

APPLIED PHYSICS 1

Chapter: Rotational Motion

Numerical Problems with Solutions – Set 3

Take $\pi = 3.14$ where required. All quantities are in SI units unless stated otherwise.

SECTION A : ROTATIONAL KINEMATICS AND TORQUE

Numerical 1. A wheel rotating at 120 rpm speeds up uniformly to 300 rpm in 6 s. Find the angular acceleration and the number of revolutions made in this time.

Given: $\omega_0 = 120 \text{ rpm}$; $\omega = 300 \text{ rpm}$; $t = 6 \text{ s}$.

To find: α and n .

Solution:

1. $\omega_0 = 120 \times 2\pi/60 = 4\pi \text{ rad/s}$; $\omega = 300 \times 2\pi/60 = 10\pi \text{ rad/s}$.
2. $\alpha = (\omega - \omega_0)/t = (10\pi - 4\pi)/6 = \pi \approx 3.14 \text{ rad/s}^2$.
3. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 4\pi \times 6 + \frac{1}{2} \times \pi \times 36 = 24\pi + 18\pi = 42\pi \text{ rad}$.
4. $n = \theta/2\pi = 42\pi/2\pi = 21$.

Answer: $\alpha \approx 3.14 \text{ rad/s}^2$; $n = 21$ revolutions.

Numerical 2. A grinding wheel of moment of inertia $0.4 \text{ kg}\cdot\text{m}^2$ is driven by a constant torque of $2 \text{ N}\cdot\text{m}$ from rest. Find the time to reach 40 rad/s and the angle turned in this time.

Given: $I = 0.4 \text{ kg}\cdot\text{m}^2$; $\tau = 2 \text{ N}\cdot\text{m}$; $\omega_0 = 0$; $\omega = 40 \text{ rad/s}$.

To find: t and θ .

Solution:

1. $\alpha = \tau/I = 2/0.4 = 5 \text{ rad/s}^2$.
2. $t = \omega/\alpha = 40/5 = 8 \text{ s}$.
3. $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 5 \times 64 = 160 \text{ rad}$ (≈ 25.5 revolutions).

Answer: $t = 8 \text{ s}$; $\theta = 160 \text{ rad}$.

Numerical 3. Two tangential forces of 15 N and 10 N act in opposite senses on the rim of a wheel of radius 0.2 m . Find the net torque. If the moment of inertia of the wheel is $0.25 \text{ kg}\cdot\text{m}^2$, find its angular acceleration.

Given: $F_1 = 15 \text{ N}$; $F_2 = 10 \text{ N}$ (opposite); $R = 0.2 \text{ m}$; $I = 0.25 \text{ kg}\cdot\text{m}^2$.

To find: τ_{net} and α .

Solution:

1. $\tau_{\text{net}} = (F_1 - F_2)R = (15 - 10) \times 0.2 = 1 \text{ N}\cdot\text{m}$.
2. $\alpha = \tau/I = 1/0.25 = 4 \text{ rad/s}^2$.

Answer: $\tau_{\text{net}} = 1 \text{ N}\cdot\text{m}$; $\alpha = 4 \text{ rad/s}^2$.

Numerical 4. A force of 12 N is applied perpendicular to a door at 0.9 m from the hinge. Find the torque. What force is needed to produce the same torque at 0.3 m from the hinge?

Given: $F = 12 \text{ N}$; $r_1 = 0.9 \text{ m}$; $r_2 = 0.3 \text{ m}$.

To find: τ and F_2 .

Solution:

1. $\tau = rF = 0.9 \times 12 = 10.8 \text{ N}\cdot\text{m}$.

2. $F_2 = \tau/r_2 = 10.8/0.3 = 36 \text{ N}$.

Answer: $\tau = 10.8 \text{ N}\cdot\text{m}$; $F_2 = 36 \text{ N}$ (three times larger).

SECTION B : ANGULAR MOMENTUM AND ITS CONSERVATION

Numerical 5. A particle of mass 0.2 kg moves in a circle of radius 0.5 m at 120 rpm. Find its angular velocity, linear speed and angular momentum about the centre.

Given: $m = 0.2 \text{ kg}$; $r = 0.5 \text{ m}$; $N = 120 \text{ rpm}$.

To find: ω , v , L .

Solution:

1. $\omega = 120 \times 2\pi/60 = 4\pi \approx 12.57 \text{ rad/s}$.

2. $v = r\omega = 0.5 \times 12.57 \approx 6.28 \text{ m/s}$.

3. $L = mvr = 0.2 \times 6.28 \times 0.5 \approx 0.628 \text{ kg}\cdot\text{m}^2/\text{s}$.

4. Check: $I = mr^2 = 0.05 \text{ kg}\cdot\text{m}^2$, $L = I\omega = 0.05 \times 12.57 \approx 0.628 \checkmark$

Answer: $\omega \approx 12.57 \text{ rad/s}$; $v \approx 6.28 \text{ m/s}$; $L \approx 0.628 \text{ kg}\cdot\text{m}^2/\text{s}$.

Numerical 6. A solid disc of mass 5 kg and radius 0.4 m rotates at 300 rpm about its central axis. Find its angular momentum. What constant torque will bring it to rest in 4 s?

Given: $M = 5 \text{ kg}$; $R = 0.4 \text{ m}$; $N = 300 \text{ rpm}$; $t = 4 \text{ s}$.

To find: L and τ .

Solution:

1. $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 5 \times 0.16 = 0.4 \text{ kg}\cdot\text{m}^2$.

2. $\omega = 300 \times 2\pi/60 = 10\pi \approx 31.4 \text{ rad/s}$.

3. $L = I\omega = 0.4 \times 31.4 \approx 12.6 \text{ kg}\cdot\text{m}^2/\text{s}$.

4. $\tau = \Delta L/\Delta t = (0 - 12.6)/4 \approx -3.14 \text{ N}\cdot\text{m}$, so the braking torque has magnitude $3.14 \text{ N}\cdot\text{m}$.

Answer: $L \approx 12.6 \text{ kg}\cdot\text{m}^2/\text{s}$; braking torque $\approx 3.14 \text{ N}\cdot\text{m}$.

Numerical 7. A diver in the straight (stretched) position has a moment of inertia of $15 \text{ kg}\cdot\text{m}^2$ and rotates at 2 rad/s on leaving the board. She tucks so that her moment of inertia becomes $6 \text{ kg}\cdot\text{m}^2$. Find her angular velocity in the tuck and the ratio of final to initial kinetic energy.

Given: $I_1 = 15 \text{ kg}\cdot\text{m}^2$; $\omega_1 = 2 \text{ rad/s}$; $I_2 = 6 \text{ kg}\cdot\text{m}^2$.

To find: ω_2 and $\text{K.E.}_2/\text{K.E.}_1$.

Solution:

1. $I_1\omega_1 = I_2\omega_2 \rightarrow \omega_2 = (15 \times 2)/6 = 5 \text{ rad/s}$.

2. $\text{K.E.}_1 = \frac{1}{2} \times 15 \times 4 = 30 \text{ J}$; $\text{K.E.}_2 = \frac{1}{2} \times 6 \times 25 = 75 \text{ J}$.

3. Ratio = $75/30 = 2.5$.

Answer: $\omega_2 = 5 \text{ rad/s}$; K.E. becomes 2.5 times the initial value.

Numerical 8. A turntable of moment of inertia $0.5 \text{ kg}\cdot\text{m}^2$ rotates freely at 3 rad/s . A lump of clay of mass 0.2 kg is dropped on it at a distance of 0.5 m from the axis and sticks. Find the new angular velocity and the loss of kinetic energy.

Given: $I_1 = 0.5 \text{ kg}\cdot\text{m}^2$; $\omega_1 = 3 \text{ rad/s}$; $m = 0.2 \text{ kg}$; $r = 0.5 \text{ m}$.

To find: ω_2 and loss of K.E.

Solution:

1. I of the clay $= mr^2 = 0.2 \times 0.25 = 0.05 \text{ kg}\cdot\text{m}^2$, so $I_2 = 0.5 + 0.05 = 0.55 \text{ kg}\cdot\text{m}^2$.
2. $I_1\omega_1 = I_2\omega_2 \rightarrow \omega_2 = (0.5 \times 3)/0.55 \approx 2.73 \text{ rad/s}$.
3. $\text{K.E.}_1 = \frac{1}{2} \times 0.5 \times 9 = 2.25 \text{ J}$; $\text{K.E.}_2 = \frac{1}{2} \times 0.55 \times (2.73)^2 \approx 2.05 \text{ J}$.
4. Loss $= 2.25 - 2.05 \approx 0.20 \text{ J}$ (converted to heat in the sticking process).

Answer: $\omega_2 \approx 2.73 \text{ rad/s}$; K.E. lost $\approx 0.2 \text{ J}$.

Numerical 9. Disc A of moment of inertia $2 \text{ kg}\cdot\text{m}^2$ rotates at 30 rad/s . It is suddenly coupled to a stationary disc B of moment of inertia $4 \text{ kg}\cdot\text{m}^2$ on the same axis. Find the common angular velocity and the loss of kinetic energy.

Given: $I_A = 2 \text{ kg}\cdot\text{m}^2$; $\omega_A = 30 \text{ rad/s}$; $I_B = 4 \text{ kg}\cdot\text{m}^2$; $\omega_B = 0$.

To find: ω and loss of K.E.

Solution:

1. $I_A\omega_A = (I_A + I_B)\omega \rightarrow \omega = (2 \times 30)/6 = 10 \text{ rad/s}$.
2. $\text{K.E.}_{\text{initial}} = \frac{1}{2} \times 2 \times 900 = 900 \text{ J}$; $\text{K.E.}_{\text{final}} = \frac{1}{2} \times 6 \times 100 = 300 \text{ J}$.
3. Loss $= 900 - 300 = 600 \text{ J}$.

Answer: $\omega = 10 \text{ rad/s}$; 600 J of kinetic energy is lost.

Numerical 10. A satellite moves in an elliptical orbit about the Earth. At perigee its distance from the Earth's centre is 7000 km and its speed is 9 km/s . Find its speed at apogee, where the distance is 10000 km .

Given: $r_1 = 7000 \text{ km}$; $v_1 = 9 \text{ km/s}$; $r_2 = 10000 \text{ km}$.

To find: v_2 .

Solution:

1. The Earth's pull gives no torque about its centre, so $L = mvr$ is constant.
2. $v_1r_1 = v_2r_2 \rightarrow v_2 = (9 \times 7000)/10000 = 6.3 \text{ km/s}$.

Answer: $v_2 = 6.3 \text{ km/s}$.

Numerical 11. The orbital angular momentum of the Earth about the Sun is $2.7 \times 10^{40} \text{ kg}\cdot\text{m}^2/\text{s}$ and its mass is $6.0 \times 10^{24} \text{ kg}$. Find the areal velocity of the Earth.

Given: $L = 2.7 \times 10^{40} \text{ kg}\cdot\text{m}^2/\text{s}$; $m = 6.0 \times 10^{24} \text{ kg}$.

To find: dA/dt .

Solution:

1. Areal velocity $dA/dt = L/2m$ (Kepler's second law: it is constant).
2. $dA/dt = (2.7 \times 10^{40})/(2 \times 6.0 \times 10^{24}) = 2.25 \times 10^{15} \text{ m}^2/\text{s}$.

Answer: $dA/dt = 2.25 \times 10^{15} \text{ m}^2/\text{s}$.

SECTION C : MOMENT OF INERTIA, RADIUS OF GYRATION AND ROTATIONAL ENERGY

Numerical 12. Two particles of mass 2 kg each are fixed at the ends of a light rod of length 1 m. Find the moment of inertia about an axis through the middle of the rod, perpendicular to it, and the radius of gyration.

Given: $m = 2$ kg each; $L = 1$ m; each particle at $r = 0.5$ m.

To find: I and K .

Solution:

1. $I = \Sigma mr^2 = 2 \times (0.5)^2 + 2 \times (0.5)^2 = 0.5 + 0.5 = 1 \text{ kg}\cdot\text{m}^2$.
2. Total mass $M = 4$ kg, so $K = \sqrt{I/M} = \sqrt{1/4} = 0.5$ m.

Answer: $I = 1 \text{ kg}\cdot\text{m}^2$; $K = 0.5$ m.

Numerical 13. A ring has mass 5 kg and radius 0.2 m. Find its moment of inertia about (a) the central axis perpendicular to its plane, (b) a diameter, (c) a tangent perpendicular to its plane.

Given: $M = 5$ kg; $R = 0.2$ m.

To find: I in each case.

Solution:

1. (a) $I = MR^2 = 5 \times 0.04 = 0.2 \text{ kg}\cdot\text{m}^2$.
2. (b) $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 0.2 = 0.1 \text{ kg}\cdot\text{m}^2$.
3. (c) $I = I_{\text{cm}} + MR^2 = 0.2 + 0.2 = 0.4 \text{ kg}\cdot\text{m}^2$. ($= 2MR^2$ ✓)

Answer: (a) 0.2; (b) 0.1; (c) 0.4 $\text{kg}\cdot\text{m}^2$.

Numerical 14. A solid sphere of mass 10 kg and radius 0.3 m is given. Find its moment of inertia about a diameter and about a tangent.

Given: $M = 10$ kg; $R = 0.3$ m.

To find: I_d and I_t .

Solution:

1. $I_d = (2/5)MR^2 = 0.4 \times 10 \times 0.09 = 0.36 \text{ kg}\cdot\text{m}^2$.
2. $I_t = I_d + MR^2 = 0.36 + 10 \times 0.09 = 1.26 \text{ kg}\cdot\text{m}^2$. ($= (7/5)MR^2$ ✓)

Answer: $I_d = 0.36 \text{ kg}\cdot\text{m}^2$; $I_t = 1.26 \text{ kg}\cdot\text{m}^2$.

Numerical 15. A hollow sphere of mass 6 kg and radius 0.5 m is given. Find its moment of inertia about a diameter and about a tangent, and its radius of gyration about the diameter.

Given: $M = 6$ kg; $R = 0.5$ m.

To find: I_d , I_t , K .

Solution:

1. $I_d = (2/3)MR^2 = (2/3) \times 6 \times 0.25 = 1.0 \text{ kg}\cdot\text{m}^2$.
2. $I_t = I_d + MR^2 = 1.0 + 6 \times 0.25 = 2.5 \text{ kg}\cdot\text{m}^2$. ($= (5/3)MR^2$ ✓)
3. $K = \sqrt{I_d/M} = \sqrt{1/6} \approx 0.408$ m. ($= R\sqrt{2/3} = 0.5 \times 0.8165$ ✓)

Answer: $I_d = 1.0 \text{ kg}\cdot\text{m}^2$; $I_t = 2.5 \text{ kg}\cdot\text{m}^2$; $K \approx 0.408$ m.

Numerical 16. A uniform rod of mass 2 kg and length 1.5 m rotates about an axis perpendicular to its length passing through a point 0.25 m from its centre. Find the moment of inertia.

Given: $M = 2 \text{ kg}$; $L = 1.5 \text{ m}$; $d = 0.25 \text{ m}$.

To find: I .

Solution:

1. $I_{\text{cm}} = ML^2/12 = 2 \times 2.25/12 = 0.375 \text{ kg}\cdot\text{m}^2$.
2. By the parallel axis theorem, $I = I_{\text{cm}} + Md^2 = 0.375 + 2 \times (0.25)^2 = 0.375 + 0.125 = 0.5 \text{ kg}\cdot\text{m}^2$.

Answer: $I = 0.5 \text{ kg}\cdot\text{m}^2$.

Numerical 17. Find the radius of gyration of a thin rod of length 1.2 m about (a) an axis through its centre and (b) an axis through one end, both perpendicular to the rod.

Given: $L = 1.2 \text{ m}$.

To find: K_c and K_e .

Solution:

1. (a) $I = ML^2/12 = MK^2 \rightarrow K_c = L/\sqrt{12} = 1.2/3.464 \approx 0.346 \text{ m}$.
2. (b) $I = ML^2/3 = MK^2 \rightarrow K_e = L/\sqrt{3} = 1.2/1.732 \approx 0.693 \text{ m}$.

Answer: $K_c \approx 0.346 \text{ m}$; $K_e \approx 0.693 \text{ m}$.

Numerical 18. A solid sphere of mass 10 kg has a radius of gyration of 0.3 m about its diameter. Find its radius and its moment of inertia.

Given: $M = 10 \text{ kg}$; $K = 0.3 \text{ m}$.

To find: R and I .

Solution:

1. $I = MK^2 = 10 \times 0.09 = 0.9 \text{ kg}\cdot\text{m}^2$.
2. For a solid sphere $I = (2/5)MR^2 \rightarrow R^2 = I/(0.4 M) = 0.9/4 = 0.225$.
3. $R = \sqrt{0.225} \approx 0.474 \text{ m}$.

Answer: $I = 0.9 \text{ kg}\cdot\text{m}^2$; $R \approx 0.474 \text{ m}$.

Numerical 19. A ring of mass 2 kg and radius 0.5 m rotates at 120 rpm about its central axis. Find its rotational kinetic energy.

Given: $M = 2 \text{ kg}$; $R = 0.5 \text{ m}$; $N = 120 \text{ rpm}$.

To find: K.E.

Solution:

1. $I = MR^2 = 2 \times 0.25 = 0.5 \text{ kg}\cdot\text{m}^2$.
2. $\omega = 120 \times 2\pi/60 = 4\pi \approx 12.57 \text{ rad/s}$.
3. $\text{K.E.} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.5 \times (12.57)^2 = 0.25 \times 158 \approx 39.5 \text{ J}$.

Answer: K.E. $\approx 39.5 \text{ J}$.

Numerical 20. A motor develops a torque of 20 N·m at 600 rpm. Find the power delivered.

Given: $\tau = 20 \text{ N}\cdot\text{m}$; $N = 600 \text{ rpm}$.

To find: P .

Solution:

1. $\omega = 600 \times 2\pi/60 = 20\pi \approx 62.8 \text{ rad/s}$.
2. $P = \tau\omega = 20 \times 62.8 \approx 1256 \text{ W} \approx 1.26 \text{ kW}$.

Answer: $P \approx 1.26 \text{ kW}$.

Numerical 21. A uniform disc of mass 2 kg has a moment of inertia 0.05 kg·m² about a diameter. Find its moment of inertia about the central axis perpendicular to its plane, its radius, and its moment of inertia about a tangent perpendicular to its plane.

Given: $M = 2 \text{ kg}$; $I_d = 0.05 \text{ kg}\cdot\text{m}^2$.

To find: I_z , R , I_t .

Solution:

1. Perpendicular axis theorem: $I_z = I_x + I_y = 2I_d = 0.10 \text{ kg}\cdot\text{m}^2$.
2. $I_z = \frac{1}{2}MR^2 \rightarrow R^2 = 2 \times 0.10/2 = 0.10$, so $R \approx 0.316 \text{ m}$.
3. Parallel axis theorem: $I_t = I_z + MR^2 = 0.10 + 2 \times 0.10 = 0.30 \text{ kg}\cdot\text{m}^2$. ($= (3/2)MR^2 \checkmark$)

Answer: $I_z = 0.10 \text{ kg}\cdot\text{m}^2$; $R \approx 0.316 \text{ m}$; $I_t = 0.30 \text{ kg}\cdot\text{m}^2$.

Numerical 22. A flywheel of moment of inertia 8 kg·m² slows from 120 rpm to 60 rpm. Find the loss of rotational kinetic energy.

Given: $I = 8 \text{ kg}\cdot\text{m}^2$; $N_1 = 120 \text{ rpm}$; $N_2 = 60 \text{ rpm}$.

To find: $\Delta \text{K.E.}$

Solution:

1. $\omega_1 = 4\pi \text{ rad/s}$; $\omega_2 = 2\pi \text{ rad/s}$.
2. $\text{K.E.}_1 = \frac{1}{2} \times 8 \times (4\pi)^2 = 64\pi^2 \approx 631.7 \text{ J}$; $\text{K.E.}_2 = \frac{1}{2} \times 8 \times (2\pi)^2 = 16\pi^2 \approx 157.9 \text{ J}$.
3. $\text{Loss} = 64\pi^2 - 16\pi^2 = 48\pi^2 \approx 473.7 \text{ J}$.

Answer: Loss of K.E. $\approx 474 \text{ J}$.