

Applied Physics 1

Chapter: Properties of Matter

Exercise: Important Questions, Solutions and Numerical Problems

This exercise covers the chapter **Properties of Matter** for engineering students. It has a quick formula sheet, important short-answer questions, broad (long-answer) questions and numerical problems with step-by-step solutions, followed by practice problems with answers. Topics covered: **Elasticity, Surface Tension, Viscosity** and **Hydrodynamics**.

How to write answers: give the definition first, then the formula with the meaning of every symbol and its unit. In numericals always write *Given, Formula, Substitution, Answer with unit*. Take $g = 9.8 \text{ m/s}^2$ unless stated otherwise.

PART 1: Quick Formula Sheet

Quantity	Formula	SI unit
Stress	$\text{Stress} = F / A$	N/m^2 (Pa)
Longitudinal strain	$\Delta L / L$	No unit
Volume strain	$\Delta V / V$	No unit
Shear strain	$\theta = x / h$	No unit (radian)
Hooke's law	$\text{Stress} \propto \text{Strain}$, i.e. $\text{Stress} = E \times \text{Strain}$	Pa
Young's modulus	$Y = (F/A) / (\Delta L/L) = FL / (A\Delta L)$	Pa
Bulk modulus	$K = -P / (\Delta V/V)$	Pa
Modulus of rigidity	$G = (F/A) / \theta$	Pa
Surface tension	$T = F / l = W / \Delta A$	N/m or J/m^2
Capillary rise	$h = 2T \cos\theta / (r \rho g)$	m
Newton's law of viscosity	$F = \eta A (dv/dx)$	N
Stokes' law	$F = 6\pi \eta r v$	N
Terminal velocity	$v = 2 r^2 (\rho - \sigma) g / (9 \eta)$	m/s
Reynolds number	$Re = \rho v D / \eta$	No unit
Equation of continuity	$A_1 v_1 = A_2 v_2$	m^3/s (flow rate)
Bernoulli's equation	$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$	Pa

PART 2: Short Answer Questions with Answers

A. Elasticity

Q1. Define elasticity. What is a perfectly elastic body?

Ans. Elasticity is the property of a body by which it regains its original shape and size when the deforming force is removed. A body which completely regains its original form is called perfectly elastic (quartz is nearly so). The opposite property is **plasticity** (e.g. putty, wet clay).

Q2. Define stress. Write its SI unit and dimensional formula.

Ans. Stress is the internal restoring force developed per unit area of a deformed body:

$\text{Stress} = F / A.$

- SI unit: N/m^2 or pascal (Pa)
- Dimensional formula: $[M L^{-1} T^{-2}]$
- Types: tensile, compressive, shear (tangential) and bulk (hydraulic) stress

Q3. Define strain. Name its types.

Ans. Strain is the ratio of the change in a dimension to the original dimension. It has no unit and no dimensions.

- Longitudinal strain = change in length / original length = $\Delta L / L$
- Volume strain = change in volume / original volume = $\Delta V / V$
- Shear strain = angle of shear $\theta = x / h$

Q4. State Hooke's law.

Ans. Within the elastic limit, stress is directly proportional to strain. $\text{Stress} \propto \text{Strain}$, so $\text{Stress} / \text{Strain} = E = \text{constant}$, where E is the modulus of elasticity of the material.

Q5. Define the three moduli of elasticity.

- **Young's modulus (Y)** = longitudinal stress / longitudinal strain
- **Bulk modulus (K)** = volume stress (pressure change) / volume strain
- **Modulus of rigidity (G)** = shear stress / shear strain
- All three are measured in N/m^2 (Pa).

Q6. What is elastic limit?

Ans. The maximum stress up to which a body completely regains its original shape and size on removing the load. Beyond it the body is permanently deformed.

Q7. Steel is more elastic than rubber. Why?

Ans. For the same stress, steel produces a much smaller strain than rubber. Hence steel has a larger Young's modulus (about 2×10^{11} Pa) than rubber, and a body with a larger modulus is more elastic.

B. Surface Tension

Q8. Define surface tension. Give its unit and dimensional formula.

Ans. Surface tension is the force acting per unit length on an imaginary line drawn on the liquid surface, perpendicular to the line and tangential to the surface. $T = F / l.$

- SI unit: N/m (equivalent to J/m^2); CGS unit: dyne/cm
- Dimensional formula: $[M T^{-2}]$
- For water at room temperature, $T \approx 0.072 \text{ N/m}$

Q9. Differentiate between cohesive and adhesive forces.

Ans. **Cohesive force** is the force of attraction between molecules of the *same* substance (e.g. water-water). **Adhesive force** is the force of attraction between molecules of *different* substances (e.g. water-glass). Water wets glass because adhesion is greater than cohesion; mercury does not wet glass because cohesion is greater.

Q10. Define angle of contact. Give its value for water-glass and mercury-glass.

Ans. The angle between the tangent to the liquid surface at the point of contact and the solid surface, measured inside the liquid. Water-glass: acute (nearly 0° to 8°). Mercury-glass: obtuse (about 135°).

Q11. Why are small liquid drops spherical?

Ans. Due to surface tension a liquid surface tries to have minimum area. For a given volume, a sphere has the minimum surface area, so small drops (where gravity is negligible) become spherical.

Q12. How does temperature affect surface tension?

Ans. Surface tension decreases as temperature rises, because the increased kinetic energy of molecules weakens cohesive forces. It becomes zero at the critical temperature.

Q13. How do impurities affect surface tension?

- Highly soluble inorganic substances (e.g. common salt) **increase** surface tension.
- Organic substances and detergents (soap, alcohol, phenol) **decrease** surface tension.
- A thin oil film or dust on the surface also lowers it.

Q14. Why do detergents clean clothes better than plain water?

Ans. Detergent lowers the surface tension of water, so the solution spreads, wets the fibres, penetrates the fabric and lifts oil and dirt particles easily.

Q15. Write the formula for capillary rise and explain the symbols.

Ans. $h = \frac{2T \cos\theta}{r \rho g}$

- h = rise (or fall) of liquid in the capillary tube
- T = surface tension of the liquid, θ = angle of contact
- r = radius of the capillary tube, ρ = density of the liquid, g = acceleration due to gravity

C. Viscosity

Q16. Define viscosity and coefficient of viscosity. Give the unit.

Ans. Viscosity is the property of a fluid by which it opposes the relative motion between its layers. The coefficient of viscosity (η) is the tangential viscous force per unit area required to maintain a unit velocity gradient between two layers.

- SI unit: $\text{N s/m}^2 = \text{Pa}\cdot\text{s} = \text{kg m}^{-1} \text{s}^{-1}$
- CGS unit: poise; $1 \text{ Pa}\cdot\text{s} = 10 \text{ poise}$
- Dimensional formula: $[\text{M L}^{-1} \text{T}^{-1}]$

Q17. What is terminal velocity?

Ans. The maximum constant velocity attained by a body falling through a fluid, when the downward weight is balanced by the upward buoyant force plus the viscous drag. Net force is zero, so acceleration is zero.

Q18. State Stokes' law.

Ans. The viscous drag on a small sphere of radius r moving with velocity v through a fluid of viscosity η is $F = 6\pi \eta r v$.

Q19. How does temperature affect the viscosity of liquids and gases?

- **Liquids:** viscosity decreases with rise in temperature (molecules gain energy and cohesive forces weaken). Example: hot honey and engine oil flow easily.
- **Gases:** viscosity increases with rise in temperature (faster molecules transfer more momentum between layers by collisions).

Q20. Why does a raindrop fall with a constant velocity after some time?

Ans. As the drop speeds up, the viscous drag of air increases. When drag plus upthrust equals the weight, the net force becomes zero and the drop falls with its terminal velocity.

D. Hydrodynamics

Q21. Define streamline flow and turbulent flow.

Ans. Streamline flow: every particle passing a point follows the same path with the same velocity as the particles before it; layers slide smoothly. **Turbulent flow:** the motion is irregular and zig-zag with eddies and whirlpools; velocity at a point changes in magnitude and direction.

Q22. What is Reynolds number? State the critical values.

Ans. Reynolds number $Re = \rho v D / \eta$ is a dimensionless number equal to the ratio of inertial force to viscous force in a flowing fluid.

- $Re < 2000$: streamline flow
- $2000 < Re < 3000$: unstable (transition) flow
- $Re > 3000$: turbulent flow

Q23. State the equation of continuity.

Ans. For steady flow of an incompressible fluid, the product of cross-sectional area and fluid speed is the same at every section of the tube: $A_1 v_1 = A_2 v_2 = \text{constant}$.

Q24. Write Bernoulli's equation and name each term.

Ans. $P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$

- P = pressure energy per unit volume
- $\frac{1}{2}\rho v^2$ = kinetic energy per unit volume
- ρgh = potential energy per unit volume

Q25. Name four applications of Bernoulli's theorem.

Ans. Venturimeter, aerofoil (lift of an aeroplane wing), atomizer (perfume sprayer) and Pitot tube.

Q26. What is an ideal fluid?

Ans. A fluid which is incompressible and non-viscous (has no internal friction) is called an ideal fluid.

PART 3: Broad (Long Answer) Questions with Answers

B1. Define stress and strain. Explain their types and state Hooke's law.

When an external force deforms a body, an equal and opposite internal restoring force develops inside it. The behaviour of the body under such forces is described by stress and strain.

- **Stress** = restoring force / area = F / A (unit N/m^2). Types: tensile stress (stretching), compressive stress (squeezing), shear stress (tangential force) and bulk stress (uniform pressure on all sides).
- **Strain** = change in dimension / original dimension (no unit).
- Longitudinal strain = $\Delta L / L$; volume strain = $\Delta V / V$; shear strain = $\theta = x / h$.

Hooke's law: within the elastic limit, stress is directly proportional to strain.

$$\text{Stress} \propto \text{Strain} \quad \text{or} \quad \text{Stress} = E \times \text{Strain}$$

The constant E is the modulus of elasticity. It depends only on the material, not on its size or shape. Hooke's law holds only up to the proportional limit; beyond that the stress-strain graph is no longer a straight line.

B2. Define the moduli of elasticity. Write their formulas and compare them.

Modulus	Type of stress and strain	Formula	Applies to
Young's modulus (Y)	Longitudinal stress and strain	$Y = (F/A) / (\Delta L/L) = FL / (A\Delta L)$	Solids only (wires, rods)
Bulk modulus (K)	Volume stress (pressure) and volume strain	$K = -P / (\Delta V/V) = -PV / \Delta V$	Solids, liquids and gases
Modulus of rigidity (G)	Shear stress and shear strain	$G = (F/A) / \theta = F / (A\theta)$	Solids only

- The negative sign in K shows that volume decreases when pressure increases.
- Reciprocal of bulk modulus is called **compressibility** ($1/K$).
- Typical values: steel $Y \approx 2 \times 10^{11}$ Pa; water $K \approx 2.2 \times 10^9$ Pa.
- Larger the modulus, more rigid (more elastic) the material.

B3. Draw and explain the stress-strain curve for a ductile material. State its significance.

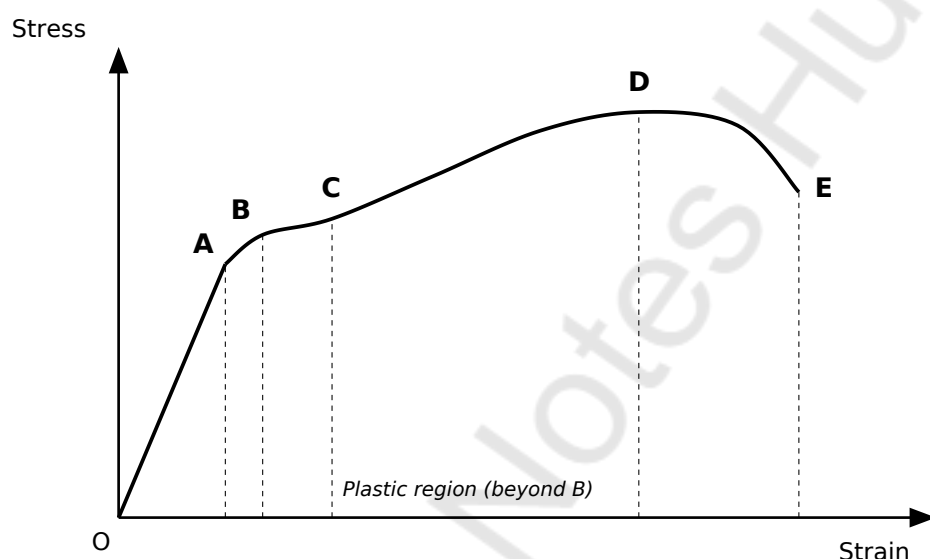


Fig.: Stress-strain curve of a ductile metal (e.g. mild steel)

- **O to A:** straight line, stress $\propto$ strain, Hooke's law holds. A is the **proportional limit**. Slope of OA gives Young's modulus.
- **A to B:** slightly curved; the wire still returns to its original length on unloading. B is the **elastic limit**.
- **B to C:** beyond the elastic limit the deformation is permanent (plastic). C is the **yield point**, where strain increases rapidly with little increase in stress.
- **C to D:** strain hardening; more stress is required for further strain. D is the **ultimate tensile strength** (maximum stress the wire can bear).
- **D to E:** the wire thins at one place (necking) and the stress falls; E is the **breaking (fracture) point**.

Significance of the curve:

- Gives Young's modulus from the slope of the straight portion.
- Shows the elastic limit and yield strength, which fix the safe working load of a machine part or structure.
- Gives the ultimate strength and breaking stress of the material.
- Distinguishes ductile materials (long plastic region, e.g. copper, mild steel) from brittle materials (break soon after the elastic limit, e.g. glass, cast iron).

- The area under the curve measures the toughness (energy absorbed before fracture).
- Helps in the selection of materials; factor of safety = breaking stress / working stress.

B4. Explain the concept of surface tension on the basis of molecular theory. How do temperature and impurities affect it?

Every molecule attracts its neighbours within a small distance called the *molecular range* (about 10^{-9} m). A molecule deep inside the liquid is attracted equally in all directions, so the net force on it is zero. A molecule at the surface has liquid molecules below it but very few above it, so a net inward cohesive force acts on it. To keep the surface area as small as possible, the surface behaves like a stretched elastic membrane. This property is surface tension.

- **Definition:** $T = F / l$, the force per unit length on a line drawn on the surface.
- **Energy form:** $T = W / \Delta A$, the work done per unit increase in surface area (J/m^2).
- **Units:** N/m (SI), dyne/cm (CGS); $1 \text{ N/m} = 10^3 \text{ dyne/cm}$. Dimensions $[M T^{-2}]$.

Effect of temperature: surface tension decreases almost linearly as temperature rises and becomes zero at the critical temperature, because greater molecular energy weakens cohesive forces.

Effect of impurities:

- Soluble salts such as NaCl increase surface tension.
- Soap, detergent, alcohol and other organic substances decrease it.
- Oil, grease or dust on the surface decrease it.

B5. Define angle of contact. Write the formula for capillary rise and discuss it.

Angle of contact (θ) is the angle between the tangent to the liquid surface at the point of contact and the solid surface, measured through the liquid. It depends on the nature of the liquid and the solid, impurities and temperature.

- $\theta < 90^\circ$ (water-glass): the liquid wets the solid, meniscus is concave, liquid rises in the tube.
- $\theta > 90^\circ$ (mercury-glass, about 135°): the liquid does not wet the solid, meniscus is convex, liquid is depressed.
- $\theta = 90^\circ$: the meniscus is plane and there is no rise or fall.

A **capillary tube** is a tube of very fine bore. When it is dipped in a liquid, the liquid rises or falls in it (capillarity). The height is given by:

$$h = 2T \cos\theta / (r \rho g)$$

- $h \propto 1/r$: the narrower the tube, the greater the rise.
- $h \propto T$: larger surface tension gives a larger rise.
- For water-glass $\cos\theta$ is positive, so h is positive (rise); for mercury-glass $\cos\theta$ is negative, so h is negative (depression).
- If the tube is not vertical, the vertical height remains the same.

B6. Explain the applications of surface tension.

- **Detergents and soaps:** lower the surface tension of water so it wets fibres and removes dirt.
- **Waterproofing:** wax or water-repellent coating increases the angle of contact; surface tension stops water passing through the fine pores of an umbrella or tent cloth.
- **Antiseptics and medicines:** have low surface tension, so they spread over wounds.
- **Lubricating oils:** low surface tension lets them spread over machine parts.
- **Soldering and welding:** flux lowers the surface tension of molten solder so it spreads and joins well.

- **Spherical drops:** raindrops, mercury drops and lead shots (made in a shot tower) are spherical.
- **Floating objects:** a needle floats on water and insects such as the water strider walk on water.
- **Capillary action:** oil rises in a lamp wick, ink in a pen nib, water in blotting paper, towels and soil; sap rises in plants.
- **Froth flotation:** used to separate ore particles from impurities.

B7. Explain viscosity. State Newton's law of viscous flow, define the coefficient of viscosity and discuss the effect of temperature.

When a fluid flows in layers, the faster layer drags the slower layer and the slower layer retards the faster one. This internal friction between fluid layers is called viscosity.

Newton's law of viscous flow: the viscous force between two layers is directly proportional to the area A of the layers and to the velocity gradient dv/dx between them.

$$F = \eta A (dv/dx)$$

- dv/dx is the change in velocity per unit perpendicular distance (unit s^{-1}).
- η is the **coefficient of viscosity**: if $A = 1$ and $dv/dx = 1$ then $\eta = F$. It is the viscous force per unit area per unit velocity gradient.
- Unit: $Pa \cdot s$ ($N s/m^2$); CGS unit poise; $1 Pa \cdot s = 10$ poise. Dimensions $[M L^{-1} T^{-1}]$.
- Viscosity of water at $20^\circ C$ is about $1 \times 10^{-3} Pa \cdot s$.

Effect of temperature: for liquids, η decreases with increase in temperature; for gases, η increases with temperature. This is why lubricating oil becomes thin in a hot engine, whereas air offers slightly more internal friction when hot.

B8. State Stokes' law. Derive an expression for the terminal velocity of a sphere falling through a viscous fluid.

Stokes' law: a small sphere of radius r moving slowly with velocity v through a fluid of coefficient of viscosity η experiences a viscous drag

$$F = 6\pi \eta r v$$

Derivation of terminal velocity: Consider a sphere of radius r and density ρ falling in a fluid of density σ . Three forces act on it:

- Weight (downward): $W = (4/3)\pi r^3 \rho g$
- Upthrust (upward): $U = (4/3)\pi r^3 \sigma g$
- Viscous drag (upward): $F = 6\pi \eta r v$

At first, v is small and the sphere accelerates. As v grows, drag grows. At terminal velocity v_t the net force is zero:

$$W = U + F \Rightarrow 6\pi \eta r v_t = (4/3)\pi r^3 (\rho - \sigma) g$$

$$v_t = \frac{2 r^2 (\rho - \sigma) g}{9 \eta}$$

- $v_t \propto r^2$: bigger drops fall faster.
- $v_t \propto (\rho - \sigma)$: if $\rho < \sigma$, v_t is negative and the body moves upward (air bubbles in a liquid).
- $v_t \propto 1/\eta$: the more viscous the fluid, the smaller the terminal velocity.
- Uses: finding η by the falling-ball method, motion of raindrops, parachutes, sedimentation of particles.

B9. Distinguish between streamline and turbulent flow. Explain Reynolds number.

Streamline (laminar) flow	Turbulent flow
Fluid moves in smooth parallel layers	Motion is irregular, with eddies and whirlpools

Streamline (laminar) flow	Turbulent flow
Velocity at a given point stays constant with time	Velocity at a point changes in magnitude and direction
Occurs at low speed, in narrow tubes, in highly viscous fluids	Occurs at high speed, in wide tubes, in less viscous fluids
Reynolds number below 2000	Reynolds number above 3000
Little energy loss	Large energy loss, noisy flow

Reynolds number: a dimensionless number which decides the type of flow in a pipe.

$$Re = \rho v D / \eta \quad (= v D / \nu, \text{ where } \nu = \eta/\rho \text{ is kinematic viscosity})$$

- ρ = density, v = average speed of the fluid, D = diameter of the pipe, η = coefficient of viscosity.
- It is the ratio of inertial force to viscous force.
- $Re < 2000$: streamline; 2000 to 3000 : unstable; $Re > 3000$: turbulent.
- Flow turns turbulent if ρ , v or D increases, or if η decreases.
- Critical velocity (at $Re = 2000$): $v_c = 2000 \eta / (\rho D)$.

B10. State and derive the equation of continuity. Give its consequences.

Consider steady flow of an incompressible, non-viscous fluid through a pipe of varying cross-section. Let the area be A_1 and speed v_1 at section 1, and A_2 , v_2 at section 2.

- Mass entering section 1 in time $\Delta t = \rho A_1 v_1 \Delta t$
- Mass leaving section 2 in time $\Delta t = \rho A_2 v_2 \Delta t$
- No fluid is created or lost in between, so the two masses are equal.

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

The product $A v$ is the **volume flow rate** Q (m^3/s). Consequences and uses:

- $v \propto 1/A$: fluid speeds up in a narrow section and slows down in a wide section.
- Water from a garden hose shoots farther when the nozzle is partly closed.
- A river flows fast in a narrow gorge and slowly where it is wide and deep.
- Blood flows slowly in capillaries because their total cross-section is large compared with the aorta.

B11. State Bernoulli's theorem with its formula. Write its assumptions and name its applications.

Statement: for the steady streamline flow of an ideal fluid, the sum of the pressure energy, kinetic energy and potential energy per unit volume is constant at every point along the flow.

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

- P = pressure (pressure energy per unit volume), $\frac{1}{2}\rho v^2$ = kinetic energy per unit volume, ρgh = potential energy per unit volume.
- Dividing by ρg : $P/\rho g + v^2/2g + h = \text{constant}$ (pressure head + velocity head + gravitational head).
- For a horizontal pipe (h constant): $P + \frac{1}{2}\rho v^2 = \text{constant}$, so where the speed is high the pressure is low and vice versa.

Assumptions:

- The fluid is incompressible and non-viscous (ideal).
- The flow is steady and streamline (no turbulence).
- No energy is added or lost by friction, pumps or heat along the path.

Applications (names only): Venturimeter, Pitot tube, aerofoil lift of an aeroplane wing, atomizer or sprayer, Bunsen burner, carburettor, filter pump, flow of liquid from an orifice in a tank (Torricelli's theorem) and swing of a spinning ball.

PART 4: Numerical Problems with Solutions

Take $g = 9.8 \text{ m/s}^2$ wherever required.

A. Elasticity

N1. A steel wire of length 2 m and diameter 1 mm is stretched by a load of 10 kg. Find the stress, strain and elongation. ($Y = 2 \times 10^{11} \text{ N/m}^2$)

Given: $L = 2 \text{ m}$, $d = 1 \text{ mm}$ so $r = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$, $m = 10 \text{ kg}$, $Y = 2 \times 10^{11} \text{ N/m}^2$

Formula: Stress = F/A ; $Y = FL / (A \Delta L)$

Solution:

- $F = mg = 10 \times 9.8 = 98 \text{ N}$
- $A = \pi r^2 = 3.14 \times (0.5 \times 10^{-3})^2 = 7.85 \times 10^{-7} \text{ m}^2$
- Stress = $F/A = 98 / 7.85 \times 10^{-7} = 1.25 \times 10^8 \text{ N/m}^2$
- $\Delta L = FL / (AY) = (98 \times 2) / (7.85 \times 10^{-7} \times 2 \times 10^{11}) = 196 / 1.57 \times 10^5 = 1.25 \times 10^{-3} \text{ m}$
- Strain = $\Delta L / L = 1.25 \times 10^{-3} / 2 = 6.24 \times 10^{-4}$

Answer: Stress = $1.25 \times 10^8 \text{ N/m}^2$, strain = 6.24×10^{-4} , elongation = 1.25 mm.

N2. A wire of length 3 m and cross-section $2 \times 10^{-6} \text{ m}^2$ extends by 1.5 mm under a load. Find the load. ($Y = 2 \times 10^{11} \text{ N/m}^2$)

Given: $L = 3 \text{ m}$, $A = 2 \times 10^{-6} \text{ m}^2$, $\Delta L = 1.5 \times 10^{-3} \text{ m}$, $Y = 2 \times 10^{11} \text{ N/m}^2$

Formula: $F = Y A \Delta L / L$

Solution:

- $F = (2 \times 10^{11} \times 2 \times 10^{-6} \times 1.5 \times 10^{-3}) / 3$
- $F = (4 \times 10^5 \times 1.5 \times 10^{-3}) / 3 = 600 / 3 = 200 \text{ N}$
- Mass = $F / g = 200 / 9.8 = 20.4 \text{ kg}$

Answer: Load = 200 N (about 20.4 kg).

N3. The pressure on water is increased by $2 \times 10^7 \text{ N/m}^2$. Find the fractional decrease in volume. (K for water = $2.2 \times 10^9 \text{ N/m}^2$)

Given: $\Delta P = 2 \times 10^7 \text{ N/m}^2$, $K = 2.2 \times 10^9 \text{ N/m}^2$

Formula: $K = \Delta P / (\Delta V/V)$, so $\Delta V/V = \Delta P / K$

Solution:

- $\Delta V/V = 2 \times 10^7 / 2.2 \times 10^9$
- $\Delta V/V = 9.09 \times 10^{-3}$

Answer: Fractional decrease in volume = 9.09×10^{-3} (about 0.91 %).

N4. A cube of side 10 cm is acted upon by a tangential force of 8×10^5 N on its top face. The top face is displaced by 0.1 mm. Find the shear stress, shear strain and modulus of rigidity.

Given: side = 0.1 m, $F = 8 \times 10^5$ N, $x = 0.1$ mm = 10^{-4} m

Formula: Shear stress = F/A ; shear strain $\theta = x/h$; $G = \text{stress} / \text{strain}$

Solution:

- $A = (0.1)^2 = 0.01 \text{ m}^2$
- Shear stress = $8 \times 10^5 / 0.01 = 8 \times 10^7 \text{ N/m}^2$
- Shear strain $\theta = 10^{-4} / 0.1 = 10^{-3}$
- $G = 8 \times 10^7 / 10^{-3} = 8 \times 10^{10} \text{ N/m}^2$

Answer: Shear stress = $8 \times 10^7 \text{ N/m}^2$, strain = 10^{-3} , $G = 8 \times 10^{10} \text{ N/m}^2$.

B. Surface Tension

N5. A soap film is formed on a rectangular frame having a sliding wire of length 5 cm. If the surface tension of the soap solution is 0.025 N/m, find the force needed to hold the wire.

Given: $l = 5$ cm = 0.05 m, $T = 0.025$ N/m; a soap film has two surfaces

Formula: $F = T \times 2l$

Solution:

- $F = 0.025 \times 2 \times 0.05$
- $F = 2.5 \times 10^{-3} \text{ N}$

Answer: $F = 2.5 \times 10^{-3} \text{ N}$.

N6. Find the work done in blowing a soap bubble from radius 2 cm to radius 4 cm. Surface tension of the soap solution is 0.03 N/m.

Given: $r_1 = 0.02$ m, $r_2 = 0.04$ m, $T = 0.03$ N/m; a bubble has two surfaces

Formula: $W = T \times \text{increase in surface area} = T \times 2 \times 4\pi (r_2^2 - r_1^2)$

Solution:

- $r_2^2 - r_1^2 = 0.0016 - 0.0004 = 0.0012 \text{ m}^2$
- Increase in area = $2 \times 4\pi \times 0.0012 = 3.016 \times 10^{-2} \text{ m}^2$
- $W = 0.03 \times 3.016 \times 10^{-2} = 9.05 \times 10^{-4} \text{ J}$

Answer: $W = 9.05 \times 10^{-4} \text{ J}$.

N7. Find the rise of water in a capillary tube of diameter 1 mm. ($T = 0.072$ N/m, $\theta = 0^\circ$, $\rho = 1000 \text{ kg/m}^3$)

Given: $d = 1$ mm so $r = 0.5 \times 10^{-3}$ m, $T = 0.072$ N/m, $\cos 0^\circ = 1$, $\rho = 1000 \text{ kg/m}^3$

Formula: $h = 2T \cos \theta / (r \rho g)$

Solution:

- $h = (2 \times 0.072 \times 1) / (0.5 \times 10^{-3} \times 1000 \times 9.8)$
- $h = 0.144 / 4.9 = 0.0294 \text{ m}$

Answer: $h = 2.94 \text{ cm}$.

N8. Find the depression of mercury in a glass capillary of radius 1 mm. ($T = 0.465 \text{ N/m}$, $\theta = 135^\circ$, $\rho = 13600 \text{ kg/m}^3$)

Given: $r = 10^{-3} \text{ m}$, $T = 0.465 \text{ N/m}$, $\theta = 135^\circ$, $\cos 135^\circ = -0.7071$, $\rho = 13600 \text{ kg/m}^3$

Formula: $h = 2T \cos \theta / (r \rho g)$

Solution:

- $h = (2 \times 0.465 \times (-0.7071)) / (10^{-3} \times 13600 \times 9.8)$
- $h = -0.6576 / 133.28 = -4.93 \times 10^{-3} \text{ m}$

Answer: The negative sign shows depression: mercury is depressed by 4.93 mm.

C. Viscosity

N9. Find the force of viscous drag on a sphere of radius 2 mm moving at 0.5 m/s in oil of coefficient of viscosity 0.2 Pa·s.

Given: $r = 2 \times 10^{-3} \text{ m}$, $v = 0.5 \text{ m/s}$, $\eta = 0.2 \text{ Pa·s}$

Formula: $F = 6\pi \eta r v$

Solution:

- $F = 6 \times 3.14 \times 0.2 \times 2 \times 10^{-3} \times 0.5$
- $F = 3.77 \times 10^{-3} \text{ N}$

Answer: $F = 3.77 \times 10^{-3} \text{ N}$.

N10. A plate of area 0.2 m^2 moves with a speed of 0.5 m/s over a fixed plate, with a liquid film 1 mm thick between them. The force required is 2 N. Find the coefficient of viscosity.

Given: $A = 0.2 \text{ m}^2$, $v = 0.5 \text{ m/s}$, $dx = 10^{-3} \text{ m}$, $F = 2 \text{ N}$

Formula: $F = \eta A (dv/dx)$, so $\eta = F dx / (A v)$

Solution:

- $\eta = (2 \times 10^{-3}) / (0.2 \times 0.5)$
- $\eta = 2 \times 10^{-3} / 0.1 = 0.02 \text{ Pa·s}$

Answer: $\eta = 0.02 \text{ Pa·s}$ (= 0.2 poise).

N11. A steel ball of radius 1 mm falls through glycerine. Find its terminal velocity.

($\rho_{\text{steel}} = 7800 \text{ kg/m}^3$, $\sigma_{\text{glycerine}} = 1260 \text{ kg/m}^3$, $\eta = 0.83 \text{ Pa·s}$)

Given: $r = 10^{-3} \text{ m}$, $\rho = 7800 \text{ kg/m}^3$, $\sigma = 1260 \text{ kg/m}^3$, $\eta = 0.83 \text{ Pa·s}$

Formula: $v = 2 r^2 (\rho - \sigma) g / (9 \eta)$

Solution:

- $\rho - \sigma = 7800 - 1260 = 6540 \text{ kg/m}^3$
- $v = (2 \times 10^{-6} \times 6540 \times 9.8) / (9 \times 0.83)$
- $v = 0.1282 / 7.47 = 1.72 \times 10^{-2} \text{ m/s}$

Answer: Terminal velocity = $1.72 \times 10^{-2} \text{ m/s}$ (1.72 cm/s).

D. Hydrodynamics

N12. Water ($\rho = 1000 \text{ kg/m}^3$, $\eta = 10^{-3} \text{ Pa}\cdot\text{s}$) flows at 0.5 m/s in a pipe of diameter 2 cm. Find the Reynolds number and state the type of flow. Also find the maximum speed for streamline flow.

Given: $\rho = 1000 \text{ kg/m}^3$, $\eta = 10^{-3} \text{ Pa}\cdot\text{s}$, $v = 0.5 \text{ m/s}$, $D = 0.02 \text{ m}$

Formula: $Re = \rho v D / \eta$; $v_c = Re_c \eta / (\rho D)$ with $Re_c = 2000$

Solution:

- $Re = (1000 \times 0.5 \times 0.02) / 10^{-3} = 10 / 10^{-3} = 10^4$
- Since $Re = 10000 > 3000$, the flow is turbulent.
- $v_c = (2000 \times 10^{-3}) / (1000 \times 0.02) = 2 / 20 = 0.1 \text{ m/s}$

Answer: $Re = 10^4$ (turbulent flow); streamline flow is possible only up to 0.1 m/s.

N13. Water flows at 2 m/s through a pipe of diameter 10 cm which narrows to a diameter of 5 cm. Find the speed in the narrow section and the volume flow rate.

Given: $d_1 = 10 \text{ cm}$, $d_2 = 5 \text{ cm}$, $v_1 = 2 \text{ m/s}$

Formula: $A_1 v_1 = A_2 v_2$; since $A \propto d^2$, $v_2 = v_1 (d_1/d_2)^2$

Solution:

- $v_2 = 2 \times (10/5)^2 = 2 \times 4 = 8 \text{ m/s}$
- $A_1 = \pi (0.05)^2 = 7.85 \times 10^{-3} \text{ m}^2$
- $Q = A_1 v_1 = 7.85 \times 10^{-3} \times 2 = 1.57 \times 10^{-2} \text{ m}^3/\text{s}$

Answer: $v_2 = 8 \text{ m/s}$; $Q = 1.57 \times 10^{-2} \text{ m}^3/\text{s}$ (15.7 litre/s).

N14. In the horizontal pipe of Problem 13 the pressure in the wide section is $2.0 \times 10^5 \text{ N/m}^2$. Find the pressure in the narrow section.

Given: $P_1 = 2.0 \times 10^5 \text{ N/m}^2$, $v_1 = 2 \text{ m/s}$, $v_2 = 8 \text{ m/s}$, $\rho = 1000 \text{ kg/m}^3$, $h_1 = h_2$

Formula: $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

Solution:

- $P_1 - P_2 = \frac{1}{2}\rho (v_2^2 - v_1^2) = \frac{1}{2} \times 1000 \times (64 - 4) = 30000 \text{ N/m}^2$
- $P_2 = 2.0 \times 10^5 - 0.3 \times 10^5 = 1.7 \times 10^5 \text{ N/m}^2$

Answer: $P_2 = 1.7 \times 10^5 \text{ N/m}^2$ (pressure drops where speed rises).

N15. Water flows in a pipe. At section 1, $P_1 = 2.5 \times 10^5 \text{ N/m}^2$, $v_1 = 2 \text{ m/s}$, height $h_1 = 0$. At section 2, height $h_2 = 5 \text{ m}$ and $v_2 = 4 \text{ m/s}$. Find P_2 .

Given: $P_1 = 2.5 \times 10^5 \text{ N/m}^2$, $v_1 = 2 \text{ m/s}$, $v_2 = 4 \text{ m/s}$, $h_1 = 0$, $h_2 = 5 \text{ m}$, $\rho = 1000 \text{ kg/m}^3$

Formula: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

Solution:

- $P_2 = P_1 + \frac{1}{2}\rho (v_1^2 - v_2^2) + \rho g (h_1 - h_2)$
- $P_2 = 250000 + 500 \times (4 - 16) + 1000 \times 9.8 \times (0 - 5)$
- $P_2 = 250000 - 6000 - 49000 = 195000 \text{ N/m}^2$

Answer: $P_2 = 1.95 \times 10^5 \text{ N/m}^2$.

N16. Find the speed of water flowing out of a small hole made 5 m below the free surface of water in a large open tank.

Given: $h = 5$ m, tank is open so both the surface and the hole are at atmospheric pressure; speed at the surface ≈ 0

Formula: Bernoulli: $P_0 + \rho gh = P_0 + \frac{1}{2}\rho v^2$, so $v = \sqrt{2gh}$

Solution:

- $v = \sqrt{2 \times 9.8 \times 5} = \sqrt{98}$
- $v = 9.9$ m/s

Answer: Speed of efflux = 9.9 m/s.

PART 5: Practice Problems (with Answers)

P1. A steel wire of length 4 m and cross-section 2 mm^2 carries a load of 40 kg. Find the elongation. ($Y = 2 \times 10^{11} \text{ N/m}^2$)

Ans. 3.92 mm

P2. A liquid of bulk modulus $2 \times 10^9 \text{ N/m}^2$ is subjected to an extra pressure of $4 \times 10^6 \text{ N/m}^2$. Find the fractional compression.

Ans. 2×10^{-3}

P3. A soap film is stretched on a frame with a movable wire of length 8 cm. If $T = 0.03 \text{ N/m}$, find the force on the wire.

Ans. $4.8 \times 10^{-3} \text{ N}$

P4. Find the rise of water in a capillary tube of radius 0.2 mm. ($T = 0.072 \text{ N/m}$, $\theta = 0^\circ$)

Ans. 7.35 cm

P5. A raindrop of radius 0.1 mm falls in air ($\eta = 1.8 \times 10^{-5} \text{ Pa}\cdot\text{s}$, $\rho = 1000 \text{ kg/m}^3$; neglect the density of air). Find its terminal velocity.

Ans. About 1.21 m/s

P6. Oil of density 900 kg/m^3 and viscosity $0.2 \text{ Pa}\cdot\text{s}$ flows at 0.5 m/s in a pipe of diameter 2 cm. Find Re and the type of flow.

Ans. $Re = 45$, streamline flow

P7. A pipe of diameter 6 cm carries water at 1.5 m/s and narrows to a diameter of 3 cm. Find the speed in the narrow part.

Ans. 6 m/s

P8. Water flows horizontally with speeds 1 m/s and 3 m/s at two sections. Find the pressure difference between them.

Ans. 4000 N/m^2

P9. Find the speed of water leaving a hole 1.8 m below the surface of a large open tank.

Ans. 5.94 m/s