

APPLIED PHYSICS 1

Chapter: Rotational Motion

Numerical Problems with Solutions – Set 4

Take $\pi = 3.14$ where required. All quantities are in SI units unless stated otherwise.

SECTION A : ROTATIONAL KINEMATICS AND TORQUE

Numerical 1. A wheel starting from rest rotates with uniform angular acceleration and makes 50 revolutions in 10 s. Find the angular acceleration and the final angular velocity (in rad/s and rpm).

Given: $\omega_0 = 0$; $n = 50$ rev; $t = 10$ s.

To find: α and ω .

Solution:

1. $\theta = 50 \times 2\pi = 100\pi$ rad.
2. $\theta = \frac{1}{2}\alpha t^2 \rightarrow \alpha = 2\theta/t^2 = (2 \times 100\pi)/100 = 2\pi \approx 6.28$ rad/s².
3. $\omega = \alpha t = 2\pi \times 10 = 20\pi \approx 62.8$ rad/s.
4. In rpm: $N = \omega \times 60/2\pi = 20\pi \times 60/2\pi = 600$ rpm.

Answer: $\alpha \approx 6.28$ rad/s²; $\omega \approx 62.8$ rad/s = 600 rpm.

Numerical 2. A flywheel slows down uniformly from 1500 rpm to 900 rpm in 12 s. Find the angular retardation and the number of revolutions made in this time.

Given: $N_1 = 1500$ rpm; $N_2 = 900$ rpm; $t = 12$ s.

To find: α and n .

Solution:

1. $\omega_0 = 1500 \times 2\pi/60 = 50\pi$ rad/s; $\omega = 900 \times 2\pi/60 = 30\pi$ rad/s.
2. $\alpha = (30\pi - 50\pi)/12 = -(5\pi/3) \approx -5.24$ rad/s².
3. $\theta = \frac{1}{2}(\omega_0 + \omega)t = \frac{1}{2} \times 80\pi \times 12 = 480\pi$ rad.
4. $n = \theta/2\pi = 480\pi/2\pi = 240$.

Answer: Retardation ≈ 5.24 rad/s²; $n = 240$ revolutions.

Numerical 3. A force of 8 N acts at a point 0.5 m from the axis. The angle between the position vector and the force is 150°. Find the torque.

Given: $F = 8$ N; $r = 0.5$ m; $\theta = 150^\circ$.

To find: τ .

Solution:

1. $\tau = rF \sin \theta = 0.5 \times 8 \times \sin 150^\circ$.
2. $\sin 150^\circ = \sin 30^\circ = 0.5$, so $\tau = 4 \times 0.5 = 2$ N·m.

Answer: $\tau = 2$ N·m.

Numerical 4. A disc of moment of inertia $0.8 \text{ kg}\cdot\text{m}^2$ rotating at 40 rad/s is stopped by a constant retarding torque of $4 \text{ N}\cdot\text{m}$. Find the time taken and the angle turned before it stops.

Given: $I = 0.8 \text{ kg}\cdot\text{m}^2$; $\omega_0 = 40 \text{ rad/s}$; $\omega = 0$; $\tau = 4 \text{ N}\cdot\text{m}$ (retarding).

To find: t and θ .

Solution:

1. $\alpha = \tau/I = 4/0.8 = 5 \text{ rad/s}^2$ (retardation).
2. $t = \omega_0/\alpha = 40/5 = 8 \text{ s}$.
3. From $\omega^2 = \omega_0^2 - 2\alpha\theta$: $\theta = \omega_0^2/2\alpha = 1600/10 = 160 \text{ rad}$.

Answer: $t = 8 \text{ s}$; $\theta = 160 \text{ rad}$ (≈ 25.5 revolutions).

SECTION B : ANGULAR MOMENTUM AND ITS CONSERVATION

Numerical 5. A particle of mass 1.5 kg is at a distance of 2 m from the origin and moves with a velocity of 4 m/s . The angle between its position vector and velocity is 30° . Find its angular momentum about the origin.

Given: $m = 1.5 \text{ kg}$; $r = 2 \text{ m}$; $v = 4 \text{ m/s}$; $\theta = 30^\circ$.

To find: L .

Solution:

1. $L = mvr \sin \theta = 1.5 \times 4 \times 2 \times \sin 30^\circ$.
2. $L = 12 \times 0.5 = 6 \text{ kg}\cdot\text{m}^2/\text{s}$.

Answer: $L = 6 \text{ kg}\cdot\text{m}^2/\text{s}$.

Numerical 6. A uniform rod of mass 3 kg and length 1 m rotates about an axis through one end perpendicular to its length at 10 rad/s . Find its moment of inertia, angular momentum and kinetic energy.

Given: $M = 3 \text{ kg}$; $L = 1 \text{ m}$; $\omega = 10 \text{ rad/s}$.

To find: I , L , $K.E.$

Solution:

1. $I = ML^2/3 = 3 \times 1/3 = 1 \text{ kg}\cdot\text{m}^2$.
2. Angular momentum $= I\omega = 1 \times 10 = 10 \text{ kg}\cdot\text{m}^2/\text{s}$.
3. $K.E. = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 1 \times 100 = 50 \text{ J}$.

Answer: $I = 1 \text{ kg}\cdot\text{m}^2$; angular momentum $= 10 \text{ kg}\cdot\text{m}^2/\text{s}$; $K.E. = 50 \text{ J}$.

Numerical 7. A wheel of moment of inertia $5 \text{ kg}\cdot\text{m}^2$ rotates at 6 rad/s . A torque of $10 \text{ N}\cdot\text{m}$ acts on it in the direction of rotation for 3 s . Find the final angular momentum and final angular velocity.

Given: $I = 5 \text{ kg}\cdot\text{m}^2$; $\omega_1 = 6 \text{ rad/s}$; $\tau = 10 \text{ N}\cdot\text{m}$; $t = 3 \text{ s}$.

To find: L_2 and ω_2 .

Solution:

1. Initial angular momentum $L_1 = I\omega_1 = 5 \times 6 = 30 \text{ kg}\cdot\text{m}^2/\text{s}$.
2. From $\tau = \Delta L/\Delta t$: $\Delta L = \tau t = 10 \times 3 = 30 \text{ kg}\cdot\text{m}^2/\text{s}$.
3. $L_2 = 30 + 30 = 60 \text{ kg}\cdot\text{m}^2/\text{s}$.
4. $\omega_2 = L_2/I = 60/5 = 12 \text{ rad/s}$.

Answer: $L_2 = 60 \text{ kg}\cdot\text{m}^2/\text{s}$; $\omega_2 = 12 \text{ rad/s}$.

Numerical 8. A dancer spinning at 60 rpm with outstretched arms has a moment of inertia of $3.6 \text{ kg}\cdot\text{m}^2$. She brings her arms in and her moment of inertia becomes $1.2 \text{ kg}\cdot\text{m}^2$. Find her new speed in rpm and the ratio of final to initial kinetic energy.

Given: $N_1 = 60 \text{ rpm}$; $I_1 = 3.6 \text{ kg}\cdot\text{m}^2$; $I_2 = 1.2 \text{ kg}\cdot\text{m}^2$.

To find: N_2 and $K.E._2/K.E._1$.

Solution:

1. $I_1 N_1 = I_2 N_2$ (the factor $2\pi/60$ cancels).
2. $N_2 = (3.6 \times 60)/1.2 = 180 \text{ rpm}$.
3. $K.E. = \frac{1}{2}I\omega^2$ and $\omega_2 = 3\omega_1$, $I_2 = I_1/3$, so $K.E._2/K.E._1 = (1/3) \times 3^2 = 3$.

Answer: $N_2 = 180 \text{ rpm}$; kinetic energy becomes 3 times larger.

Numerical 9. A horizontal platform in the form of a uniform disc of mass 120 kg and radius 2 m rotates freely at 12 rpm with a man of mass 60 kg standing at its centre. The man walks to a point 1 m from the centre. Find the new speed of rotation.

Given: $M = 120 \text{ kg}$; $R = 2 \text{ m}$; $m = 60 \text{ kg}$; $N_1 = 12 \text{ rpm}$; $r = 1 \text{ m}$.

To find: N_2 .

Solution:

1. I of the platform $= \frac{1}{2}MR^2 = \frac{1}{2} \times 120 \times 4 = 240 \text{ kg}\cdot\text{m}^2$.
2. Man at the centre contributes ≈ 0 , so $I_1 = 240 \text{ kg}\cdot\text{m}^2$.
3. Man at 1 m: $I_2 = 240 + 60 \times 1^2 = 300 \text{ kg}\cdot\text{m}^2$.
4. $I_1 N_1 = I_2 N_2 \rightarrow N_2 = (240 \times 12)/300 = 9.6 \text{ rpm}$.

Answer: $N_2 = 9.6 \text{ rpm}$.

Numerical 10. A uniform rod of mass 1 kg and length 1 m rotates freely about a vertical axis through its centre at 6 rad/s. Two small beads of 0.1 kg each, initially at the centre, slide outwards along the rod to its ends. Find the final angular velocity.

Given: $M = 1 \text{ kg}$; $L = 1 \text{ m}$; $\omega_1 = 6 \text{ rad/s}$; $m = 0.1 \text{ kg}$ each.

To find: ω_2 .

Solution:

1. $I_{\text{rod}} = ML^2/12 = 1/12 \approx 0.0833 \text{ kg}\cdot\text{m}^2$. Beads at the centre add ≈ 0 , so $I_1 = 0.0833 \text{ kg}\cdot\text{m}^2$.
2. At the ends each bead is 0.5 m away: $I_{\text{beads}} = 2 \times 0.1 \times (0.5)^2 = 0.05 \text{ kg}\cdot\text{m}^2$. So $I_2 = 0.0833 + 0.05 = 0.1333 \text{ kg}\cdot\text{m}^2$.
3. $I_1 \omega_1 = I_2 \omega_2 \rightarrow \omega_2 = (0.0833 \times 6)/0.1333 = 3.75 \text{ rad/s}$.

Answer: $\omega_2 = 3.75 \text{ rad/s}$.

Numerical 11. A comet moves round the Sun. At a distance of 0.6 AU its speed is 50 km/s. Find its speed when it is 4.8 AU from the Sun.

Given: $r_1 = 0.6 \text{ AU}$; $v_1 = 50 \text{ km/s}$; $r_2 = 4.8 \text{ AU}$.

To find: v_2 .

Solution:

1. The Sun's attraction exerts no torque about the Sun, so mvr is constant (at the two points of nearest and farthest approach).
2. $v_1 r_1 = v_2 r_2 \rightarrow v_2 = (50 \times 0.6)/4.8 = 6.25 \text{ km/s}$.

Answer: $v_2 = 6.25 \text{ km/s}$.

Numerical 12. Two rotating bodies have moments of inertia in the ratio 1 : 4. Find the ratio of their angular momenta if (a) they have the same kinetic energy, (b) they have the same angular velocity.

Given: $I_1 : I_2 = 1 : 4$.

To find: $L_1 : L_2$ in each case.

Solution:

1. (a) $K.E. = L^2/2I \rightarrow L = \sqrt{2I \cdot K.E.}$. With equal K.E., $L \propto \sqrt{I}$.
2. $L_1/L_2 = \sqrt{I_1/I_2} = \sqrt{1/4} = 1/2$.
3. (b) $L = I\omega$. With equal ω , $L_1/L_2 = I_1/I_2 = 1/4$.

Answer: (a) 1 : 2; (b) 1 : 4.

SECTION C : MOMENT OF INERTIA, RADIUS OF GYRATION AND ROTATIONAL ENERGY

Numerical 13. Four particles of masses 1 kg, 2 kg, 3 kg and 4 kg are placed along a line at 0, 1 m, 2 m and 3 m from the origin. Find the moment of inertia of the system about an axis through the origin perpendicular to the line, and the radius of gyration.

Given: $m = 1, 2, 3, 4$ kg at $r = 0, 1, 2, 3$ m.

To find: I and K .

Solution:

1. $I = \sum mr^2 = 1 \times 0 + 2 \times 1^2 + 3 \times 2^2 + 4 \times 3^2 = 0 + 2 + 12 + 36 = 50 \text{ kg}\cdot\text{m}^2$.
2. $M = 1 + 2 + 3 + 4 = 10$ kg.
3. $K = \sqrt{I/M} = \sqrt{5} \approx 2.24$ m.

Answer: $I = 50 \text{ kg}\cdot\text{m}^2$; $K \approx 2.24$ m.

Numerical 14. A ring and a disc each have mass 3 kg and radius 0.4 m. Find the moment of inertia of each about a tangent lying in its own plane.

Given: $M = 3$ kg; $R = 0.4$ m.

To find: I_{ring} and I_{disc} about a tangent in the plane.

Solution:

1. Ring: $I = (3/2)MR^2 = 1.5 \times 3 \times 0.16 = 0.72 \text{ kg}\cdot\text{m}^2$. (From $I_d = \frac{1}{2}MR^2 = 0.24$, plus $MR^2 = 0.48$.)
2. Disc: $I = (5/4)MR^2 = 1.25 \times 3 \times 0.16 = 0.60 \text{ kg}\cdot\text{m}^2$. (From $I_d = \frac{1}{4}MR^2 = 0.12$, plus $MR^2 = 0.48$.)

Answer: $I_{\text{ring}} = 0.72 \text{ kg}\cdot\text{m}^2$; $I_{\text{disc}} = 0.60 \text{ kg}\cdot\text{m}^2$.

Numerical 15. A solid sphere of mass 5 kg and radius 0.2 m is given. Find its moment of inertia and radius of gyration about (a) a diameter and (b) a tangent.

Given: $M = 5$ kg; $R = 0.2$ m.

To find: I and K in each case.

Solution:

1. (a) $I_d = (2/5)MR^2 = 0.4 \times 5 \times 0.04 = 0.08 \text{ kg}\cdot\text{m}^2$. $K_d = \sqrt{(0.08/5)} = \sqrt{0.016} \approx 0.126$ m.
2. (b) $I_t = I_d + MR^2 = 0.08 + 5 \times 0.04 = 0.28 \text{ kg}\cdot\text{m}^2$. $K_t = \sqrt{(0.28/5)} = \sqrt{0.056} \approx 0.237$ m.

Answer: (a) $0.08 \text{ kg}\cdot\text{m}^2$, 0.126 m; (b) $0.28 \text{ kg}\cdot\text{m}^2$, 0.237 m.

Numerical 16. A uniform disc of radius 0.6 m has a moment of inertia $0.36 \text{ kg}\cdot\text{m}^2$ about its central axis perpendicular to its plane. Find its mass, its radius of gyration and its moment of inertia about a tangent in its plane.

Given: $R = 0.6 \text{ m}$; $I = 0.36 \text{ kg}\cdot\text{m}^2$.

To find: M , K , I_t .

Solution:

1. $I = \frac{1}{2}MR^2 \rightarrow M = 2I/R^2 = (2 \times 0.36)/0.36 = 2 \text{ kg}$.
2. $K = R/\sqrt{2} = 0.6/1.414 \approx 0.424 \text{ m}$.
3. I_t (tangent in plane) $= (5/4)MR^2 = 1.25 \times 2 \times 0.36 = 0.9 \text{ kg}\cdot\text{m}^2$.

Answer: $M = 2 \text{ kg}$; $K \approx 0.424 \text{ m}$; $I_t = 0.9 \text{ kg}\cdot\text{m}^2$.

Numerical 17. Two identical thin rods, each of mass 1 kg and length 1 m, are joined at their centres at right angles to form a cross. Find the moment of inertia of the cross about an axis through the common centre and perpendicular to its plane.

Given: $M = 1 \text{ kg}$ and $L = 1 \text{ m}$ for each rod.

To find: I of the cross.

Solution:

1. Each rod about the axis through its centre, perpendicular to its length: $I = ML^2/12 = 1/12 \text{ kg}\cdot\text{m}^2$.
2. The axis is perpendicular to the plane of both rods, so the moments of inertia add: $I = 2 \times 1/12 = 1/6 \approx 0.167 \text{ kg}\cdot\text{m}^2$.

Answer: $I \approx 0.167 \text{ kg}\cdot\text{m}^2$.

Numerical 18. A uniform rod of mass 2 kg and length 1.2 m has a particle of mass 0.5 kg attached at each end. Find the moment of inertia of the system about an axis through the centre of the rod perpendicular to its length.

Given: $M = 2 \text{ kg}$; $L = 1.2 \text{ m}$; $m = 0.5 \text{ kg}$ at each end.

To find: I .

Solution:

1. $I_{\text{rod}} = ML^2/12 = 2 \times 1.44/12 = 0.24 \text{ kg}\cdot\text{m}^2$.
2. Each particle is 0.6 m from the axis: $I_{\text{particles}} = 2 \times 0.5 \times (0.6)^2 = 0.36 \text{ kg}\cdot\text{m}^2$.
3. $I = 0.24 + 0.36 = 0.60 \text{ kg}\cdot\text{m}^2$.

Answer: $I = 0.60 \text{ kg}\cdot\text{m}^2$.

Numerical 19. A disc of mass 2 kg and radius 0.2 m rotates at 300 rpm about its central axis. Find its rotational kinetic energy and angular momentum.

Given: $M = 2 \text{ kg}$; $R = 0.2 \text{ m}$; $N = 300 \text{ rpm}$.

To find: K.E. and L .

Solution:

1. $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 2 \times 0.04 = 0.04 \text{ kg}\cdot\text{m}^2$.
2. $\omega = 300 \times 2\pi/60 = 10\pi \approx 31.4 \text{ rad/s}$.
3. K.E. $= \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.04 \times (31.4)^2 \approx 0.02 \times 986 \approx 19.7 \text{ J}$.
4. $L = I\omega = 0.04 \times 31.4 \approx 1.26 \text{ kg}\cdot\text{m}^2/\text{s}$.

Answer: K.E. $\approx 19.7 \text{ J}$; $L \approx 1.26 \text{ kg}\cdot\text{m}^2/\text{s}$.

Numerical 20. A constant torque of 5 N·m turns a wheel of moment of inertia 2 kg·m² through 10 revolutions starting from rest. Find the work done and the final angular velocity.

Given: $\tau = 5 \text{ N·m}$; $I = 2 \text{ kg·m}^2$; $\theta = 10 \text{ rev}$; $\omega_0 = 0$.

To find: W and ω .

Solution:

1. $\theta = 10 \times 2\pi = 20\pi \approx 62.8 \text{ rad}$.
2. $W = \tau\theta = 5 \times 62.8 = 314 \text{ J}$.
3. Work done = gain in rotational K.E.: $\frac{1}{2}I\omega^2 = 314 \rightarrow \omega^2 = 314 \rightarrow \omega \approx 17.7 \text{ rad/s}$.

Answer: $W = 314 \text{ J}$; $\omega \approx 17.7 \text{ rad/s}$.

Numerical 21. A uniform rod of mass 4 kg and length 1.5 m rotates about an axis through one end, perpendicular to the rod, at 5 rad/s. Find its rotational kinetic energy.

Given: $M = 4 \text{ kg}$; $L = 1.5 \text{ m}$; $\omega = 5 \text{ rad/s}$.

To find: K.E.

Solution:

1. $I = ML^2/3 = 4 \times 2.25/3 = 3 \text{ kg·m}^2$.
2. K.E. = $\frac{1}{2}I\omega^2 = \frac{1}{2} \times 3 \times 25 = 37.5 \text{ J}$.

Answer: K.E. = 37.5 J.

Numerical 22. A circular disc has a moment of inertia 0.04 kg·m² about a diameter, and its radius is 0.2 m. Find its mass and its moment of inertia about the central axis perpendicular to its plane.

Given: $I_d = 0.04 \text{ kg·m}^2$; $R = 0.2 \text{ m}$.

To find: M and I_z .

Solution:

1. $I_d = \frac{1}{4}MR^2 \rightarrow M = 4I_d/R^2 = (4 \times 0.04)/0.04 = 4 \text{ kg}$.
2. Perpendicular axis theorem: $I_z = 2I_d = 0.08 \text{ kg·m}^2$. (Check: $\frac{1}{2}MR^2 = \frac{1}{2} \times 4 \times 0.04 = 0.08 \checkmark$)

Answer: $M = 4 \text{ kg}$; $I_z = 0.08 \text{ kg·m}^2$.

Numerical 23. Two uniform discs have equal mass, and their radii are R and $2R$. Find the ratio of their moments of inertia and of their radii of gyration about their central axes perpendicular to their planes.

Given: $M_1 = M_2$; $R_1 = R$; $R_2 = 2R$.

To find: $I_1 : I_2$ and $K_1 : K_2$.

Solution:

1. $I = \frac{1}{2}MR^2$, so $I_1/I_2 = R^2/(2R)^2 = 1/4$.
2. $K = R/\sqrt{2}$, so $K_1/K_2 = R/2R = 1/2$.

Answer: $I_1 : I_2 = 1 : 4$; $K_1 : K_2 = 1 : 2$.