

Applied Physics 1

Chapter: Properties of Matter

Exercise Set 2: Important Questions, Solutions and Numerical Problems

This is the second set of exercise questions for the chapter **Properties of Matter**. All questions are new and do not repeat Set 1. It has short answer questions, broad (long-answer) questions, numerical problems with step-by-step solutions and practice problems with answers. Topics: **Elasticity, Surface Tension, Viscosity and Hydrodynamics**.

How to write answers: start with the definition, then the formula with every symbol and its unit. In numericals write *Given, Formula, Substitution, Answer with unit*. Take $g = 9.8 \text{ m/s}^2$ and $\pi = 3.14$ unless stated otherwise.

PART 1: Short Answer Questions with Answers

A. Elasticity

Q1. Differentiate between deforming force and restoring force.

Ans. A **deforming force** is the external force applied on a body which changes its shape or size. The **restoring force** is the internal force developed in the body which opposes the deformation and tries to bring the body back to its original state. Within the elastic limit the two are equal in magnitude and opposite in direction.

Q2. Differentiate between stress and pressure.

Stress	Pressure
Internal restoring force per unit area inside a deformed body	External force per unit area applied normally on a surface
Can be tensile, compressive or shear	Always acts normal (perpendicular) to the surface
Both have the same unit, N/m^2 (Pa)	Both have the same unit, N/m^2 (Pa)

Q3. Define breaking stress and factor of safety.

Ans. **Breaking stress** is the minimum stress at which a material fractures. **Factor of safety** = breaking stress / working (permissible) stress. It is always greater than 1, so that structures work far below their breaking limit.

Q4. What are ductile and brittle materials? Give examples.

Ans. **Ductile** materials show a large plastic region before breaking and can be drawn into wires (copper, mild steel, aluminium). **Brittle** materials break soon after the elastic limit with almost no plastic deformation (glass, cast iron, concrete).

Q5. Define compressibility. How is it related to bulk modulus?

Ans. Compressibility is the fractional decrease in volume per unit increase in pressure. It is the reciprocal of the bulk modulus: $\text{compressibility} = 1 / K$. Its unit is Pa^{-1} .

Q6. Why do liquids and gases have only bulk modulus and not Young's modulus or modulus of rigidity?

Ans. Liquids and gases cannot sustain shear and have no fixed shape, so they have no modulus of rigidity and no Young's modulus. They can resist change in volume, so only the bulk modulus applies to them.

Q7. Why are steel ropes preferred to copper ropes for cranes and lifts?

Ans. Steel has a larger Young's modulus (about 2×10^{11} Pa against about 1.2×10^{11} Pa for copper) and a much higher elastic limit and breaking stress. So for the same load, a steel rope stretches less and remains elastic, which makes it safer.

B. Surface Tension

Q8. Show that the unit N/m is equivalent to J/m².

Ans. $1 \text{ J/m}^2 = 1 (\text{N}\cdot\text{m}) / \text{m}^2 = 1 \text{ N/m}$. Hence surface tension (force per unit length) is numerically equal to surface energy per unit area.

Q9. Define surface energy.

Ans. Surface energy is the extra potential energy possessed by the surface molecules of a liquid per unit area of its surface. It equals the work done to increase the surface area by one unit and is numerically equal to surface tension.

Q10. A paint brush has separated hairs inside water but they cling together when taken out. Why?

Ans. Inside water the hairs are surrounded by liquid on all sides, so the surface of the water does not act on them. When the brush is taken out, the thin film of water between the hairs tries to contract to minimum surface area due to surface tension and pulls the hairs together.

Q11. Why does hot water clean clothes better than cold water?

Ans. Surface tension decreases with rise in temperature. Hot water, having lower surface tension, wets and penetrates the fabric more easily and removes grease and dirt more effectively.

Q12. State two factors on which the angle of contact depends.

- The nature of the liquid and the solid in contact (e.g. water-glass, mercury-glass).
- Impurities in the liquid or on the solid surface, and the temperature.

Q13. Why does water rise and mercury fall in a glass capillary tube?

Ans. For water the adhesive force with glass is greater than the cohesive force, so θ is acute and the meniscus is concave, giving a rise. For mercury cohesion is greater than adhesion, so θ is obtuse and the meniscus is convex, giving a depression.

Q14. Why can a greased needle or a razor blade float on water?

Ans. The greased surface is not wetted by water, so the water surface is pushed down and behaves like a stretched elastic sheet. The upward component of surface tension balances the weight of the needle.

C. Viscosity

Q15. Define velocity gradient. Give its unit.

Ans. Velocity gradient is the rate of change of velocity with perpendicular distance between fluid layers, dv/dx . Its SI unit is s^{-1} .

Q16. Distinguish between viscous force and solid friction.

Viscous force	Solid friction
Acts between layers of a fluid	Acts between surfaces of solids in contact
Depends on area of layers and velocity gradient	Does not depend on area of contact
Is zero when the fluid is at rest	Static friction exists even at rest

Q17. Why does a small air bubble rise slowly in a liquid?

Ans. The density of air is less than that of the liquid, so the upthrust is greater than the weight and the bubble moves up. As it rises, the viscous drag ($6\pi \eta r v$) grows until it balances the net upward force. Because r is very small, the terminal velocity ($\propto r^2$) is small.

Q18. State the conditions for the validity of Stokes' law.

- The body is a small, smooth sphere moving at low speed (streamline flow, small Reynolds number).
- The fluid is homogeneous and extends very far compared to the size of the sphere (no wall effect).

Q19. Why is oil used as a lubricant in engines instead of water?

Ans. Oil has a much higher viscosity and low volatility, so it forms a stable thin film between moving parts that reduces friction and wear. Water is too thin and would be squeezed out.

Q20. Which has the greater viscosity, honey or water? What happens when honey is heated?

Ans. Honey is far more viscous than water. On heating, the viscosity of honey decreases and it flows more easily.

D. Hydrodynamics

Q21. What is steady flow?

Ans. The flow in which the velocity, pressure and density at every fixed point of the fluid do not change with time.

Q22. What is a streamline? Why can two streamlines never cross?

Ans. A streamline is an imaginary line in a flowing fluid such that the tangent at any point gives the direction of the fluid velocity at that point. If two streamlines crossed, the fluid at the crossing point would have two directions of velocity at the same time, which is impossible.

Q23. What is critical velocity?

Ans. The maximum speed up to which the flow of a fluid in a pipe remains streamline. Above it the flow becomes turbulent. $v_c = \text{Re}_c \eta / (\rho D)$.

Q24. Define volume flow rate. Give its unit.

Ans. The volume of fluid crossing a section of the pipe per unit time: $Q = A v$. Its SI unit is m^3/s .

Q25. Why does a stream of water from a tap become narrower as it falls?

Ans. As the water falls, its speed increases due to gravity. By the equation of continuity ($A v = \text{constant}$), a higher speed means a smaller cross-sectional area, so the stream narrows.

Q26. Why do two boats moving close to and parallel to each other tend to collide?

Ans. The water in the narrow gap between the boats moves faster than the water outside, so by Bernoulli's principle the pressure in the gap is lower. The higher outside pressure pushes the boats towards each other.

PART 2: Broad (Long Answer) Questions with Answers

B1. Explain elastic and plastic behaviour of materials. Define elastic limit, breaking stress and factor of safety, and compare ductile and brittle materials.

A material is **elastic** if it regains its original shape after the load is removed, and **plastic** if it keeps the deformation permanently. Every material is elastic up to a certain stress and plastic beyond it.

- **Elastic limit:** the maximum stress up to which the body fully recovers on unloading.
- **Breaking stress:** the stress at which the material fractures.
- **Factor of safety:** breaking stress / working stress. Engineers keep it above 1 (for example 3 to 6 for many structures) to allow for defects and sudden loads.

Ductile material	Brittle material
Large plastic region on the stress-strain curve	Very small or no plastic region
Gives warning (visible stretching) before failure	Fails suddenly without warning
Can be drawn into wires, hammered into sheets	Cannot be drawn or hammered; breaks into pieces
Examples: copper, mild steel, aluminium	Examples: glass, cast iron, concrete

B2. Explain the different types of stress with engineering examples. Distinguish stress from pressure.

- **Tensile stress:** a force tries to elongate the body. Examples: lift cables, tie rods, cables of a suspension bridge.
- **Compressive stress:** a force tries to shorten the body. Examples: pillars, columns, foundations.
- **Shear (tangential) stress:** a force acts parallel to a surface. Examples: rivets and bolts in a joint, cutting with scissors or a punching machine.
- **Bulk (hydraulic) stress:** equal pressure acts on all sides. Examples: submarine hull in deep water, hydraulic cylinders.

Stress is the internal restoring force per unit area developed inside a body, while pressure is the external normal force per unit area on a surface. Both are measured in N/m^2 (pascal), but stress can act along any direction (tension, compression, shear) whereas pressure always acts normal to the surface.

B3. Explain the importance of elasticity in engineering.

- **Design below the elastic limit:** machine parts and structures are designed so that working stress stays below the elastic limit, so they do not deform permanently.
- **Choice of material:** beams and frames need a high Young's modulus for less sag; springs need a high elastic limit.
- **Cables and ropes:** extension of a cable is found from $\Delta L = FL / (AY)$ and the cable size is chosen using the factor of safety.
- **Bridges and buildings:** columns and girders are designed for known tensile and compressive stresses.
- **Shock absorbers and springs:** use elastic materials to absorb energy and regain shape.
- **Measuring instruments:** spring balances and pressure gauges work on Hooke's law.

B4. What is surface energy? Show that surface tension is equal to surface energy per unit area.

Surface energy is the extra potential energy of the surface molecules per unit area. A molecule brought from inside the liquid to the surface works against the inward cohesive force, so energy is stored in the surface.

Proof: Take a soap film on a rectangular frame with a movable wire of length l . The film has two surfaces, so the inward pull on the wire is $F = 2Tl$.

- Pull the wire outward through a small distance x . Work done $W = F \times x = 2Tl \times x$.
- Increase in surface area (both surfaces) $\Delta A = 2l \times x$.
- Therefore $W = T \times \Delta A$, which gives **$T = W / \Delta A$** .

So surface tension is numerically equal to the work done (energy stored) per unit increase in area, measured in J/m^2 , which is the same as N/m . This is why liquid surfaces always contract to the minimum area, and why small drops are spherical.

B5. Explain wetting and non-wetting liquids on the basis of cohesive and adhesive forces and angle of contact.

If the adhesive force between the liquid and the solid is greater than the cohesive force inside the liquid, the liquid **wets** the solid. If cohesion is greater, the liquid does **not wet** the solid.

Property	Water on clean glass	Mercury on glass
Cohesion and adhesion	Adhesion > cohesion	Cohesion > adhesion
Angle of contact	Acute (nearly 0° to 8°)	Obtuse (about 135°)
Shape of meniscus	Concave	Convex
Behaviour in a capillary	Rises	Depressed
Wetting	Wets the glass	Does not wet the glass

- Adding soap or detergent to water lowers the angle of contact, so it wets better.
- Wax or grease on a surface increases the angle of contact, which is the principle of waterproofing.
- The rise or fall in a capillary tube is given by $h = 2T \cos\theta / (r \rho g)$.

B6. What is capillarity? Write the formula for capillary rise, state the factors affecting it and give its applications.

The rise or fall of a liquid in a tube of very fine bore dipped in it is called **capillarity** (capillary action).

$$h = 2T \cos\theta / (r \rho g)$$

- The rise is greater in a narrower tube ($h \propto 1/r$).
- It increases with surface tension ($h \propto T$) and decreases for a denser liquid ($h \propto 1/\rho$).
- It depends on the angle of contact: rise if $\theta < 90^\circ$, fall if $\theta > 90^\circ$.

Applications and examples:

- Oil rises through the wick of a lamp; ink flows to the nib of a pen.
- Sap and water rise from roots to leaves in plants.
- Blotting paper, towels and chalk absorb liquids through fine pores.
- **Damp-proof course** in building walls prevents ground water from rising through brick pores.
- Ploughing breaks soil capillaries, so moisture is retained in the field.

B7. Explain velocity gradient and viscous force between fluid layers. Define the coefficient of viscosity and derive its dimensional formula. How is it used in lubrication?

In streamline flow, adjacent layers move with different speeds. The layer touching a fixed wall is at rest and the speed increases with distance from the wall. The rate of change of speed with distance, dv/dx , is the **velocity gradient** (unit s^{-1}). Each layer exerts a viscous (tangential) force on the next layer, given by $F = \eta A (dv/dx)$.

Coefficient of viscosity: $\eta = F / (A dv/dx) = F dx / (A v)$. It is the viscous force per unit area per unit velocity gradient.

Dimensions:

$$[\eta] = [F][dx] / ([A][v]) = [M L T^{-2}][L] / ([L^2][L T^{-1}]) = [M L^{-1} T^{-1}]$$

- SI unit: $Pa \cdot s = N s/m^2$; CGS unit: poise ($1 Pa \cdot s = 10$ poise).
- Viscosity of liquids falls and that of gases rises with temperature.
- **Lubrication:** a viscous oil film between moving metal parts replaces solid friction by fluid friction, reducing wear and heat. Oils are graded by viscosity, and the grade is chosen for the working temperature of the machine.

B8. Describe the falling ball method of determining the coefficient of viscosity. State the limitations of Stokes' law.

Principle: a small sphere falling in a viscous liquid reaches terminal velocity $v_t = 2 r^2 (\rho - \sigma) g / (9 \eta)$. Measuring v_t gives η .

Method:

- Fill a tall, wide glass jar with the liquid (such as glycerine) and mark two lines A and B, with A well below the surface so that the ball has reached terminal velocity by then.
- Measure the radius r of a small steel ball with a screw gauge, and note its density ρ and the density σ of the liquid.
- Drop the ball gently at the centre of the jar and note the time t taken to travel the distance d between A and B.
- Terminal velocity $v = d / t$.

$$\eta = 2 r^2 (\rho - \sigma) g / (9 v)$$

Precautions: use a very small smooth ball, wet it before dropping, avoid air bubbles, keep the temperature constant and drop the ball along the axis of the jar.

Limitations of Stokes' law:

- Valid only for small spheres moving at low speed (small Reynolds number).
- Fluid must be homogeneous and of very large extent (no wall effect).
- Not valid for irregular bodies or when turbulence forms behind the body.

B9. Explain the different types of fluid flow. State the properties of streamlines and the importance of Reynolds number and critical velocity.

- **Steady flow:** velocity at a fixed point does not change with time. **Unsteady flow:** it changes with time.
- **Streamline (laminar) flow:** smooth layers, no mixing, low speed.
- **Turbulent flow:** irregular motion with eddies, high speed.

Properties of streamlines:

- The tangent at any point gives the direction of fluid velocity there.
- Two streamlines never intersect.
- Streamlines crowd together where the speed is high and spread out where it is low.
- A bundle of streamlines forms a *tube of flow*; no fluid crosses its walls.

Reynolds number $Re = \rho v D / \eta$ decides the type of flow: below 2000 streamline, 2000 to 3000 unstable, above 3000 turbulent. **Critical velocity** is the speed at which Re reaches its critical value: $v_c = Re_c \eta / (\rho D)$.

Importance in engineering: pipe and pump design (turbulence needs more pumping power), flow of blood in arteries, motion of aircraft and ships, and testing scale models in wind tunnels, since two flows with the same Reynolds number behave alike.

B10. Explain the equation of continuity for a pipe system with branches. Give its practical examples.

For steady flow of an incompressible fluid, the volume of fluid entering a pipe system per second equals the volume leaving it per second.

$$A_1 v_1 = A_2 v_2$$

- **For a main pipe splitting into branches:** $Q_{\text{main}} = Q_1 + Q_2 + Q_3 + \dots$, that is $A v = A_1 v_1 + A_2 v_2 + \dots$
- **For circular pipes:** $A \propto D^2$, so $v_2 = v_1 (D_1 / D_2)^2$.
- Speed is inversely proportional to the cross-sectional area.

Examples: a hose with a narrow nozzle throws water faster and farther; a river flows slowly in a wide, deep part and fast in a narrow gorge; blood flows fast in the aorta and slowly in the capillaries, whose total cross-section is much larger; the narrowing stream from a tap.

B11. Write Bernoulli's equation and deduce its special cases. How is it used in numerical problems?

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

Special cases (formulas):

- **Fluid at rest** ($v = 0$): $P_2 - P_1 = \rho g h$, the pressure increase with depth h .
- **Horizontal flow** (h constant): $P_1 - P_2 = \frac{1}{2}\rho (v_2^2 - v_1^2)$. Pressure is lower where the speed is higher.
- **Speed of efflux** from a hole at depth h below the open surface of a tank: $v = \sqrt{2 g h}$.
- **Venturi flow:** with $v_2 = v_1 (A_1/A_2)$, the pressure difference is $P_1 - P_2 = \frac{1}{2}\rho v_1^2 [(A_1/A_2)^2 - 1]$.

Steps to solve a numerical: (i) choose two points on the same streamline, (ii) write P , v , h at both points, (iii) use the continuity equation to find the unknown speed, (iv) substitute in Bernoulli's equation in SI units and solve.

Applications (names only): Venturimeter, Pitot tube, aerofoil, atomizer, Bunsen burner, carburettor and filter pump.

PART 3: Numerical Problems with Solutions

Take $g = 9.8 \text{ m/s}^2$ wherever required.

A. Elasticity

N1. What load is needed to stretch a copper wire of length 3 m and radius 0.4 mm by 1 mm? ($Y = 1.2 \times 10^{11} \text{ N/m}^2$)

Given: $L = 3 \text{ m}$, $r = 0.4 \times 10^{-3} \text{ m}$, $\Delta L = 10^{-3} \text{ m}$, $Y = 1.2 \times 10^{11} \text{ N/m}^2$

Formula: $Y = FL / (A \Delta L)$, so $F = Y A \Delta L / L$

Solution:

- $A = \pi r^2 = 3.14 \times (0.4 \times 10^{-3})^2 = 5.02 \times 10^{-7} \text{ m}^2$
- $F = (1.2 \times 10^{11} \times 5.02 \times 10^{-7} \times 10^{-3}) / 3 = 60.2 / 3 = 20.1 \text{ N}$
- $\text{Mass} = F / g = 20.1 / 9.8 = 2.05 \text{ kg}$

Answer: $F = 20.1 \text{ N}$ (a load of about 2.05 kg).

N2. Two wires A and B of the same material carry the same load. The length of A is twice that of B and the diameter of A is half that of B. Find the ratio of their extensions.

Given: $L_A / L_B = 2$, $r_A / r_B = 1/2$, same F and Y

Formula: $\Delta L = F L / (\pi r^2 Y)$, so $\Delta L \propto L / r^2$

Solution:

- $\Delta L_A / \Delta L_B = (L_A / L_B) \times (r_B / r_A)^2$
- $= 2 \times (2)^2 = 2 \times 4 = 8$

Answer: $\Delta L_A : \Delta L_B = 8 : 1$.

N3. A steel cable of diameter 2 cm supports a load of 4000 kg. Find the stress in the cable and the factor of safety if the breaking stress of steel is $4 \times 10^8 \text{ N/m}^2$.

Given: $d = 2 \text{ cm}$ so $r = 0.01 \text{ m}$, $m = 4000 \text{ kg}$, breaking stress $= 4 \times 10^8 \text{ N/m}^2$

Formula: Stress $= F / A$; factor of safety $= \text{breaking stress} / \text{working stress}$

Solution:

- $F = mg = 4000 \times 9.8 = 39200 \text{ N}$
- $A = \pi r^2 = 3.14 \times (0.01)^2 = 3.14 \times 10^{-4} \text{ m}^2$
- Stress $= 39200 / 3.14 \times 10^{-4} = 1.25 \times 10^8 \text{ N/m}^2$
- Factor of safety $= 4 \times 10^8 / 1.25 \times 10^8 = 3.2$

Answer: Stress $= 1.25 \times 10^8 \text{ N/m}^2$; factor of safety $= 3.2$.

N4. A body of volume 100 cm^3 is subjected to an increase of pressure of $1 \times 10^7 \text{ N/m}^2$ and its volume decreases by 0.01 cm^3 . Find the bulk modulus and compressibility.

Given: $V = 100 \text{ cm}^3$, $\Delta V = 0.01 \text{ cm}^3$, $\Delta P = 10^7 \text{ N/m}^2$

Formula: $K = \Delta P \times V / \Delta V$ (magnitude); compressibility $= 1 / K$

Solution:

- $K = (10^7 \times 100) / 0.01 = 10^{11} \text{ N/m}^2$
- Compressibility $= 1 / K = 10^{-11} \text{ m}^2/\text{N}$

Answer: $K = 10^{11} \text{ N/m}^2$; compressibility $= 10^{-11} \text{ m}^2/\text{N}$.

N5. A rubber cube of side 5 cm has a tangential force of 50 N applied on its top face and the face is displaced by 1 mm. Find the modulus of rigidity.

Given: $h = 0.05 \text{ m}$, $F = 50 \text{ N}$, $x = 10^{-3} \text{ m}$

Formula: $G = (F / A) / \theta$, where $\theta = x / h$

Solution:

- $A = (0.05)^2 = 2.5 \times 10^{-3} \text{ m}^2$
- Shear stress $= 50 / 2.5 \times 10^{-3} = 2 \times 10^4 \text{ N/m}^2$
- Shear strain $\theta = 10^{-3} / 0.05 = 0.02$
- $G = 2 \times 10^4 / 0.02 = 10^6 \text{ N/m}^2$

Answer: $G = 1 \times 10^6 \text{ N/m}^2$.

B. Surface Tension

N6. A needle of length 4 cm floats on water. If the surface tension of water is 0.072 N/m, find the maximum weight the needle can have (take the surface tension to act vertically on both sides).

Given: $l = 4 \text{ cm} = 0.04 \text{ m}$, $T = 0.072 \text{ N/m}$

Formula: Weight = $2 T l$ (surface tension acts along both long sides)

Solution:

- $W = 2 \times 0.072 \times 0.04$
- $W = 5.76 \times 10^{-3} \text{ N}$

Answer: Maximum weight = $5.76 \times 10^{-3} \text{ N}$ (mass about 0.59 g).

N7. Find the work done in splitting a water drop of radius 1 cm into 8 identical small drops. ($T = 0.07 \text{ N/m}$)

Given: $R = 1 \text{ cm} = 10^{-2} \text{ m}$, $T = 0.07 \text{ N/m}$

Formula: Equal volume: $(4/3)\pi R^3 = 8 \times (4/3)\pi r^3$; $W = T \times \text{increase in area}$

Solution:

- $r^3 = R^3 / 8$, so $r = R / 2 = 0.5 \times 10^{-2} \text{ m}$
- Increase in area = $8 \times 4\pi r^2 - 4\pi R^2 = 4\pi (8 \times 2.5 \times 10^{-5} - 10^{-4})$
- $= 4\pi \times 10^{-4} = 1.257 \times 10^{-3} \text{ m}^2$
- $W = 0.07 \times 1.257 \times 10^{-3} = 8.8 \times 10^{-5} \text{ J}$

Answer: $W = 8.8 \times 10^{-5} \text{ J}$.

N8. Water rises to a height of 4 cm in a capillary tube. Find the radius of the tube. ($T = 0.072 \text{ N/m}$, $\theta = 0^\circ$, $\rho = 1000 \text{ kg/m}^3$)

Given: $h = 0.04 \text{ m}$, $T = 0.072 \text{ N/m}$, $\cos 0^\circ = 1$, $\rho = 1000 \text{ kg/m}^3$

Formula: $h = 2T \cos\theta / (r \rho g)$, so $r = 2T \cos\theta / (h \rho g)$

Solution:

- $r = (2 \times 0.072 \times 1) / (0.04 \times 1000 \times 9.8)$
- $r = 0.144 / 392 = 3.67 \times 10^{-4} \text{ m}$

Answer: $r = 0.367 \text{ mm}$.

N9. Find the rise of alcohol in a capillary tube of radius 0.4 mm. ($T = 0.022 \text{ N/m}$, $\rho = 800 \text{ kg/m}^3$, $\theta = 0^\circ$)

Given: $r = 0.4 \times 10^{-3} \text{ m}$, $T = 0.022 \text{ N/m}$, $\rho = 800 \text{ kg/m}^3$, $\cos 0^\circ = 1$

Formula: $h = 2T \cos\theta / (r \rho g)$

Solution:

- $h = (2 \times 0.022) / (0.4 \times 10^{-3} \times 800 \times 9.8)$
- $h = 0.044 / 3.136 = 1.40 \times 10^{-2} \text{ m}$

Answer: $h = 1.40 \text{ cm}$.

C. Viscosity

N10. An iron ball of diameter 4 mm falls through oil of density 900 kg/m^3 and viscosity $0.15 \text{ Pa}\cdot\text{s}$. Find its terminal velocity. (density of iron = 7800 kg/m^3)

Given: $r = 2 \times 10^{-3} \text{ m}$, $\rho = 7800 \text{ kg/m}^3$, $\sigma = 900 \text{ kg/m}^3$, $\eta = 0.15 \text{ Pa}\cdot\text{s}$

Formula: $v = \frac{2}{9} r^2 (\rho - \sigma) g / \eta$

Solution:

- $\rho - \sigma = 6900 \text{ kg/m}^3$
- $v = \frac{2 \times 4 \times 10^{-6} \times 6900 \times 9.8}{9 \times 0.15} = 0.541 / 1.35$
- $v = 0.40 \text{ m/s}$

Answer: Terminal velocity = 0.40 m/s .

N11. A ball of radius 1 mm and density 2500 kg/m^3 falls steadily through 0.3 m of a liquid of density 1200 kg/m^3 in 5 s. Find the coefficient of viscosity of the liquid.

Given: $r = 10^{-3} \text{ m}$, $\rho = 2500 \text{ kg/m}^3$, $\sigma = 1200 \text{ kg/m}^3$, $d = 0.3 \text{ m}$, $t = 5 \text{ s}$

Formula: $v = d / t$; $\eta = \frac{2}{9} r^2 (\rho - \sigma) g / v$

Solution:

- $v = 0.3 / 5 = 0.06 \text{ m/s}$
- $\eta = \frac{2 \times 10^{-6} \times 1300 \times 9.8}{9 \times 0.06} = 0.02548 / 0.54$
- $\eta = 0.047 \text{ Pa}\cdot\text{s}$

Answer: $\eta = 0.047 \text{ Pa}\cdot\text{s}$.

N12. Two spheres of the same material have radii in the ratio 1 : 2. Find the ratio of their terminal velocities in the same liquid.

Given: $r_1 : r_2 = 1 : 2$, same ρ , σ and η

Formula: $v \propto r^2$ (since $v = \frac{2}{9} r^2 (\rho - \sigma) g / \eta$)

Solution:

- $v_1 / v_2 = (r_1 / r_2)^2 = (1/2)^2 = 1/4$

Answer: $v_1 : v_2 = 1 : 4$.

N13. A square plate of side 0.5 m rests on an oil film 2 mm thick over a fixed surface. Find the speed of the plate when a force of 2 N pulls it. (η of oil = $0.5 \text{ Pa}\cdot\text{s}$)

Given: $A = 0.5 \times 0.5 = 0.25 \text{ m}^2$, $dx = 2 \times 10^{-3} \text{ m}$, $F = 2 \text{ N}$, $\eta = 0.5 \text{ Pa}\cdot\text{s}$

Formula: $F = \eta A v / dx$, so $v = F dx / (\eta A)$

Solution:

- $v = \frac{2 \times 2 \times 10^{-3}}{0.5 \times 0.25} = 4 \times 10^{-3} / 0.125$
- $v = 0.032 \text{ m/s}$

Answer: Speed = 0.032 m/s (3.2 cm/s).

D. Hydrodynamics

N14. Find the maximum rate of flow of water through a pipe of diameter 1 cm so that the flow remains streamline. ($\rho = 1000 \text{ kg/m}^3$, $\eta = 10^{-3} \text{ Pa}\cdot\text{s}$, $Re_c = 2000$)

Given: $D = 0.01 \text{ m}$, $\rho = 1000 \text{ kg/m}^3$, $\eta = 10^{-3} \text{ Pa}\cdot\text{s}$, $Re_c = 2000$

Formula: $v_c = Re_c \eta / (\rho D)$; $Q = A v_c$

Solution:

- $v_c = \frac{2000 \times 10^{-3}}{1000 \times 0.01} = 2 / 10 = 0.2 \text{ m/s}$
- $A = \pi (0.005)^2 = 7.85 \times 10^{-5} \text{ m}^2$
- $Q = 7.85 \times 10^{-5} \times 0.2 = 1.57 \times 10^{-5} \text{ m}^3/\text{s}$

Answer: $Q = 1.57 \times 10^{-5} \text{ m}^3/\text{s}$ (15.7 mL/s).

N15. Blood ($\rho = 1060 \text{ kg/m}^3$, $\eta = 3 \times 10^{-3} \text{ Pa}\cdot\text{s}$) flows at 0.3 m/s in an aorta of diameter 2.5 cm. Find the Reynolds number and comment on the flow.

Given: $\rho = 1060 \text{ kg/m}^3$, $\eta = 3 \times 10^{-3} \text{ Pa}\cdot\text{s}$, $v = 0.3 \text{ m/s}$, $D = 0.025 \text{ m}$

Formula: $Re = \rho v D / \eta$

Solution:

- $Re = (1060 \times 0.3 \times 0.025) / (3 \times 10^{-3}) = 7.95 / 3 \times 10^{-3}$
- $Re = 2650$

Answer: $Re \approx 2650$. It lies between 2000 and 3000, so the flow is in the unstable (transition) region.

N16. Water flows at 4 m/s in a main pipe of diameter 6 cm which divides into three identical pipes of diameter 2 cm each. Find the speed of water in each branch.

Given: $D_1 = 6 \text{ cm}$, $v_1 = 4 \text{ m/s}$, three branches with $D_2 = 2 \text{ cm}$

Formula: $A_1 v_1 = 3 A_2 v_2$; $A \propto D^2$

Solution:

- $v_2 = v_1 D_1^2 / (3 D_2^2)$
- $v_2 = 4 \times 36 / (3 \times 4) = 144 / 12 = 12 \text{ m/s}$

Answer: Speed in each branch = 12 m/s.

N17. A horizontal pipe of diameter 12 cm has a throat of diameter 4 cm. Water flows at 1.5 m/s in the wide part. Find the pressure difference between the wide part and the throat.

Given: $D_1 = 12 \text{ cm}$, $D_2 = 4 \text{ cm}$, $v_1 = 1.5 \text{ m/s}$, $\rho = 1000 \text{ kg/m}^3$

Formula: $v_2 = v_1 (D_1/D_2)^2$; $P_1 - P_2 = \frac{1}{2}\rho (v_2^2 - v_1^2)$

Solution:

- $v_2 = 1.5 \times (12/4)^2 = 1.5 \times 9 = 13.5 \text{ m/s}$
- $P_1 - P_2 = \frac{1}{2} \times 1000 \times (13.5^2 - 1.5^2) = 500 \times (182.25 - 2.25)$
- $= 500 \times 180 = 90000 \text{ N/m}^2$

Answer: Pressure difference = $9 \times 10^4 \text{ N/m}^2$.

N18. Water flows in a uniform pipe at 1 m/s. The pressure at the ground floor is $3 \times 10^5 \text{ N/m}^2$. Find the pressure at a point 10 m higher.

Given: $P_1 = 3 \times 10^5 \text{ N/m}^2$, $v_1 = v_2 = 1 \text{ m/s}$ (same pipe), $h = 10 \text{ m}$, $\rho = 1000 \text{ kg/m}^3$

Formula: $P_1 + \rho g h_1 = P_2 + \rho g h_2$ (speed terms cancel)

Solution:

- $P_2 = P_1 - \rho g (h_2 - h_1) = 3 \times 10^5 - 1000 \times 9.8 \times 10$
- $P_2 = 300000 - 98000 = 202000 \text{ N/m}^2$

Answer: $P_2 = 2.02 \times 10^5 \text{ N/m}^2$.

N19. A closed tank holds water with air above it at a gauge pressure of $2 \times 10^5 \text{ N/m}^2$ (above atmospheric). Find the speed of efflux from a small hole 4 m below the water surface.

Given: Excess pressure $\Delta P = 2 \times 10^5 \text{ N/m}^2$, $h = 4 \text{ m}$, $\rho = 1000 \text{ kg/m}^3$; speed at the surface ≈ 0

Formula: $\Delta P + \rho g h = \frac{1}{2} \rho v^2$, so $v = \sqrt{2 (\Delta P / \rho + g h)}$

Solution:

- $\Delta P / \rho = 2 \times 10^5 / 1000 = 200 \text{ m}^2/\text{s}^2$; $g h = 9.8 \times 4 = 39.2$
- $v = \sqrt{2 \times (200 + 39.2)} = \sqrt{478.4}$
- $v = 21.9 \text{ m/s}$

Answer: Speed of efflux = 21.9 m/s.

PART 4: Practice Problems (with Answers)

P1. A steel wire of length 2.5 m and radius 0.7 mm carries a load of 20 kg. Find the elongation. ($Y = 2 \times 10^{11} \text{ N/m}^2$)

Ans. 1.59 mm

P2. Find the fractional decrease in the volume of water at a depth of 4 km in the sea. ($K = 2.2 \times 10^9 \text{ N/m}^2$, $\rho = 1000 \text{ kg/m}^3$)

Ans. 1.78×10^{-2} (about 1.8 %)

P3. Find the work done in increasing the area of a soap film by 50 cm^2 . ($T = 0.03 \text{ N/m}$, two surfaces)

Ans. $3 \times 10^{-4} \text{ J}$

P4. Find the rise of water in a capillary tube of radius 0.25 mm. ($T = 0.072 \text{ N/m}$, $\theta = 0^\circ$)

Ans. 5.88 cm

P5. Two raindrops of radii 1 mm and 2 mm fall through air. Find the ratio of their terminal velocities.

Ans. 1 : 4

P6. A ball of radius 1.5 mm and density 2700 kg/m^3 falls in a liquid of density 1000 kg/m^3 and viscosity $0.2 \text{ Pa}\cdot\text{s}$. Find the terminal velocity.

Ans. 4.17 cm/s

P7. Water flows at 3 m/s in a pipe of diameter 4 cm that narrows to 2 cm. Find the speed in the narrow part and the flow rate.

Ans. 12 m/s; $3.77 \times 10^{-3} \text{ m}^3/\text{s}$

P8. Water ($\rho = 1000 \text{ kg/m}^3$, $\eta = 10^{-3} \text{ Pa}\cdot\text{s}$) flows at 0.05 m/s in a pipe of diameter 3 cm. Find Re and the type of flow.

Ans. Re = 1500, streamline flow

P9. Water flows horizontally with speeds 2 m/s and 6 m/s at two sections. Find the pressure difference between them.

Ans. $1.6 \times 10^4 \text{ N/m}^2$