 h/day , 30 days

Solution:

$$\text{Energy per day} = 1.5 \times 2 = 3 \text{ kWh}$$

$$\text{In 30 days} = 3 \times 30 = 90 \text{ kWh}$$

$$\text{Cost} = 90 \times 7 = ₹630$$

Answer: 90 kWh; ₹630

P8. A waterfall 20 m high delivers 50 kg of water per second to a turbine of 60 % efficiency. Find the power available and the electrical output.

Given: $h = 20 \text{ m}$, $m/t = 50 \text{ kg/s}$, $\eta = 0.6$

Solution:

$$P = (m/t) g h = 50 \times 9.8 \times 20 = 9800 \text{ W}$$

$$\text{Output} = 0.6 \times 9800 = 5880 \text{ W}$$

Answer: Available = 9800 W; output = 5880 W

P9. A man pushes a cart with a 100 N force at 1.5 m/s for 2 minutes. Find his power and the work done.

Given: $F = 100 \text{ N}$, $v = 1.5 \text{ m/s}$, $t = 120 \text{ s}$

Solution:

$$P = F v = 100 \times 1.5 = 150 \text{ W}$$

$$W = P t = 150 \times 120 = 18000 \text{ J}$$

Answer: $P = 150 \text{ W}$; $W = 18000 \text{ J}$

Exam tip: Marks are given for the formula, steps, units and the final answer. Show every step.