

APPLIED PHYSICS 1

Chapter: Rotational Motion

Exercise & Question Bank with Solutions (Short, Long and Numerical Questions)

Topics covered:

- Translational and rotational motion with examples
- Torque and angular momentum – definitions and their relation
- Conservation of angular momentum (quantitative) and its applications
- Moment of inertia, its physical significance and radius of gyration
- Theorems of parallel and perpendicular axes (statements only)
- Moment of inertia of rod, ring, disc and sphere (hollow and solid) – formulae only
- Simple numerical problems

PART A : QUICK REVISION NOTES

1. Translational and Rotational Motion

In **translational motion** every particle of a body undergoes the same displacement in the same time, so the whole body can be treated as a single point (its centre of mass). In **rotational motion** every particle moves in a circle about a fixed line called the **axis of rotation**. All particles have the same angular displacement (θ), angular velocity (ω) and angular acceleration (α), but their linear speeds differ according to $v = r\omega$. Rolling motion is a combination of both.

- **Translational examples:** a car on a straight road, a freely falling stone, a lift moving up.
- **Rotational examples:** a ceiling fan, a spinning top, the Earth about its axis, a wheel on a fixed axle.
- **Combined (rolling) examples:** a wheel moving along a road, a ball rolling down an incline.
- **Link between linear and angular quantities:** $s = r\theta$, $v = r\omega$, $a_t = r\alpha$, $a_c = \omega^2 r$.
- **Equations of rotation (constant α):** $\omega = \omega_0 + \alpha t$; $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$; $\omega^2 = \omega_0^2 + 2\alpha\theta$.

2. Torque and Angular Momentum

Torque is the turning effect of a force about an axis. It is the rotational analogue of force. **Angular momentum** is the rotational analogue of linear momentum. Torque is the time rate of change of angular momentum, just as force is the time rate of change of linear momentum.

- **Torque:** $\tau = \mathbf{r} \times \mathbf{F}$, magnitude $\tau = rF \sin \theta$ (θ = angle between $\mathbf{r}$ and $\mathbf{F}$). SI unit: N·m. Dimensions: $[ML^2T^{-2}]$.
- **Angular momentum of a particle:** $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, magnitude $L = mvr \sin \theta$. SI unit: kg·m²/s (or J·s). Dimensions: $[ML^2T^{-1}]$.
- **For a rigid body** rotating about a fixed axis: $L = I\omega$.
- **Relation:** $\tau = dL/dt$, and for a rigid body $\tau = I\alpha$.

3. Conservation of Angular Momentum

If the net external torque acting on a system is zero, its total angular momentum remains constant in magnitude and direction. For a rotating body whose moment of inertia changes from I_1 to I_2 , this gives $I_1\omega_1 = I_2\omega_2$. When I decreases, ω increases, and vice versa.

- **Applications:** ice skater / ballet dancer, diver somersaulting, planetary motion (Kepler's second law), person on a rotating stool with dumbbells, neutron star formation, helicopter tail rotor,

gyroscope and stability of a spinning bicycle wheel.

4. Moment of Inertia (M.I.) and Radius of Gyration

Moment of inertia is the rotational analogue of mass. It measures a body's opposition to any change in its rotational motion. For a system of particles, $I = \sum m_i r_i^2$, where r_i is the perpendicular distance of the i -th particle from the axis. SI unit: $\text{kg}\cdot\text{m}^2$. Dimensions: $[\text{ML}^2]$. I depends on (i) the mass of the body, (ii) the distribution of mass about the axis, and (iii) the position and orientation of the axis. It does **not** depend on the angular velocity.

Radius of gyration (K) is the distance from the axis at which the entire mass of the body, if concentrated at a point, would have the same moment of inertia as the actual body. $I = MK^2$, so $K = \sqrt{I/M}$. SI unit: metre.

Theorems (statements only):

- **Parallel axis theorem:** The M.I. of a body about any axis is equal to its M.I. about a parallel axis through the centre of mass plus the product of its mass and the square of the distance between the two axes. $I = I_{\text{cm}} + Md^2$
- **Perpendicular axis theorem:** For a plane (laminar) body, the M.I. about an axis perpendicular to its plane equals the sum of its M.I. about two mutually perpendicular axes in the plane, all three axes meeting at one point. $I_z = I_x + I_y$

Moment of inertia of standard bodies (formulae only):

Body	Axis	Moment of inertia (I)	Radius of gyration (K)
Thin rod (mass M, length L)	Through centre, perpendicular to length	$ML^2/12$	$L/\sqrt{12}$
Thin rod	Through one end, perpendicular to length	$ML^2/3$	$L/\sqrt{3}$
Ring (mass M, radius R)	Through centre, perpendicular to plane	MR^2	R
Ring	Diameter	$\frac{1}{2}MR^2$	$R/\sqrt{2}$
Ring	Tangent, in plane of ring	$(3/2)MR^2$	$R\sqrt{3/2}$
Ring	Tangent, perpendicular to plane	$2MR^2$	$R\sqrt{2}$
Disc (mass M, radius R)	Through centre, perpendicular to plane	$\frac{1}{2}MR^2$	$R/\sqrt{2}$
Disc	Diameter	$\frac{1}{4}MR^2$	$R/2$
Disc	Tangent, in plane of disc	$(5/4)MR^2$	$R\sqrt{5/2}$
Solid sphere (mass M, radius R)	Diameter	$(2/5)MR^2$	$R\sqrt{2/5}$
Solid sphere	Tangent	$(7/5)MR^2$	$R\sqrt{7/5}$
Hollow sphere (mass M, radius R)	Diameter	$(2/3)MR^2$	$R\sqrt{2/3}$
Hollow sphere	Tangent	$(5/3)MR^2$	$R\sqrt{5/3}$

PART B : SHORT ANSWER QUESTIONS (2 - 3 Marks)

Q1. Define translational motion and rotational motion. Give two examples of each.

Ans. Translational motion: motion in which every particle of the body moves through the same distance in the same direction in the same time (all points follow identical paths). Examples: a car moving on a straight road; a stone falling freely.

Rotational motion: motion in which every particle of the body moves in a circle about a fixed axis. Examples: a rotating ceiling fan; a spinning top.

Q2. Define torque. Write its SI unit and dimensional formula.

Ans. Torque is the moment of a force about an axis, i.e. the turning effect produced by a force. $\tau = \mathbf{r} \times \mathbf{F} = rF \sin \theta$.

SI unit = N·m; Dimensional formula = $[ML^2T^{-2}]$.

Q3. Define angular momentum. Write its SI unit and dimensional formula.

Ans. Angular momentum of a particle about a point is the moment of its linear momentum about that point: $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, with magnitude $L = mvr \sin \theta$. For a rigid body, $L = I\omega$.

SI unit = $\text{kg}\cdot\text{m}^2\text{s}^{-1}$; Dimensional formula = $[ML^2T^{-1}]$.

Q4. What is the relation between torque and angular momentum?

Ans. Torque equals the rate of change of angular momentum: $\tau = dL/dt$. For a rigid body $L = I\omega$, hence $\tau = I(d\omega/dt) = I\alpha$.

Q5. State the law of conservation of angular momentum.

Ans. If no external torque acts on a system, its total angular momentum remains constant. If $\tau_{\text{ext}} = 0$, then $L = I\omega = \text{constant}$, i.e. $I_1\omega_1 = I_2\omega_2$.

Q6. Why does an ice skater spin faster when she pulls her arms close to her body?

Ans. There is no external torque, so angular momentum $I\omega$ is conserved. Pulling the arms in reduces the moment of inertia I . To keep $I\omega$ constant, the angular velocity ω must increase.

Q7. Why does a diver curl (tuck) the body while performing somersaults in the air?

Ans. Curling reduces the moment of inertia, so by conservation of angular momentum the angular velocity increases and more somersaults can be completed. The diver stretches out before entering the water to increase I and reduce ω .

Q8. A planet moves faster when it is nearer to the Sun. Explain.

Ans. The gravitational force of the Sun acts along the line joining the Sun and planet, so its torque about the Sun is zero. Angular momentum $L = mvr$ is therefore constant. When r is small, v must be large (Kepler's second law: equal areas in equal times).

Q9. Define moment of inertia. Give its SI unit and dimensional formula.

Ans. The moment of inertia of a body about an axis is the sum of the products of the mass of each particle and the square of its perpendicular distance from the axis: $I = \sum mr^2$.

SI unit = $\text{kg}\cdot\text{m}^2$; Dimensional formula = $[ML^2]$.

Q10. What is the physical significance of moment of inertia?

Ans.

- It is the rotational analogue of mass: it measures the inertia of a body in rotational motion.
- A body with larger I is harder to start rotating or to stop; it needs a larger torque for a given angular acceleration ($\tau = I\alpha$).
- It depends on mass, the way mass is distributed about the axis, and the position of the axis.
- Rotational kinetic energy, $\frac{1}{2}I\omega^2$, and angular momentum, $I\omega$, both depend on it.

Q11. Define radius of gyration. Write its relation with moment of inertia.

Ans. The radius of gyration is the distance from the axis of rotation at which the whole mass of the body could be assumed to be concentrated so that the moment of inertia remains unchanged. $I = MK^2$, so $K = \sqrt{I/M}$. Its SI unit is the metre.

Q12. State the theorem of parallel axes.

Ans. The moment of inertia of a body about any axis equals its moment of inertia about a parallel axis through its centre of mass plus Md^2 , where M is the mass and d is the perpendicular distance between the axes: $I = I_{cm} + Md^2$.

Q13. State the theorem of perpendicular axes.

Ans. For a plane lamina, the moment of inertia about an axis perpendicular to its plane is equal to the sum of the moments of inertia about two mutually perpendicular axes lying in the plane and intersecting the first axis at the same point: $I_z = I_x + I_y$.

Q14. Why is most of the mass of a flywheel concentrated at its rim?

Ans. Placing the mass at the rim gives a large moment of inertia for a given mass. A large I means the wheel resists changes in its angular speed, so it stores more rotational energy ($\frac{1}{2}I\omega^2$) and keeps the engine running smoothly.

Q15. Why is it easier to open a door by pushing near the handle (far from the hinge) than near the hinge?

Ans. Torque $\tau = rF \sin \theta$. For the same force F , a larger distance r from the hinge gives a larger torque, so less force is needed to open the door.

Q16. Why does a helicopter need a small tail rotor?

Ans. The main rotor gives the helicopter body an angular momentum in the opposite sense. To keep the total angular momentum zero (no external torque), the tail rotor produces a counter torque and prevents the body from spinning.

Q17. Can a force act on a body and still produce zero torque? Explain.

Ans. Yes. Since $\tau = rF \sin \theta$, the torque is zero if the line of action of the force passes through the axis ($\theta = 0^\circ$ or 180°). Example: a force applied along the axis or radially through the hinge of a door.

PART C : LONG ANSWER QUESTIONS (5 - 6 Marks)

Q1. Distinguish between translational and rotational motion. Write the analogous quantities and equations of both.

Ans. Difference:

Translational motion	Rotational motion
All particles move along parallel, identical paths.	All particles move in circles about a fixed axis.
Described by displacement s , velocity v , acceleration a .	Described by angular displacement θ , angular velocity ω , angular acceleration α .
Inertia is measured by mass m .	Inertia is measured by moment of inertia I .
Cause is force F .	Cause is torque τ .

Analogous quantities:

Quantity	Linear (translational)	Angular (rotational)
Displacement	s	θ
Velocity	$v = ds/dt$	$\omega = d\theta/dt$
Acceleration	$a = dv/dt$	$\alpha = d\omega/dt$
Inertia	mass, m	moment of inertia, I
Force / Torque	$F = ma$	$\tau = I\alpha$
Momentum	$p = mv$	$L = I\omega$
Kinetic energy	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$
Work	$W = Fs$	$W = \tau\theta$
Power	$P = Fv$	$P = \tau\omega$
Equations of motion	$v = u + at$; $s = ut + \frac{1}{2}at^2$; $v^2 = u^2 + 2as$	$\omega = \omega_0 + \alpha t$; $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$; $\omega^2 = \omega_0^2 + 2\alpha\theta$

Q2. Define torque and angular momentum. Derive the relation between them and hence show that $\tau = I\alpha$.

Ans.

Torque: $\tau = \mathbf{r} \times \mathbf{F}$, magnitude $\tau = rF \sin \theta$.

Angular momentum: $\mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathbf{r} \times m\mathbf{v}$, magnitude $L = mvr \sin \theta$.

Derivation:

1. Start with $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ and differentiate with respect to time: $d\mathbf{L}/dt = (d\mathbf{r}/dt) \times \mathbf{p} + \mathbf{r} \times (d\mathbf{p}/dt)$.
2. Here $d\mathbf{r}/dt = \mathbf{v}$ and $\mathbf{p} = m\mathbf{v}$, so the first term is $\mathbf{v} \times m\mathbf{v} = 0$ (a vector crossed with itself is zero).
3. From Newton's second law $d\mathbf{p}/dt = \mathbf{F}$, so the second term is $\mathbf{r} \times \mathbf{F} = \boldsymbol{\tau}$.
4. Therefore $\boldsymbol{\tau} = d\mathbf{L}/dt$. (Torque equals the rate of change of angular momentum.)
5. **For a rigid body** about a fixed axis, each particle has $v_i = r_i\omega$, so $L = \sum m_i v_i r_i = (\sum m_i r_i^2)\omega = I\omega$.
6. Then $\tau = dL/dt = d(I\omega)/dt = I(d\omega/dt) = I\alpha$ (I is constant for a rigid body).

Q3. State and prove the law of conservation of angular momentum. Explain any three applications.

Statement: If the resultant external torque on a system is zero, the total angular momentum of the system remains constant.

Proof: We know $\tau = dL/dt$. If $\tau_{\text{ext}} = 0$, then $dL/dt = 0$, so $L = \text{constant}$. For a rigid body $L = I\omega$, hence $I\omega = \text{constant}$, or $I_1\omega_1 = I_2\omega_2$.

Applications:

1. **Ice skater / ballet dancer:** With arms stretched, I is large and ω is small. When the arms are pulled in, I decreases, so ω increases (spin becomes faster). Stretching the arms again slows the spin.
2. **Diver:** The diver curls up after leaving the board (small I , large ω) to perform somersaults, and stretches out before entry (large I , small ω) for a clean dive.
3. **Planetary motion (Kepler's second law):** The Sun's gravitational force exerts no torque on a planet, so $L = mvr$ is constant. The planet moves faster at perihelion (small r) and slower at aphelion (large r). The areal velocity $dA/dt = L/2m$ is constant.
4. **Other examples:** a person on a rotating stool holding dumbbells; a collapsing star forming a fast-spinning neutron star; a helicopter's tail rotor; stability of a spinning top, gyroscope and bicycle wheel.

Q4. Define moment of inertia and radius of gyration. Discuss the physical significance of moment of inertia. Derive the expression for rotational kinetic energy.

Ans. Moment of inertia: $I = \sum m_i r_i^2$ (unit $\text{kg}\cdot\text{m}^2$, dimensions ML^2).

Radius of gyration: $K = \sqrt{I/M}$, i.e. $I = MK^2$ (unit m).

Physical significance:

- I plays the same role in rotation as mass does in translation ($\tau = I\alpha$ compared with $F = ma$).
- It depends on the mass, how that mass is distributed about the axis, and the position of the axis. The farther the mass from the axis, the greater is I .
- Large I is useful for storing energy (flywheel) and for stability (a tightrope walker carries a long pole).

Rotational kinetic energy: Consider a rigid body rotating with angular velocity ω about a fixed axis.

1. The i -th particle of mass m_i at distance r_i from the axis has linear speed $v_i = r_i\omega$.
2. Its kinetic energy is $\frac{1}{2}m_i v_i^2 = \frac{1}{2}m_i r_i^2 \omega^2$.
3. Total K.E. = $\sum \frac{1}{2}m_i r_i^2 \omega^2 = \frac{1}{2}(\sum m_i r_i^2)\omega^2$.
4. Therefore **K.E.** = $\frac{1}{2}I\omega^2$. Also, since $L = I\omega$, K.E. = $L^2/2I$.

Q5. State the parallel and perpendicular axes theorems. Use them to find the moment of inertia of (a) a thin rod about one end, (b) a disc about its diameter, (c) a ring about a tangent in its plane.

Parallel axis theorem: $I = I_{cm} + Md^2$ (d = distance between the two parallel axes).

Perpendicular axis theorem: $I_z = I_x + I_y$ (for a plane lamina only).

(a) Rod about one end (using parallel axes):

1. $I_{cm} = ML^2/12$ and $d = L/2$.
2. $I = ML^2/12 + M(L/2)^2 = ML^2/12 + ML^2/4 = \mathbf{ML^2/3}$.

(b) Disc about a diameter (using perpendicular axes):

1. About the axis through the centre perpendicular to the plane, $I_z = \frac{1}{2}MR^2$.
2. By symmetry, the two perpendicular diameters have equal M.I.: $I_x = I_y = I_d$.
3. $I_z = I_x + I_y$ gives $\frac{1}{2}MR^2 = 2I_d$, so $\mathbf{I_d = \frac{1}{4}MR^2}$.

(c) Ring about a tangent in its plane (using both theorems):

1. About the central axis perpendicular to the plane, $I_z = MR^2$. By symmetry $I_x = I_y$, so each diameter gives $I_d = \frac{1}{2}MR^2$.
2. The tangent in the plane is parallel to a diameter at distance $d = R$. By the parallel axis theorem, $I = \frac{1}{2}MR^2 + MR^2 = \mathbf{(3/2)MR^2}$.

Q6. Explain the terms translational, rotational and rolling motion. Why does a hollow cylinder/ring have a larger moment of inertia than a solid disc of the same mass and radius?

Ans. In translational motion all particles move alike; in rotational motion all particles circle around an axis; in rolling, the body translates and rotates at the same time (e.g. a wheel on a road).

In a ring, all the mass lies at the maximum distance R from the axis, so $I = MR^2$. In a solid disc the mass is spread from the centre to the rim, so on average it lies closer to the axis and $I = \frac{1}{2}MR^2$.

Since $I = \sum mr^2$, more mass at larger r means larger I . Hence $I_{ring} > I_{disc}$. Similarly, a hollow sphere $((2/3)MR^2)$ has a larger I than a solid sphere $((2/5)MR^2)$.

PART D : NUMERICAL PROBLEMS WITH SOLUTIONS

Use $\pi = 3.14$ (or 3.142) where required. All answers are in SI units unless stated.

Numerical 1. A wheel starts from rest and is uniformly accelerated to 120 rpm in 10 s. Find the angular acceleration and the number of revolutions made in this time.

Given: $\omega_0 = 0$; final speed = 120 rpm; $t = 10$ s.

To find: α and number of revolutions n .

Solution:

1. Convert: $\omega = 120 \text{ rpm} = 120 \times 2\pi / 60 = 4\pi \text{ rad/s} \approx 12.57 \text{ rad/s}$.
2. $\alpha = (\omega - \omega_0)/t = 4\pi / 10 = 0.4\pi \approx 1.26 \text{ rad/s}^2$.
3. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2} \times 0.4\pi \times (10)^2 = 20\pi \text{ rad}$.
4. Number of revolutions $n = \theta/2\pi = 20\pi / 2\pi = 10$.

Answer: $\alpha \approx 1.26 \text{ rad/s}^2$; the wheel makes 10 revolutions.

Numerical 2. A force of 20 N acts at a point 0.5 m from the axis of rotation, making an angle of 30° with the line joining the point and the axis. Find the torque. What is the torque if the force is perpendicular to this line?

Given: $F = 20 \text{ N}$; $r = 0.5 \text{ m}$; $\theta = 30^\circ$.

To find: Torque τ .

Solution:

1. $\tau = rF \sin \theta = 0.5 \times 20 \times \sin 30^\circ$.
2. $\sin 30^\circ = 0.5$, so $\tau = 0.5 \times 20 \times 0.5 = 5 \text{ N}\cdot\text{m}$.
3. If the force is perpendicular ($\theta = 90^\circ$): $\tau = 0.5 \times 20 \times 1 = 10 \text{ N}\cdot\text{m}$.

Answer: $5 \text{ N}\cdot\text{m}$ at 30° ; $10 \text{ N}\cdot\text{m}$ (maximum) at 90° .

Numerical 3. A flywheel of moment of inertia $2 \text{ kg}\cdot\text{m}^2$ is acted upon by a constant torque of $10 \text{ N}\cdot\text{m}$. Find (a) the angular acceleration, (b) the time to reach 50 rad/s from rest, (c) the angle turned in this time.

Given: $I = 2 \text{ kg}\cdot\text{m}^2$; $\tau = 10 \text{ N}\cdot\text{m}$; $\omega_0 = 0$; $\omega = 50 \text{ rad/s}$.

To find: α , t , θ .

Solution:

1. $\alpha = \tau/I = 10/2 = 5 \text{ rad/s}^2$.
2. $t = (\omega - \omega_0)/\alpha = 50/5 = 10 \text{ s}$.
3. $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 5 \times 10^2 = 250 \text{ rad}$. (Check: $\omega^2 = 2\alpha\theta \rightarrow 2500 = 2 \times 5 \times 250 \checkmark$)

Answer: $\alpha = 5 \text{ rad/s}^2$; $t = 10 \text{ s}$; $\theta = 250 \text{ rad}$ (≈ 39.8 revolutions).

Numerical 4. (a) A particle of mass 2 kg moves in a circle of radius 0.4 m with a speed of 5 m/s. Find its angular momentum about the centre. (b) A uniform disc of mass 2 kg and radius 0.5 m rotates at 10 rad/s about its axis. Find its angular momentum.

Given: (a) $m = 2 \text{ kg}$; $r = 0.4 \text{ m}$; $v = 5 \text{ m/s}$. (b) $M = 2 \text{ kg}$; $R = 0.5 \text{ m}$; $\omega = 10 \text{ rad/s}$.

To find: L in each case.

Solution:

1. (a) The velocity is perpendicular to the radius, so $L = mvr = 2 \times 5 \times 0.4 = 4 \text{ kg}\cdot\text{m}^2/\text{s}$.
2. (Check: $\omega = v/r = 12.5 \text{ rad/s}$, $I = mr^2 = 0.32$, $L = I\omega = 0.32 \times 12.5 = 4 \checkmark$)
3. (b) $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 2 \times (0.5)^2 = 0.25 \text{ kg}\cdot\text{m}^2$.
4. $L = I\omega = 0.25 \times 10 = 2.5 \text{ kg}\cdot\text{m}^2/\text{s}$.

Answer: (a) $4 \text{ kg}\cdot\text{m}^2/\text{s}$; (b) $2.5 \text{ kg}\cdot\text{m}^2/\text{s}$.

Numerical 5. A skater spinning at 3 rad/s with her arms outstretched has a moment of inertia $4 \text{ kg}\cdot\text{m}^2$. She pulls her arms in so that the moment of inertia becomes $1.5 \text{ kg}\cdot\text{m}^2$. Find her new angular velocity and the change in kinetic energy.

Given: $I_1 = 4 \text{ kg}\cdot\text{m}^2$; $\omega_1 = 3 \text{ rad/s}$; $I_2 = 1.5 \text{ kg}\cdot\text{m}^2$.

To find: ω_2 and $\Delta \text{K.E.}$

Solution:

1. By conservation of angular momentum: $I_1 \omega_1 = I_2 \omega_2$.
2. $\omega_2 = (4 \times 3)/1.5 = 8 \text{ rad/s}$.
3. $\text{K.E.}_1 = \frac{1}{2} I_1 \omega_1^2 = \frac{1}{2} \times 4 \times 9 = 18 \text{ J}$.
4. $\text{K.E.}_2 = \frac{1}{2} I_2 \omega_2^2 = \frac{1}{2} \times 1.5 \times 64 = 48 \text{ J}$.
5. $\Delta \text{K.E.} = 48 - 18 = 30 \text{ J}$ (the extra energy comes from the work done by the skater's muscles).

Answer: $\omega_2 = 8 \text{ rad/s}$; kinetic energy increases by 30 J.

Numerical 6. A horizontal platform of moment of inertia $200 \text{ kg}\cdot\text{m}^2$ rotates freely at 10 rpm. A man of mass 60 kg standing at its edge (radius 2 m) walks to the centre. Find the new speed of rotation.

Given: $I_{\text{platform}} = 200 \text{ kg}\cdot\text{m}^2$; $\omega_1 = 10 \text{ rpm}$; $m = 60 \text{ kg}$; $r = 2 \text{ m}$.

To find: ω_2 .

Solution:

1. Initial M.I. (man at edge): $I_1 = 200 + mr^2 = 200 + 60 \times 2^2 = 440 \text{ kg}\cdot\text{m}^2$.
2. Final M.I. (man at centre, treated as a point): $I_2 = 200 \text{ kg}\cdot\text{m}^2$.
3. $I_1 \omega_1 = I_2 \omega_2 \rightarrow \omega_2 = (440 \times 10)/200 = 22 \text{ rpm}$.

Answer: 22 rpm (the platform speeds up).

Numerical 7. A wheel of moment of inertia $4 \text{ kg}\cdot\text{m}^2$ rotating at 300 rpm is brought to rest in 20 s by a constant braking torque. Find the torque.

Given: $I = 4 \text{ kg}\cdot\text{m}^2$; $\omega_0 = 300 \text{ rpm}$; $\omega = 0$; $t = 20 \text{ s}$.

To find: Braking torque τ .

Solution:

1. $\omega_0 = 300 \times 2\pi/60 = 10\pi \text{ rad/s} \approx 31.4 \text{ rad/s}$.
2. $\alpha = (0 - 10\pi)/20 = -0.5\pi \approx -1.57 \text{ rad/s}^2$ (retardation).
3. $\tau = I\alpha = 4 \times 1.57 \approx 6.28 \text{ N}\cdot\text{m}$ (magnitude).

Answer: $\tau \approx 6.28 \text{ N}\cdot\text{m}$ ($= 2\pi \text{ N}\cdot\text{m}$), opposing the motion.

Numerical 8. Calculate the moment of inertia about the axis stated: (a) a ring of mass 2 kg, radius 0.5 m, about the axis through its centre perpendicular to its plane; (b) a disc of mass 4 kg, radius 0.3 m, about the same type of axis; (c) a solid sphere of mass 5 kg, radius 0.2 m, about a diameter; (d) a hollow sphere of mass 3 kg, radius 0.1 m, about a diameter.

Given: As listed above.

To find: I in each case.

Solution:

1. (a) Ring: $I = MR^2 = 2 \times (0.5)^2 = 0.5 \text{ kg}\cdot\text{m}^2$.
2. (b) Disc: $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 4 \times (0.3)^2 = 0.18 \text{ kg}\cdot\text{m}^2$.
3. (c) Solid sphere: $I = \frac{2}{5}MR^2 = 0.4 \times 5 \times (0.2)^2 = 0.08 \text{ kg}\cdot\text{m}^2$.
4. (d) Hollow sphere: $I = \frac{2}{3}MR^2 = \frac{2}{3} \times 3 \times (0.1)^2 = 0.02 \text{ kg}\cdot\text{m}^2$.

Answer: (a) 0.5; (b) 0.18; (c) 0.08; (d) 0.02 $\text{kg}\cdot\text{m}^2$.

Numerical 9. Using the parallel axis theorem find the moment of inertia of (a) a uniform rod of mass 3 kg and length 1.2 m about an axis through one end perpendicular to its length; (b) a ring of mass 1 kg and radius 0.4 m about an axis through a point on its rim, perpendicular to its plane; (c) a solid sphere of mass 2 kg and radius 0.1 m about a tangent.

Given: (a) $M = 3 \text{ kg}$, $L = 1.2 \text{ m}$; (b) $M = 1 \text{ kg}$, $R = 0.4 \text{ m}$; (c) $M = 2 \text{ kg}$, $R = 0.1 \text{ m}$.

To find: I in each case.

Solution:

1. (a) $I_{\text{cm}} = ML^2/12 = 3 \times 1.44/12 = 0.36 \text{ kg}\cdot\text{m}^2$; $d = L/2 = 0.6 \text{ m}$. $I = 0.36 + 3 \times (0.6)^2 = 0.36 + 1.08 = 1.44 \text{ kg}\cdot\text{m}^2$. (Check: $ML^2/3 = 3 \times 1.44/3 = 1.44 \checkmark$)
2. (b) $I_{\text{cm}} = MR^2 = 1 \times (0.4)^2 = 0.16$; $d = R = 0.4 \text{ m}$. $I = 0.16 + 1 \times (0.4)^2 = 0.32 \text{ kg}\cdot\text{m}^2$. ($= 2MR^2 \checkmark$)
3. (c) $I_{\text{cm}} = \frac{2}{5}MR^2 = 0.4 \times 2 \times 0.01 = 0.008$; $d = R = 0.1 \text{ m}$. $I = 0.008 + 2 \times (0.1)^2 = 0.008 + 0.02 = 0.028 \text{ kg}\cdot\text{m}^2$. ($= \frac{7}{5}MR^2 \checkmark$)

Answer: (a) 1.44 $\text{kg}\cdot\text{m}^2$; (b) 0.32 $\text{kg}\cdot\text{m}^2$; (c) 0.028 $\text{kg}\cdot\text{m}^2$.

Numerical 10. A uniform disc has mass 6 kg and radius 0.5 m. Find its moment of inertia about the axis through its centre perpendicular to its plane, and then, using the perpendicular axes theorem, about a diameter.

Given: $M = 6 \text{ kg}$; $R = 0.5 \text{ m}$.

To find: I_z and I_d .

Solution:

1. $I_z = \frac{1}{2}MR^2 = \frac{1}{2} \times 6 \times (0.5)^2 = 0.75 \text{ kg}\cdot\text{m}^2$.
2. By symmetry $I_x = I_y = I_d$, and $I_z = I_x + I_y = 2I_d$.
3. $I_d = I_z/2 = 0.75/2 = 0.375 \text{ kg}\cdot\text{m}^2$. (Check: $\frac{1}{4}MR^2 = 0.25 \times 6 \times 0.25 = 0.375 \checkmark$)

Answer: $I_z = 0.75 \text{ kg}\cdot\text{m}^2$; $I_d = 0.375 \text{ kg}\cdot\text{m}^2$.

Numerical 11. (a) A body of mass 10 kg has a moment of inertia $40 \text{ kg}\cdot\text{m}^2$ about an axis. Find its radius of gyration. (b) Find the radius of gyration of a solid sphere of radius 0.5 m about its diameter.

Given: (a) $M = 10 \text{ kg}$; $I = 40 \text{ kg}\cdot\text{m}^2$. (b) $R = 0.5 \text{ m}$.

To find: K in each case.

Solution:

- (a) $K = \sqrt{I/M} = \sqrt{(40/10)} = \sqrt{4} = 2 \text{ m}$.
- (b) For a solid sphere $I = (2/5)MR^2$, so $K^2 = I/M = (2/5)R^2$.
- $K = R\sqrt{(2/5)} = 0.5 \times 0.632 \approx 0.316 \text{ m}$.

Answer: (a) 2 m; (b) $\approx 0.316 \text{ m}$.

Numerical 12. A solid sphere of mass 10 kg and radius 0.1 m rotates at 600 rpm about its diameter. Find its moment of inertia, angular momentum and rotational kinetic energy.

Given: $M = 10 \text{ kg}$; $R = 0.1 \text{ m}$; $N = 600 \text{ rpm}$.

To find: I , L and $K.E.$

Solution:

- $I = (2/5)MR^2 = 0.4 \times 10 \times (0.1)^2 = 0.04 \text{ kg}\cdot\text{m}^2$.
- $\omega = 600 \times 2\pi/60 = 20\pi \approx 62.8 \text{ rad/s}$.
- $L = I\omega = 0.04 \times 62.8 \approx 2.51 \text{ kg}\cdot\text{m}^2/\text{s}$.
- $K.E. = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.04 \times (62.8)^2 = 0.02 \times 3944 \approx 78.9 \text{ J}$.

Answer: $I = 0.04 \text{ kg}\cdot\text{m}^2$; $L \approx 2.51 \text{ kg}\cdot\text{m}^2/\text{s}$; $K.E. \approx 79 \text{ J}$.

PART E : MULTIPLE CHOICE QUESTIONS

1. The SI unit of torque is

(a) N	(b) $\text{N}\cdot\text{m}$	(c) $\text{kg}\cdot\text{m}^2$	(d) $\text{J}\cdot\text{s}$
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2. The dimensional formula of angular momentum is

(a) $[\text{MLT}^{-1}]$	(b) $[\text{ML}^2\text{T}^{-2}]$	(c) $[\text{ML}^2\text{T}^{-1}]$	(d) $[\text{ML}^2]$
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3. Torque is the rate of change of

(a) linear momentum	(b) angular momentum	(c) angular velocity	(d) kinetic energy
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4. If the net external torque on a system is zero, which quantity is conserved?

(a) Linear momentum only	(b) Angular momentum	(c) Kinetic energy	(d) Angular velocity
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5. When a skater pulls her arms in, her kinetic energy

(a) increases	(b) decreases	(c) remains constant	(d) becomes zero
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6. Moment of inertia of a thin rod (mass M , length L) about an axis through its centre, perpendicular to its length, is

(a) $ML^2/3$	(b) $ML^2/12$	(c) $ML^2/2$	(d) $ML^2/6$
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7. Moment of inertia of a ring about its central axis perpendicular to its plane is

(a) MR^2	(b) $\frac{1}{2}MR^2$	(c) $\frac{1}{4}MR^2$	(d) $(2/5)MR^2$
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8. Moment of inertia of a solid sphere about its diameter is

(a) $(2/3)MR^2$	(b) $(2/5)MR^2$	(c) $\frac{1}{2}MR^2$	(d) $(7/5)MR^2$
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9. Moment of inertia of a hollow sphere about its diameter is

(a) $(2/5)MR^2$	(b) MR^2	(c) $(2/3)MR^2$	(d) $(5/3)MR^2$
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10. The perpendicular axis theorem is applicable to

(a) any 3-D body	(b) a plane lamina	(c) a sphere only	(d) a rod only
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11. Radius of gyration of a ring about its central axis perpendicular to its plane is

(a) R	(b) $R/\sqrt{2}$	(c) $R/2$	(d) 2R
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12. Rotational kinetic energy of a body is

(a) $\frac{1}{2}I\omega$	(b) $\frac{1}{2}I\omega^2$	(c) $I\omega^2$	(d) $\frac{1}{2}I^2\omega$
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Answer Key:

1 - (b); 2 - (c); 3 - (b); 4 - (b); 5 - (a); 6 - (b); 7 - (a); 8 - (b); 9 - (c); 10 - (b); 11 - (a); 12 - (b)