

APPLIED PHYSICS 1

Chapter: Rotational Motion

Numerical Problems with Solutions – Set 2

Take $\pi = 3.14$ where required. All quantities are in SI units unless stated otherwise.

SECTION A : ROTATIONAL KINEMATICS AND TORQUE

Numerical 1. A ceiling fan rotating at 600 rpm is switched off and comes to rest uniformly in 20 s. Find the angular retardation and the number of revolutions made before it stops.

Given: $\omega_0 = 600$ rpm; $\omega = 0$; $t = 20$ s.

To find: α and number of revolutions n .

Solution:

1. $\omega_0 = 600 \times 2\pi/60 = 20\pi$ rad/s ≈ 62.8 rad/s.
2. $\alpha = (\omega - \omega_0)/t = (0 - 20\pi)/20 = -\pi \approx -3.14$ rad/s².
3. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 20\pi \times 20 - \frac{1}{2} \times \pi \times 400 = 400\pi - 200\pi = 200\pi$ rad.
4. $n = \theta/2\pi = 200\pi/2\pi = 100$.

Answer: Retardation = 3.14 rad/s²; the fan makes 100 revolutions.

Numerical 2. A wheel of radius 0.4 m rotates with an angular velocity of 5 rad/s and an angular acceleration of 2 rad/s². Find the linear speed, centripetal acceleration, tangential acceleration and the magnitude of the resultant acceleration of a point on its rim.

Given: $r = 0.4$ m; $\omega = 5$ rad/s; $\alpha = 2$ rad/s².

To find: v , a_c , a_t , a .

Solution:

1. $v = r\omega = 0.4 \times 5 = 2$ m/s.
2. $a_c = \omega^2 r = 25 \times 0.4 = 10$ m/s².
3. $a_t = r\alpha = 0.4 \times 2 = 0.8$ m/s².
4. Since a_c and a_t are perpendicular, $a = \sqrt{(10^2 + 0.8^2)} = \sqrt{100.64} \approx 10.03$ m/s².

Answer: $v = 2$ m/s; $a_c = 10$ m/s²; $a_t = 0.8$ m/s²; $a \approx 10.03$ m/s².

Numerical 3. A force of 40 N is applied perpendicular to a spanner at a distance of 0.25 m from the nut. Find the torque. What force would give the same torque if the spanner were 0.5 m long (force applied at its end)?

Given: $F = 40$ N; $r_1 = 0.25$ m; $r_2 = 0.5$ m; $\theta = 90^\circ$.

To find: τ and the new force F_2 .

Solution:

1. $\tau = rF \sin 90^\circ = 0.25 \times 40 \times 1 = 10$ N·m.
2. For the same torque, $F_2 = \tau/r_2 = 10/0.5 = 20$ N.

Answer: $\tau = 10$ N·m; $F_2 = 20$ N (a longer spanner needs less force).

Numerical 4. A force of 30 N acts at a point whose position vector from the axis has magnitude 0.6 m. The angle between the force and the position vector is 60° . Find the torque about the axis.

Given: $F = 30 \text{ N}$; $r = 0.6 \text{ m}$; $\theta = 60^\circ$.

To find: τ .

Solution:

1. $\tau = rF \sin \theta = 0.6 \times 30 \times \sin 60^\circ$.
2. $\sin 60^\circ = 0.866$, so $\tau = 18 \times 0.866 \approx 15.6 \text{ N}\cdot\text{m}$.

Answer: $\tau \approx 15.6 \text{ N}\cdot\text{m}$.

Numerical 5. A uniform disc of mass 4 kg and radius 0.5 m is free to rotate about its central axis. A tangential force of 10 N is applied at its rim starting from rest. Find the torque, angular acceleration and the angular velocity after 3 s.

Given: $M = 4 \text{ kg}$; $R = 0.5 \text{ m}$; $F = 10 \text{ N}$; $\omega_0 = 0$; $t = 3 \text{ s}$.

To find: τ , α , ω .

Solution:

1. $\tau = FR = 10 \times 0.5 = 5 \text{ N}\cdot\text{m}$.
2. $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 4 \times (0.5)^2 = 0.5 \text{ kg}\cdot\text{m}^2$.
3. $\alpha = \tau/I = 5/0.5 = 10 \text{ rad/s}^2$.
4. $\omega = \omega_0 + \alpha t = 0 + 10 \times 3 = 30 \text{ rad/s}$.

Answer: $\tau = 5 \text{ N}\cdot\text{m}$; $\alpha = 10 \text{ rad/s}^2$; $\omega = 30 \text{ rad/s}$.

SECTION B : ANGULAR MOMENTUM AND ITS CONSERVATION

Numerical 6. A particle of mass 0.5 kg moves in a straight line with a speed of 4 m/s. The line of motion is at a perpendicular distance of 3 m from the origin. Find the angular momentum of the particle about the origin.

Given: $m = 0.5 \text{ kg}$; $v = 4 \text{ m/s}$; perpendicular distance $d = 3 \text{ m}$.

To find: L .

Solution:

1. $L = mvr \sin \theta$, and $r \sin \theta$ is the perpendicular distance d of the line of motion from the origin.
2. $L = m v d = 0.5 \times 4 \times 3 = 6 \text{ kg}\cdot\text{m}^2/\text{s}$.

Answer: $L = 6 \text{ kg}\cdot\text{m}^2/\text{s}$ (constant, since the particle moves freely).

Numerical 7. Find the angular momentum of the Earth due to its rotation about its own axis. (Moment of inertia of the Earth = $8.0 \times 10^{37} \text{ kg}\cdot\text{m}^2$; period of rotation = 24 hours.)

Given: $I = 8.0 \times 10^{37} \text{ kg}\cdot\text{m}^2$; $T = 24 \text{ h} = 86400 \text{ s}$.

To find: L .

Solution:

1. $\omega = 2\pi/T = 2 \times 3.14/86400 \approx 7.27 \times 10^{-5} \text{ rad/s}$.
2. $L = I\omega = 8.0 \times 10^{37} \times 7.27 \times 10^{-5} \approx 5.8 \times 10^{33} \text{ kg}\cdot\text{m}^2/\text{s}$.

Answer: $L \approx 5.8 \times 10^{33} \text{ kg}\cdot\text{m}^2/\text{s}$.

Numerical 8. The angular momentum of a rotating body changes from $20 \text{ kg}\cdot\text{m}^2/\text{s}$ to $50 \text{ kg}\cdot\text{m}^2/\text{s}$ in 5 s . Find the average torque acting on it. If its moment of inertia is $3 \text{ kg}\cdot\text{m}^2$, find the angular acceleration.

Given: $L_1 = 20$; $L_2 = 50 \text{ kg}\cdot\text{m}^2/\text{s}$; $\Delta t = 5 \text{ s}$; $I = 3 \text{ kg}\cdot\text{m}^2$.

To find: τ and α .

Solution:

1. $\tau = \Delta L / \Delta t = (50 - 20) / 5 = 6 \text{ N}\cdot\text{m}$.
2. $\alpha = \tau / I = 6 / 3 = 2 \text{ rad/s}^2$.

Answer: $\tau = 6 \text{ N}\cdot\text{m}$; $\alpha = 2 \text{ rad/s}^2$.

Numerical 9. A skater spinning at 4 rad/s has a moment of inertia of $5 \text{ kg}\cdot\text{m}^2$. She pulls in her arms and her angular velocity becomes 10 rad/s . Find the new moment of inertia and the increase in her kinetic energy.

Given: $I_1 = 5 \text{ kg}\cdot\text{m}^2$; $\omega_1 = 4 \text{ rad/s}$; $\omega_2 = 10 \text{ rad/s}$.

To find: I_2 and $\Delta \text{K.E.}$

Solution:

1. Conservation of angular momentum: $I_1 \omega_1 = I_2 \omega_2$.
2. $I_2 = (5 \times 4) / 10 = 2 \text{ kg}\cdot\text{m}^2$.
3. $\text{K.E.}_1 = \frac{1}{2} \times 5 \times 4^2 = 40 \text{ J}$; $\text{K.E.}_2 = \frac{1}{2} \times 2 \times 10^2 = 100 \text{ J}$.
4. $\Delta \text{K.E.} = 100 - 40 = 60 \text{ J}$.

Answer: $I_2 = 2 \text{ kg}\cdot\text{m}^2$; K.E. increases by 60 J .

Numerical 10. A merry-go-round is a uniform disc of mass 100 kg and radius 2 m rotating freely at 2 rad/s . A child of mass 25 kg , initially at rest, steps onto its rim. Find the new angular velocity.

Given: $M = 100 \text{ kg}$; $R = 2 \text{ m}$; $\omega_1 = 2 \text{ rad/s}$; $m = 25 \text{ kg}$.

To find: ω_2 .

Solution:

1. I of the disc $= \frac{1}{2}MR^2 = \frac{1}{2} \times 100 \times 2^2 = 200 \text{ kg}\cdot\text{m}^2$.
2. I of the child at the rim $= mR^2 = 25 \times 2^2 = 100 \text{ kg}\cdot\text{m}^2$, so $I_2 = 200 + 100 = 300 \text{ kg}\cdot\text{m}^2$.
3. Conservation of angular momentum: $200 \times 2 = 300 \times \omega_2$.
4. $\omega_2 = 400 / 300 \approx 1.33 \text{ rad/s}$.

Answer: $\omega_2 \approx 1.33 \text{ rad/s}$ (the merry-go-round slows down).

Numerical 11. A student sits on a rotating stool holding a 3 kg mass in each hand, arms stretched so that each mass is 0.8 m from the axis. The system rotates at 2 rad/s. The moment of inertia of the student and stool (without the masses) is 4 kg·m². He pulls the masses to 0.2 m from the axis. Find the new angular velocity and the final kinetic energy.

Given: $m = 3$ kg each; $r_1 = 0.8$ m; $r_2 = 0.2$ m; $\omega_1 = 2$ rad/s; $I_0 = 4$ kg·m².

To find: ω_2 and K.E.₂.

Solution:

1. $I_1 = 4 + 2 \times 3 \times (0.8)^2 = 4 + 3.84 = 7.84$ kg·m².
2. $I_2 = 4 + 2 \times 3 \times (0.2)^2 = 4 + 0.24 = 4.24$ kg·m².
3. $\omega_2 = I_1 \omega_1 / I_2 = (7.84 \times 2) / 4.24 \approx 3.70$ rad/s.
4. $K.E._2 = \frac{1}{2} I_2 \omega_2^2 = \frac{1}{2} \times 4.24 \times (3.70)^2 \approx 29.0$ J. (Initial K.E. = $\frac{1}{2} \times 7.84 \times 4 = 15.7$ J.)

Answer: $\omega_2 \approx 3.70$ rad/s; final K.E. ≈ 29 J.

Numerical 12. A planet moves round the Sun. At its nearest point the distance is 1.0×10^{11} m and its speed is 30 km/s. Find its speed at the farthest point, where the distance is 1.5×10^{11} m.

Given: $r_1 = 1.0 \times 10^{11}$ m; $v_1 = 30$ km/s; $r_2 = 1.5 \times 10^{11}$ m.

To find: v_2 .

Solution:

1. The Sun's force gives no torque on the planet, so $L = mvr$ is constant: $v_1 r_1 = v_2 r_2$ (at these points v is perpendicular to r).
2. $v_2 = v_1 r_1 / r_2 = (30 \times 1.0 \times 10^{11}) / (1.5 \times 10^{11}) = 20$ km/s.

Answer: $v_2 = 20$ km/s.

Numerical 13. A uniform spherical star rotates once in 20 days. It collapses to one-fourth of its original radius without losing mass. Find its new period of rotation.

Given: $T_1 = 20$ days; $R_2 = R_1/4$; mass constant.

To find: T_2 .

Solution:

1. $I = (2/5)MR^2$, so $I_2/I_1 = (R_2/R_1)^2 = 1/16$.
2. $I_1 \omega_1 = I_2 \omega_2 \rightarrow \omega_2 = 16\omega_1$.
3. Since $T = 2\pi/\omega$, $T_2 = T_1/16 = 20/16 = 1.25$ days.

Answer: $T_2 = 1.25$ days (≈ 30 hours).

SECTION C : MOMENT OF INERTIA, RADIUS OF GYRATION AND ROTATIONAL ENERGY

Numerical 14. Three particles of masses 2 kg, 3 kg and 5 kg are at perpendicular distances 1 m, 2 m and 0.5 m from an axis. Find the moment of inertia of the system and its radius of gyration.

Given: $m_1 = 2 \text{ kg}$, $r_1 = 1 \text{ m}$; $m_2 = 3 \text{ kg}$, $r_2 = 2 \text{ m}$; $m_3 = 5 \text{ kg}$, $r_3 = 0.5 \text{ m}$.

To find: I and K .

Solution:

1. $I = \sum mr^2 = 2 \times 1^2 + 3 \times 2^2 + 5 \times (0.5)^2 = 2 + 12 + 1.25 = 15.25 \text{ kg}\cdot\text{m}^2$.
2. Total mass $M = 2 + 3 + 5 = 10 \text{ kg}$.
3. $K = \sqrt{I/M} = \sqrt{(15.25/10)} = \sqrt{1.525} \approx 1.235 \text{ m}$.

Answer: $I = 15.25 \text{ kg}\cdot\text{m}^2$; $K \approx 1.24 \text{ m}$.

Numerical 15. Four particles, each of mass 1 kg, are placed at the corners of a square of side 2 m. Find the moment of inertia of the system about an axis through the centre of the square and perpendicular to its plane. (Neglect the mass of the connecting frame.)

Given: $m = 1 \text{ kg}$ (four); side $a = 2 \text{ m}$.

To find: I .

Solution:

1. Half the diagonal $= a\sqrt{2}/2 = 2 \times 1.414/2 = \sqrt{2} \text{ m}$. So $r^2 = 2 \text{ m}^2$ for every particle.
2. $I = \sum mr^2 = 4 \times 1 \times 2 = 8 \text{ kg}\cdot\text{m}^2$.

Answer: $I = 8 \text{ kg}\cdot\text{m}^2$.

Numerical 16. A ring and a disc each have mass 2 kg and radius 0.25 m. Find the moment of inertia of each about the axis through its centre perpendicular to its plane, and the ratio

$I_{\text{ring}} : I_{\text{disc}}$

Given: $M = 2 \text{ kg}$; $R = 0.25 \text{ m}$.

To find: I_{ring} , I_{disc} , ratio.

Solution:

1. $I_{\text{ring}} = MR^2 = 2 \times (0.25)^2 = 0.125 \text{ kg}\cdot\text{m}^2$.
2. $I_{\text{disc}} = \frac{1}{2}MR^2 = \frac{1}{2} \times 2 \times (0.25)^2 = 0.0625 \text{ kg}\cdot\text{m}^2$.
3. Ratio $= 0.125/0.0625 = 2$.

Answer: $I_{\text{ring}} = 0.125 \text{ kg}\cdot\text{m}^2$; $I_{\text{disc}} = 0.0625 \text{ kg}\cdot\text{m}^2$; ratio $= 2 : 1$.

Numerical 17. A solid sphere and a hollow sphere each have mass 4 kg and radius 0.1 m. Find the moment of inertia of each about a diameter and compare them.

Given: $M = 4 \text{ kg}$; $R = 0.1 \text{ m}$.

To find: I_{solid} , I_{hollow} .

Solution:

1. $I_{\text{solid}} = (2/5)MR^2 = 0.4 \times 4 \times (0.1)^2 = 0.016 \text{ kg}\cdot\text{m}^2$.
2. $I_{\text{hollow}} = (2/3)MR^2 = (2/3) \times 4 \times 0.01 \approx 0.0267 \text{ kg}\cdot\text{m}^2$.
3. $I_{\text{hollow}}/I_{\text{solid}} = (2/3)/(2/5) = 5/3 \approx 1.67$.

Answer: $I_{\text{solid}} = 0.016 \text{ kg}\cdot\text{m}^2$; $I_{\text{hollow}} \approx 0.0267 \text{ kg}\cdot\text{m}^2$; the hollow sphere has 1.67 times larger I .

Numerical 18. A uniform rod of mass 6 kg and length 2 m is given. Find its moment of inertia (a) about an axis through its centre perpendicular to its length and (b) about an axis through one end perpendicular to its length. Verify (b) using the parallel axis theorem.

Given: $M = 6 \text{ kg}$; $L = 2 \text{ m}$.

To find: I_{cm} and I_{end} .

Solution:

1. (a) $I_{\text{cm}} = ML^2/12 = 6 \times 4/12 = 2 \text{ kg}\cdot\text{m}^2$.
2. (b) $I_{\text{end}} = ML^2/3 = 6 \times 4/3 = 8 \text{ kg}\cdot\text{m}^2$.
3. Verification: $d = L/2 = 1 \text{ m}$, so $I = I_{\text{cm}} + Md^2 = 2 + 6 \times 1^2 = 8 \text{ kg}\cdot\text{m}^2 \checkmark$

Answer: (a) $2 \text{ kg}\cdot\text{m}^2$; (b) $8 \text{ kg}\cdot\text{m}^2$.

Numerical 19. A uniform disc of mass 4 kg and radius 0.2 m is given. Find its moment of inertia about (a) a tangent perpendicular to its plane and (b) a tangent lying in its plane.

Given: $M = 4 \text{ kg}$; $R = 0.2 \text{ m}$.

To find: I in each case.

Solution:

1. (a) $I_{\text{cm}} = \frac{1}{2}MR^2 = \frac{1}{2} \times 4 \times 0.04 = 0.08 \text{ kg}\cdot\text{m}^2$. Parallel axis ($d = R = 0.2 \text{ m}$): $I = 0.08 + 4 \times 0.04 = 0.24 \text{ kg}\cdot\text{m}^2$. ($= (3/2)MR^2 \checkmark$)
2. (b) $I_{\text{diameter}} = \frac{1}{4}MR^2 = \frac{1}{4} \times 4 \times 0.04 = 0.04 \text{ kg}\cdot\text{m}^2$. Parallel axis ($d = R$): $I = 0.04 + 4 \times 0.04 = 0.20 \text{ kg}\cdot\text{m}^2$. ($= (5/4)MR^2 \checkmark$)

Answer: (a) $0.24 \text{ kg}\cdot\text{m}^2$; (b) $0.20 \text{ kg}\cdot\text{m}^2$.

Numerical 20. Find the radius of gyration of (a) a disc of radius 0.4 m about its central axis perpendicular to its plane, and (b) a hollow sphere of radius 0.3 m about its diameter.

Given: (a) $R = 0.4 \text{ m}$. (b) $R = 0.3 \text{ m}$.

To find: K in each case.

Solution:

1. (a) $I = \frac{1}{2}MR^2 = MK^2 \rightarrow K = R/\sqrt{2} = 0.4/1.414 \approx 0.283 \text{ m}$.
2. (b) $I = (2/3)MR^2 = MK^2 \rightarrow K = R\sqrt{(2/3)} = 0.3 \times 0.8165 \approx 0.245 \text{ m}$.

Answer: (a) $K \approx 0.283 \text{ m}$; (b) $K \approx 0.245 \text{ m}$.

Numerical 21. A thin ring has mass 2 kg and radius 0.3 m. Using the perpendicular axis theorem, find its moment of inertia about a diameter. Then, using the parallel axis theorem, find it about a tangent in the plane of the ring.

Given: $M = 2 \text{ kg}$; $R = 0.3 \text{ m}$.

To find: I_d and I_{tangent} .

Solution:

1. $I_z = MR^2 = 2 \times 0.09 = 0.18 \text{ kg}\cdot\text{m}^2$.
2. By symmetry $I_x = I_y = I_d$ and $I_z = I_x + I_y = 2I_d$, so $I_d = 0.18/2 = 0.09 \text{ kg}\cdot\text{m}^2$.
3. Tangent in the plane is parallel to a diameter at $d = R$: $I = I_d + MR^2 = 0.09 + 2 \times 0.09 = 0.27 \text{ kg}\cdot\text{m}^2$. ($= (3/2)MR^2 \checkmark$)

Answer: $I_d = 0.09 \text{ kg}\cdot\text{m}^2$; $I_{\text{tangent}} = 0.27 \text{ kg}\cdot\text{m}^2$.

Numerical 22. A flywheel in the form of a uniform disc of mass 50 kg and radius 0.5 m rotates at 1200 rpm. Find its moment of inertia and the kinetic energy stored in it.

Given: $M = 50 \text{ kg}$; $R = 0.5 \text{ m}$; $N = 1200 \text{ rpm}$.

To find: I and K.E.

Solution:

1. $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 50 \times (0.5)^2 = 6.25 \text{ kg}\cdot\text{m}^2$.
2. $\omega = 1200 \times 2\pi/60 = 40\pi \approx 125.6 \text{ rad/s}$.
3. $\text{K.E.} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 6.25 \times (125.6)^2 = 3.125 \times 15775 \approx 4.93 \times 10^4 \text{ J}$. (Using $\pi = 3.1416$: $(40\pi)^2 = 15791$, giving 49,348 J.)

Answer: $I = 6.25 \text{ kg}\cdot\text{m}^2$; $\text{K.E.} \approx 4.9 \times 10^4 \text{ J}$ (about 49 kJ).

Numerical 23. How much work is required to increase the angular velocity of a wheel of moment of inertia $2 \text{ kg}\cdot\text{m}^2$ from 10 rad/s to 20 rad/s ?

Given: $I = 2 \text{ kg}\cdot\text{m}^2$; $\omega_1 = 10 \text{ rad/s}$; $\omega_2 = 20 \text{ rad/s}$.

To find: Work W .

Solution:

1. By the work-energy theorem, $W = \text{change in rotational K.E.} = \frac{1}{2}I(\omega_2^2 - \omega_1^2)$.
2. $W = \frac{1}{2} \times 2 \times (400 - 100) = 300 \text{ J}$.

Answer: $W = 300 \text{ J}$.

Numerical 24. A ring and a disc of equal mass and equal radius rotate about their central axes (perpendicular to their planes) with the same angular velocity. Find the ratio of their (a) angular momenta and (b) kinetic energies.

Given: M , R and ω are the same for both.

To find: $L_{\text{ring}}/L_{\text{disc}}$ and $\text{K.E.}_{\text{ring}}/\text{K.E.}_{\text{disc}}$.

Solution:

1. $I_{\text{ring}} = MR^2$ and $I_{\text{disc}} = \frac{1}{2}MR^2$, so $I_{\text{ring}}/I_{\text{disc}} = 2$.
2. (a) $L = I\omega$, with the same ω : $L_{\text{ring}}/L_{\text{disc}} = I_{\text{ring}}/I_{\text{disc}} = 2$.
3. (b) $\text{K.E.} = \frac{1}{2}I\omega^2$, with the same ω : $\text{ratio} = I_{\text{ring}}/I_{\text{disc}} = 2$.

Answer: (a) $2 : 1$; (b) $2 : 1$.

Numerical 25. A rotating body has a kinetic energy of 100 J and an angular momentum of $20 \text{ kg}\cdot\text{m}^2/\text{s}$. Find its moment of inertia and angular velocity.

Given: $\text{K.E.} = 100 \text{ J}$; $L = 20 \text{ kg}\cdot\text{m}^2/\text{s}$.

To find: I and ω .

Solution:

1. $\text{K.E.} = L^2/2I$, so $I = L^2/(2 \times \text{K.E.}) = 400/(2 \times 100) = 2 \text{ kg}\cdot\text{m}^2$.
2. $\omega = L/I = 20/2 = 10 \text{ rad/s}$.
3. Check: $\text{K.E.} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 2 \times 100 = 100 \text{ J} \checkmark$

Answer: $I = 2 \text{ kg}\cdot\text{m}^2$; $\omega = 10 \text{ rad/s}$.