

DC MACHINES AND TRANSFORMERS

CHAPTER 1 GENERAL INTRODUCTION OF ROTATING MACHINE

Topic 1.1

Mechanism of Electro-Mechanical Energy Conversion for Generator and Motor Mode

Includes

Detailed theory in paragraph, list and table form

Basic laws, generator mode, motor mode, energy balance

Diagrams, formulas and solved numericals

15 MCQs with answer key and 10 broad questions

Study notes for Engineering and Diploma students

Poly Notes Hub

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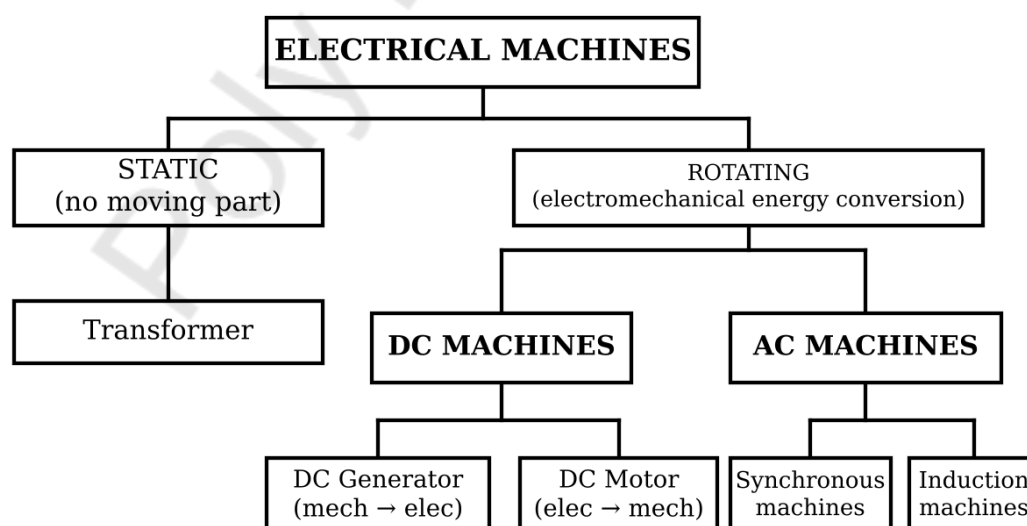
1.1.1 Introduction to Electrical Machines

An **electrical machine** is a device that converts energy from one form to another, or changes the level of an electrical quantity such as voltage and current, using the principles of electromagnetism. Machines that contain moving parts and convert energy between the electrical and mechanical forms are called **rotating electrical machines**. Machines that have no moving part but transfer electrical energy from one circuit to another at the same frequency (usually with a change of voltage level) are called **static machines**, the best example being the transformer.

The subject **DC Machines and Transformers** begins with the study of how a rotating machine performs **electromechanical energy conversion (EMEC)**. Before studying the construction, characteristics and applications of a DC generator or a DC motor, a student must understand the common energy-conversion mechanism that sits at the heart of every rotating machine.

Table 1.1 Broad classification of energy-conversion devices

Class of machine	Energy conversion	Moving part	Common examples
Generator	Mechanical → Electrical	Yes (rotor / armature)	DC generator, alternator
Motor	Electrical → Mechanical	Yes (rotor / armature)	DC motor, induction motor, synchronous motor
Transformer	Electrical → Electrical (voltage level changes, frequency same)	No (static)	Power transformer, distribution transformer, instrument transformer
Converter / Rectifier	AC ↔ DC	No (static, electronic)	Rectifier, inverter



Every rotating machine can work in generator mode or motor mode

Fig. 1.1 Classification of electrical machines

Why study rotating machines first?

- Nearly all electrical energy in the world is produced by rotating generators driven by turbines or engines.
- A very large share of the electrical energy produced is consumed again by motors (fans, pumps, compressors, traction, lifts, machine tools).
- The same physical laws (Faraday's law and the Lorentz force) explain generators, motors and, with small modification, transformers.
- Understanding the *energy balance* of a machine helps in analysing losses, efficiency and heating.

1.1.2 Concept of Electromechanical Energy Conversion

Electromechanical energy conversion is the process of converting electrical energy into mechanical energy (motor action) or mechanical energy into electrical energy (generator action) through the medium of a **coupling field**, which is almost always a **magnetic field**. The process follows the **law of conservation of energy**: energy is neither created nor destroyed, it only changes its form. The input energy is therefore equal to the useful output energy plus the energy stored in the field plus the energy lost as heat.

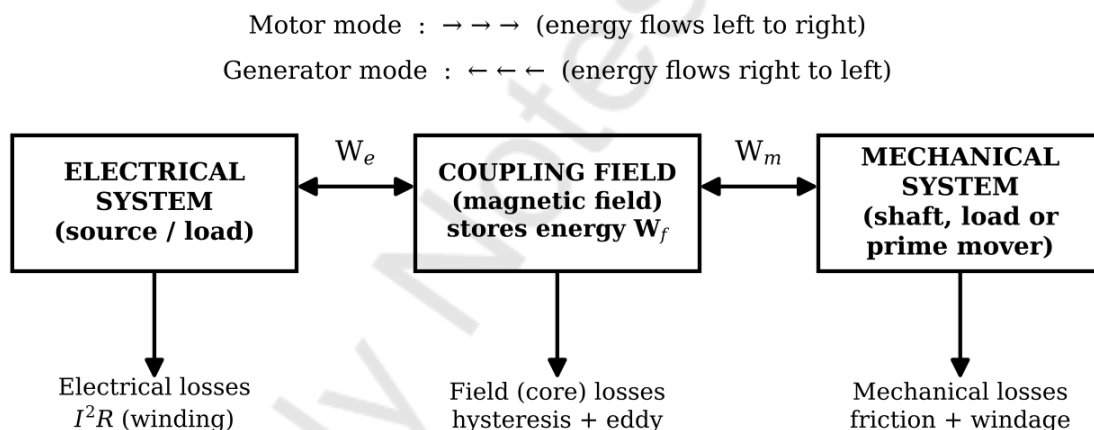


Fig. 1.2 Block diagram of an electromechanical energy conversion system

Three parts of an EMEC system

1. **Electrical system** - the source (supply, battery) or the load (lamps, heaters, grid) connected to the winding terminals. It exchanges electrical energy W_e with the coupling field.
2. **Coupling field** - the magnetic field in the air-gap and iron path. It acts as the *medium* that transfers energy between the two systems and also stores a small amount of energy W_f .
3. **Mechanical system** - the shaft, the rotor or armature, and the mechanical load or prime mover. It exchanges mechanical energy W_m with the coupling field.

Why is the magnetic field used as the coupling medium?

Both electric and magnetic fields can store energy, but the amount that can be stored per unit volume is very different. The energy density of an electric field is $\frac{1}{2}\epsilon_0 E^2$ and that of a magnetic field is $B^2/2\mu_0$. Air breaks down electrically at about 3 MV/m, while iron-cored machines easily work at 1 tesla or more.

Table 1.2 Electric versus magnetic field as a coupling medium

Property	Electric field coupling	Magnetic field coupling
Energy density formula	$\frac{1}{2} \epsilon_0 E^2$	$B^2 / (2 \mu_0)$
Practical limiting value	$E \approx 3 \times 10^6$ V/m (breakdown of air)	$B \approx 1$ to 2 T (saturation of iron)
Energy density at the limit	≈ 40 J/m ³	$\approx 4 \times 10^5$ J/m ³ (at 1 T)
Ratio	1	$\approx 10^4$ times larger
Result	Needs huge, impractical machine size	Compact, practical machines of high power

Hence, practical motors and generators of every power rating use the **magnetic field** as the coupling medium. Electric-field coupling is used only in tiny devices such as electrostatic microphones, capacitor sensors and MEMS actuators.

Types of electromechanical energy conversion devices

Table 1.3 Classification of EMEC devices

Type	Nature of motion	Typical devices
Transducers	Small displacement, low power, linear relation between input and output	Microphone, loudspeaker, pick-up, tacho-generator, sensors
Force-producing devices	Limited mechanical motion, produce force or torque	Solenoid, relay, electromagnet, lifting magnet, circuit-breaker trip coil
Continuous energy conversion devices	Continuous rotary motion, large power	DC and AC generators, DC and AC motors

1.1.3 Basic Laws and Principles Used

Two fundamental laws of electromagnetism explain the whole mechanism of electromechanical energy conversion. Faraday's law of electromagnetic induction explains **generator action** and Lorentz's law (force on a current-carrying conductor in a magnetic field) explains **motor action**. Lenz's law ties them together by giving the direction of the induced effect so that energy is conserved.

(a) Faraday's law of electromagnetic induction

First law: Whenever the magnetic flux linked with a circuit changes, an emf is induced in it.

Second law: The magnitude of the induced emf is equal to the rate of change of flux linkage.

$$e = - d\lambda/dt = - N d\phi/dt$$

Here $\lambda = N\phi$ is the flux linkage (weber-turns), N the number of turns and ϕ the flux in webers. Induced emf is of two kinds:

- **Statically induced emf** – the conductor is stationary and the flux changes with time (transformer action, self-induction and mutual induction).
- **Dynamically induced emf** – the conductor moves in a steady magnetic field and cuts the flux (generator action). The magnitude is given by the motional emf equation:

$$e = B l v \sin \theta$$

where B is the flux density (T), l is the active length of the conductor in the field (m), v is the velocity of the conductor (m/s) and θ is the angle between the direction of motion and the field. When the conductor moves at right angles to the field, $\theta = 90^\circ$ and $e = B l v$.

(b) Lenz's law

The direction of the induced emf (and current) is always such that it **opposes the cause producing it**. In a generator the induced current produces a force that opposes the motion of the conductor, so the prime mover must supply mechanical energy continuously. The mechanical work done against this opposing force is exactly the energy that appears as electrical energy. Lenz's law is therefore a statement of conservation of energy.

(c) Force on a current-carrying conductor (Lorentz force)

When a conductor carrying current I is placed in a magnetic field B , it experiences a mechanical force:

$$F = B I l \sin \theta$$

For a conductor at right angles to the field, $F = B I l$ newtons. The torque on a rotor of radius r is $T = F \times r$ (N·m). This force is the **motor action** and it is responsible for the rotation of a motor.

(d) Fleming's rules for finding directions

Table 1.4 Fleming's rules (the three fingers are mutually perpendicular)

Rule	Applied to	Thumb	Forefinger	Middle finger
Fleming's right-hand rule	Generator (induced emf)	Direction of motion	Direction of magnetic field	Direction of induced emf / current
Fleming's left-hand rule	Motor (force on conductor)	Direction of force / motion	Direction of magnetic field	Direction of current

Table 1.5 Laws underlying electromechanical energy conversion

Law / principle	Statement in brief	Equation	Relevant machine mode
Faraday's law	Induced emf = rate of change of flux linkage	$e = - d\lambda/dt$	Generator, transformer
Motional emf	Emf due to conductor cutting flux	$e = B l v$	Generator
Lenz's law	Induced effect opposes its cause	Negative sign in Faraday's law	Both
Lorentz force	Force on current-carrying conductor	$F = B I l$	Motor
Conservation of energy	Input = output + stored + losses	$dW_e = dW_f + dW_m$	Both

Exam tip

Remember the pair: **Right-hand rule** → **Generator** → **emf** and **Left-hand rule** → **Motor** → **force**. A simple memory aid is “GRE: Generator, Right, Emf” and “MLF: Motor, Left, Force”.

1.1.4 Generator Mode of Operation

1.1.4.1 Principle

A generator converts **mechanical energy into electrical energy**. Its working is based on Faraday's law: when a conductor is moved (by a prime mover such as a turbine or diesel engine) across a magnetic field, an emf $e = B l v$ is induced in it. If the conductor circuit is closed through a load, a current flows and electrical power is delivered.

GENERATOR MODE : conductor driven by prime mover

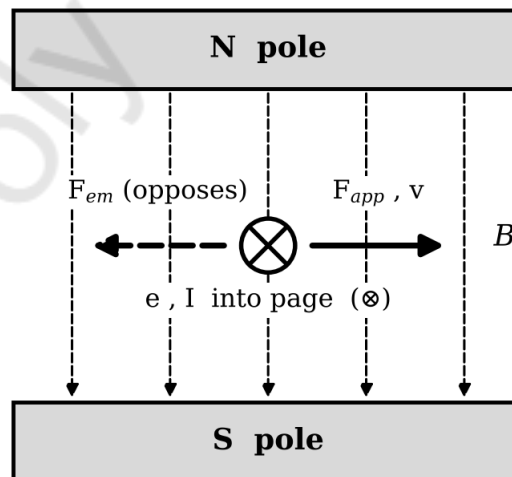


Fig. 1.3 Generator mode: conductor moved in a magnetic field (cross-sectional view)

1.1.4.2 Step-by-step mechanism

1. The prime mover applies a mechanical force F_{app} on the conductor and moves it with velocity v across the magnetic field B set up between the N and S poles.

2. The conductor cuts the flux and an emf $e = B l v$ is induced in it. Its direction is obtained by Fleming's right-hand rule (for the figure: into the page).
3. When the circuit is closed through a load, the emf drives a current I . Electrical power $e \cdot I$ is delivered to the load.
4. The current-carrying conductor now lies in the magnetic field, so it experiences an electromagnetic force $F_{em} = B I l$. By Lenz's law this force **opposes** the motion.
5. The prime mover must therefore supply a force larger than F_{em} to keep the conductor moving. The mechanical power $F \cdot v$ is converted to electrical power $e \cdot I$ through the magnetic field.
6. If the load current rises, F_{em} rises, and the prime mover must supply more mechanical power. The generator *draws* mechanical input in proportion to the electrical output.

1.1.4.3 Power balance in generator mode

In the ideal (lossless) case the mechanical power supplied by the prime mover equals the electrical power generated:

$$F \cdot v = (B l v) \cdot I = e \cdot I$$

For a DC generator the armature develops an emf E_g and delivers a terminal voltage V to the load:

$$E_g = V + I_a R_a \text{ and } E_g I_a = V I_a + I_a^2 R_a$$

The product $E_g I_a$ is the **electrical power developed in the armature**. It is equal to the mechanical power converted, that is $T \cdot \omega$, where T is the shaft torque developed and ω is the angular speed in rad/s. A part of this power is lost in the armature resistance and the remainder is delivered to the load.

- **Condition for generator action:** the induced emf E_g is greater than the terminal voltage V , so that current flows *out* of the machine into the system.
- **Direction of electromagnetic torque:** opposite to the direction of rotation (it is a braking torque).
- **Role of the commutator (DC generator):** the emf induced in an armature conductor is alternating in nature; the commutator and brushes act as a mechanical rectifier and give a unidirectional (DC) voltage at the terminals.

1.1.5 Motor Mode of Operation

1.1.5.1 Principle

A motor converts **electrical energy into mechanical energy**. Its working is based on the Lorentz force law: when a conductor carrying current is placed in a magnetic field, it experiences a force $F = B I l$. This force produces a torque on the rotor and makes the rotor rotate, driving the mechanical load connected to the shaft.

MOTOR MODE : current-carrying conductor in field

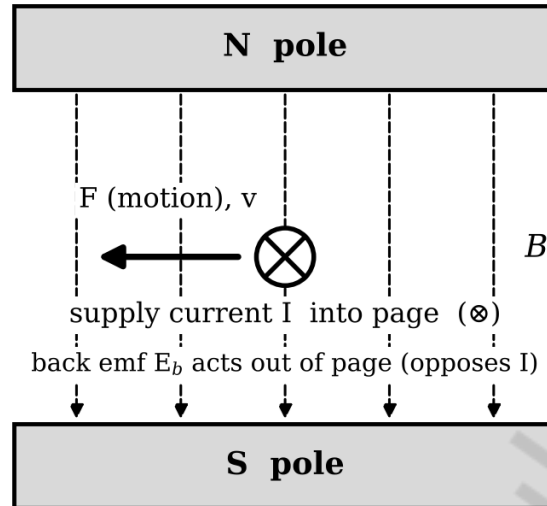


Fig. 1.4 Motor mode: current-carrying conductor in a magnetic field (cross-sectional view)

1.1.5.2 Step-by-step mechanism

1. A supply voltage V is connected to the armature winding and a current I flows through the armature conductors, which lie in the magnetic field set up by the field poles.
2. Each conductor experiences a force $F = B I l$. The direction of the force is given by Fleming's left-hand rule.
3. The forces on the conductors of the armature produce a torque $T = F \times r$ on the rotor, and the rotor begins to rotate in the direction of the force.
4. As soon as the conductors move, they cut the magnetic flux and an emf is induced in them (generator action occurs inside the motor). By Lenz's law, this emf opposes the supply voltage and is called the **back emf E_b** .
5. The armature current is now limited by the difference between supply voltage and back emf: $I_a = (V - E_b) / R_a$.
6. The electrical power drawn from the supply is converted, through the magnetic field, into mechanical power $E_b \cdot I_a$ available at the shaft (before mechanical losses).

1.1.5.3 Power balance in motor mode

$$V = E_b + I_a R_a \text{ and } V I_a = E_b I_a + I_a^2 R_a$$

$$E_b I_a = T_a \omega \text{ (mechanical power developed by the armature)}$$

- **Condition for motor action:** the back emf E_b is smaller than the supply voltage V , so that current flows *into* the machine from the supply.
- **Direction of electromagnetic torque:** same as the direction of rotation (it is a driving torque).
- **Role of the commutator (DC motor):** it reverses the direction of current in the armature conductor as it passes from under one pole to the next, so that the torque always acts in the same direction (acts as a mechanical inverter).

1.1.5.4 Significance of back emf

The back emf is the *proof* that energy conversion is taking place inside the motor. It acts as a **self-regulating mechanism** that automatically adjusts the armature current to the mechanical load:

- When the load on the motor increases, the speed falls slightly. The back emf ($E_b \propto \phi N$) falls, the difference ($V - E_b$) increases, so the armature current rises and a larger torque is developed to meet the load.
- When the load decreases, the speed rises slightly, E_b rises, armature current falls and the torque falls accordingly.
- At the instant of starting, speed is zero, so $E_b = 0$ and the armature current is very high ($I_a = V/R_a$). This is why a starter is required for DC motors.

Remember

In both modes the **same two effects occur together** in a machine: an emf is induced (generator effect) and a force is produced on a current-carrying conductor (motor effect). Which effect is *useful* and which is *reaction* decides whether the machine is a generator or a motor.

1.1.6 Comparison of Generator Mode and Motor Mode

Table 1.6 Generator mode versus motor mode

Basis	Generator mode	Motor mode
Energy conversion	Mechanical → Electrical	Electrical → Mechanical
Governing law	Faraday's law, $e = B l v$	Lorentz force, $F = B I l$
Direction rule	Fleming's right-hand rule	Fleming's left-hand rule
Primary input	Mechanical torque from a prime mover	Electrical power from a supply
Useful output	Electrical power to the load	Mechanical power at the shaft
Induced emf	Generated emf E_g drives the load current	Back emf E_b opposes the supply voltage
Armature current	Flows in the direction of the induced emf	Flows against the induced emf
Voltage equation	$E_g = V + I_a R_a$	$V = E_b + I_a R_a$
Emf versus terminal voltage	$E_g > V$	$E_b < V$
Electromagnetic torque	Opposes rotation (braking torque)	Aids rotation (driving torque)
Power developed in armature	$E_g I_a$ (electrical)	$E_b I_a$ (converted to mechanical)
Efficiency	$\eta = \text{Output (electrical)} / \text{Input (mechanical)}$	$\eta = \text{Output (mechanical)} / \text{Input (electrical)}$

Reversibility. Every rotating machine is **reversible**: the same machine can work as a generator when it is driven mechanically and as a motor when it is connected to an electrical supply. The mode simply depends on the direction of power flow, that is on whether the induced emf is greater or lesser than the terminal voltage. This property is used in regenerative braking of electric trains, where the traction motors act as generators and return energy to the supply while slowing the train.

1.1.7 Energy Balance in Electromechanical Energy Conversion

To analyse the conversion quantitatively, consider a **singly excited** magnetic system, that is, one with a single winding connected to an electrical source and a movable mechanical part. The coupling field is assumed to be **lossless** (all losses are lumped into the external electrical and mechanical systems), so the energy balance can be written as a simple conservation statement.

Word statement of the energy balance

Table 1.7 Energy balance statements

Mode	Energy balance
Motoring	Electrical energy input = Mechanical energy output + Increase in energy stored in the field + Energy lost as heat (copper, core, friction)
Generating	Mechanical energy input = Electrical energy output + Increase in energy stored in the field + Energy lost as heat (copper, core, friction)

Mathematical form

For a lossless coupling field, in a small time interval dt :

$$dW_e = dW_m + dW_f$$

where dW_e is the net electrical energy entering the field, dW_m is the mechanical energy delivered to the mechanical system and dW_f is the increase of field energy. The electrical energy input is $dW_e = e \cdot i \cdot dt$ and, from Faraday's law $e = d\lambda/dt$, we get:

$$dW_e = i d\lambda \text{ and } dW_m = f_e dx$$

$$dW_f = i d\lambda - f_e dx$$

Here f_e is the electromagnetic force and x the displacement. For rotational systems, replace f_e by the torque T_e and dx by the angle $d\theta$.

Table 1.8 Sign convention of energy terms

Operating mode	dW_e (electrical)	dW_m (mechanical)	Meaning
Motoring	Positive (drawn from the source)	Positive (output to load)	Electrical $\rightarrow$ Mechanical
Generating	Negative (returned to the source / load)	Negative (input from prime mover)	Mechanical $\rightarrow$ Electrical
Standstill, field build-up	Positive	Zero	All input stored as field energy

Field energy and coenergy

Since the coupling field is conservative, the stored field energy W_f depends only on the *present state* of the system, that is on the flux linkage λ and the position x , and not on the path by which that state was reached. Holding x fixed ($dx = 0$):

$$W_f(\lambda, x) = \int i \, d\lambda \text{ (area between the } \lambda\text{-}i \text{ curve and the } \lambda\text{-axis)}$$

$$\text{Coenergy: } W'_f(i, x) = \int \lambda \, di = \lambda i - W_f \text{ (area between the curve and the } i\text{-axis)}$$

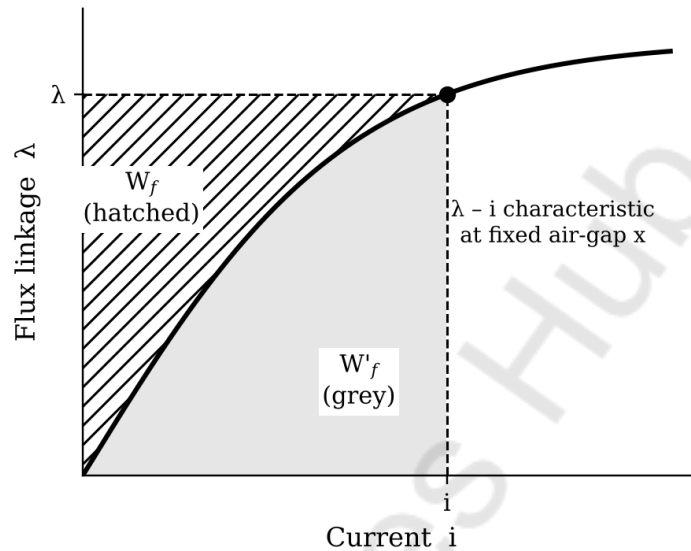


Fig. 1.5 Field energy W_f and coenergy W'_f on the λ - i characteristic

Force and torque expressions

Table 1.9 Electromagnetic force and torque from energy functions

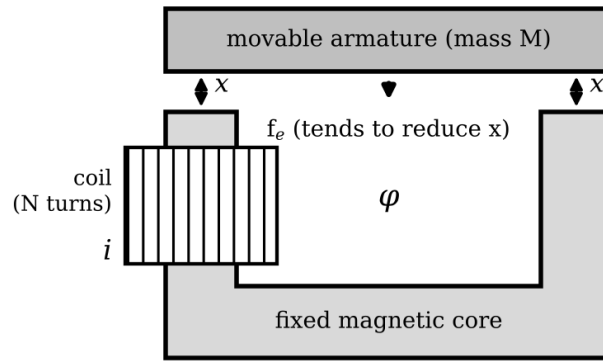
Quantity held constant	Linear motion	Rotary motion
Flux linkage λ (use field energy)	$f_e = - \partial W_f(\lambda, x) / \partial x$	$T_e = - \partial W_f(\lambda, \theta) / \partial \theta$
Current i (use coenergy)	$f_e = + \partial W'_f(i, x) / \partial x$	$T_e = + \partial W'_f(i, \theta) / \partial \theta$

Linear magnetic system

When the iron is unsaturated, flux linkage is directly proportional to current, $\lambda = L(x) \cdot i$, where $L(x)$ is the inductance, which depends on the position x . Then the λ - i curve is a straight line and field energy and coenergy are equal:

$$W_f = W'_f = \frac{1}{2} L i^2 = \frac{\lambda^2}{2L}$$

$$f_e = \frac{1}{2} i^2 \frac{dL}{dx} \text{ and } T_e = \frac{1}{2} i^2 \frac{dL}{d\theta}$$



Singly-excited electromagnet (relay type)

Fig. 1.6 Singly excited electromagnet: the force acts so as to decrease the air-gap (increase inductance)

- The force always acts in the direction that tends to **increase the inductance** (reduce the reluctance) of the magnetic circuit. In the electromagnet above, the armature is pulled so that the air-gap x decreases.
- The torque of a rotating machine is produced by the same tendency: the rotor tries to align itself with the stator field so as to attain the position of minimum reluctance (maximum inductance) or of maximum flux linkage.
- The force is proportional to the **square of the current**; hence reversing the current does not reverse the force in a singly excited system.

1.1.8 Torque Production and Emf in a DC Machine

In a practical rotating machine, torque is produced by the interaction of two magnetic fields: the field produced by the **stator** (field poles) and the field produced by the **rotor** (armature winding). These fields try to align with each other, and the resulting tendency to align produces the electromagnetic torque. In a DC machine, the commutator keeps the armature field fixed in space at about 90° to the main field, so that the torque is maximum and unidirectional.

For a DC machine with P poles, Z total armature conductors, A parallel paths, flux per pole ϕ (Wb) and speed N (rpm), the two basic equations (obtained from the laws above) are:

$$\text{Emf: } E = (P \phi Z N) / (60 A) \text{ volts} = K_a \phi \omega$$

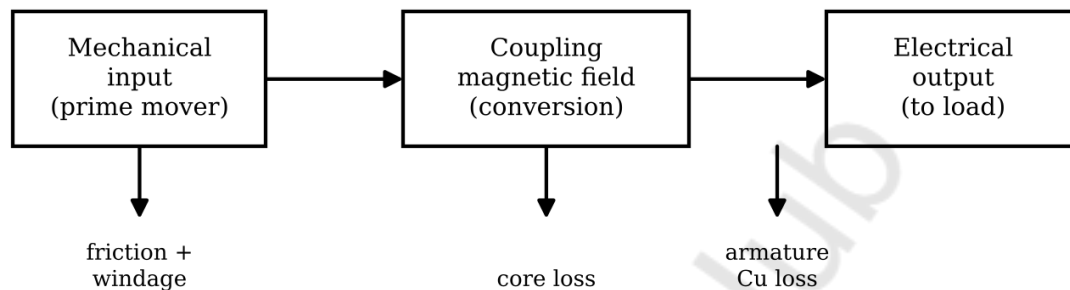
$$\text{Torque: } T_a = (P Z \phi I_a) / (2\pi A) \text{ N}\cdot\text{m} = K_a \phi I_a$$

- The **same machine constant K_a** appears in both the emf and the torque equation, which shows that generator action and motor action are two sides of the same physical process.
- Since $E I_a = K_a \phi \omega I_a = T_a \omega$, the electrical power developed in the armature equals the mechanical power developed – the proof of ideal energy conversion.
- In a generator, E is the *output* quantity depending on speed; in a motor, T is the output quantity depending on armature current.

1.1.9 Losses and Efficiency in Energy Conversion

No real machine converts energy perfectly. A part of the input energy is dissipated as heat and appears as losses. These losses reduce the efficiency and raise the temperature of the machine.

(a) Generator mode : mechanical → electrical



(b) Motor mode : electrical → mechanical

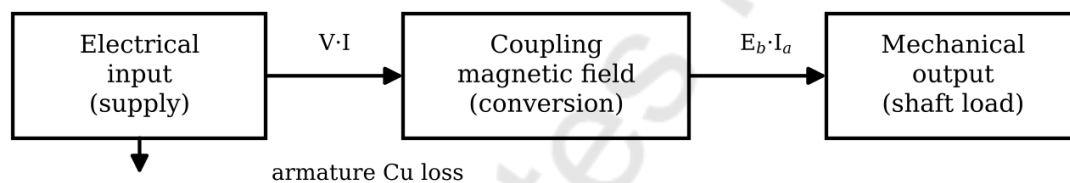


Fig. 1.7 Power flow in generator mode and in motor mode

Table 1.10 Losses in a rotating machine

Type of loss	Cause	Nature	Approx. dependence
Armature copper loss	$I_a^2 R_a$ in armature winding	Electrical, variable	Square of load current
Field copper loss	$I_f^2 R_f$ in field winding	Electrical, constant (shunt) or variable (series)	Field current
Hysteresis loss	Reversal of magnetisation in the iron core	Magnetic (core loss), constant	Flux density and frequency
Eddy current loss	Currents induced in the iron core	Magnetic (core loss), constant	Square of B and frequency
Friction loss	Bearings and brushes on commutator	Mechanical, constant	Speed
Windage loss	Air resistance to rotating parts and fan	Mechanical, constant	Cube of speed (approx.)
Stray load loss	Distortion of flux, minor effects	Miscellaneous	Load

Efficiency

$$\eta = \text{Output} / \text{Input} = (\text{Input} - \text{Losses}) / \text{Input} = \text{Output} / (\text{Output} + \text{Losses})$$

Table 1.11 Efficiency in the two modes

Mode	Input	Output	Efficiency
Generator	Mechanical power from prime mover	Electrical power $V \cdot I_L$ to the load	$\eta = V I_L / (V I_L + \text{total losses})$
Motor	Electrical power $V \cdot I_L$ from the supply	Mechanical power at the shaft	$\eta = (V I_L - \text{total losses}) / (V I_L)$

The efficiency is **maximum** when the variable loss (armature copper loss) equals the constant loss (core + mechanical + shunt field loss). A well-designed machine reaches its maximum efficiency at or near its rated load.

1.1.10 Solved Numerical Problems

Example 1 - Motional emf, force and conservation of energy

A conductor of length 0.5 m moves at 20 m/s at right angles to a uniform field of 0.8 T and carries a current of 10 A. Find the induced emf, the force on it and the mechanical power.

Solution. $e = B l v = 0.8 \times 0.5 \times 20 = 8 \text{ V}$. Force $F = B I l = 0.8 \times 10 \times 0.5 = 4 \text{ N}$. Mechanical power = $F v = 4 \times 20 = 80 \text{ W}$. Electrical power = $e I = 8 \times 10 = 80 \text{ W}$. The two powers are equal, which verifies the conversion of mechanical power into electrical power (conservation of energy).

Example 2 - DC motor

A DC motor is connected to a 220 V supply. Its armature resistance is 0.5Ω and it draws an armature current of 20 A while running at 1500 rpm. Find the back emf, the electrical input to the armature, armature copper loss, mechanical power developed and the torque.

Solution. $E_b = V - I_a R_a = 220 - 20 \times 0.5 = 210 \text{ V}$. Armature input = $V I_a = 220 \times 20 = 4400 \text{ W}$. Copper loss = $I_a^2 R_a = 400 \times 0.5 = 200 \text{ W}$. Power developed = $E_b I_a = 210 \times 20 = 4200 \text{ W}$ (check: $4400 - 200 = 4200 \text{ W}$). $\omega = 2\pi \times 1500 / 60 = 157.08 \text{ rad/s}$, so $T = 4200 / 157.08 = 26.74 \text{ N}\cdot\text{m}$.

Example 3 - DC generator

A DC generator delivers 100 A at 230 V. The armature resistance is 0.1Ω . The sum of core, friction and windage losses is 1 kW (field loss neglected). Find the generated emf, the mechanical input and the efficiency.

Solution. $E_g = V + I_a R_a = 230 + 100 \times 0.1 = 240 \text{ V}$. Electrical power developed = $E_g I_a = 240 \times 100 = 24 \text{ kW}$. Output = $230 \times 100 = 23 \text{ kW}$. Armature copper loss = $100^2 \times 0.1 = 1 \text{ kW}$. Mechanical input = $23 + 1 + 1 = 25 \text{ kW}$. Efficiency $\eta = 23 / 25 = 92 \%$.

Example 4 - Force in a singly excited electromagnet

The inductance of an electromagnet is $L(x) = k/x$, where $k = 2 \times 10^{-4} \text{ H}\cdot\text{m}$ and x is the air-gap in metres. Find the stored energy and force when $i = 2 \text{ A}$ and $x = 2 \text{ mm}$.

Solution. $L = k/x = 2 \times 10^{-4} / 0.002 = 0.1 \text{ H}$. Stored energy $W_f = \frac{1}{2} L i^2 = \frac{1}{2} \times 0.1 \times 4 = 0.2 \text{ J}$. Force $f_e = \frac{1}{2} i^2 dL/dx = \frac{1}{2} \times 4 \times (-k/x^2) = \frac{1}{2} \times 4 \times (-2 \times 10^{-4} / 4 \times 10^{-6}) = -100 \text{ N}$. The negative sign shows that the force is **attractive** and tends to reduce the air-gap x .

1.1.11 Summary of Important Formulas and Points

Table 1.12 Formula sheet

Quantity	Formula
Faraday's law	$e = - d\lambda/dt = - N d\phi/dt$
Motional emf	$e = B l v \sin \theta$
Force on a conductor	$F = B I l \sin \theta$
Torque	$T = F \times r$
Energy balance	$dW_e = dW_m + dW_f$
Field energy; coenergy	$W_f = \int i d\lambda ; W'_f = \int \lambda di = \lambda i - W_f$
Force (linear system)	$f_e = \frac{1}{2} i^2 dL/dx; T_e = \frac{1}{2} i^2 dL/d\theta$
Generator (DC)	$E_g = V + I_a R_a$
Motor (DC)	$V = E_b + I_a R_a$
Power converted	$P_{conv} = E I_a = T \omega$
DC machine emf	$E = P \phi Z N / (60 A)$
DC machine torque	$T_a = P Z \phi I_a / (2\pi A)$
Efficiency	$\eta = \text{Output} / \text{Input}$

Points to remember

- Rotating machines use the **magnetic field** as coupling medium because its energy density is about 10^4 times that of the electric field.
- Generator: **mechanical** → **electrical**, Faraday's law, Fleming's right-hand rule, $E_g > V$, torque opposes rotation.
- Motor: **electrical** → **mechanical**, Lorentz force, Fleming's left-hand rule, $E_b < V$, torque aids rotation.
- Both effects (emf generation and force production) happen in every machine; only the direction of power flow changes.
- In a lossless coupling field: **electrical input = mechanical output + increase in stored field energy**.
- Force or torque acts so as to **increase inductance** (reduce reluctance) of the magnetic circuit.
- Back emf regulates the armature current of a motor automatically as the load changes.

PRACTICE SECTION

A. Multiple Choice Questions (15 MCQs)

Choose the most appropriate option. The answer key is given after the questions.

Q1. In practical electromechanical energy conversion devices, the most commonly used coupling medium is the:

- (a) Electric field
- (b) Magnetic field
- (c) Gravitational field
- (d) Thermal field

Q2. The emf induced in a conductor moving through a magnetic field is explained by:

- (a) Ohm's law
- (b) Coulomb's law
- (c) Faraday's law of electromagnetic induction
- (d) Gauss's law

Q3. The direction of force on a current-carrying conductor placed in a magnetic field (motor action) is found by:

- (a) Fleming's left-hand rule
- (b) Fleming's right-hand rule
- (c) Coulomb's law
- (d) Kirchhoff's current law

Q4. In generator mode, the electromagnetic torque developed in the machine:

- (a) Acts in the direction of rotation
- (b) Is zero
- (c) Acts perpendicular to the shaft
- (d) Opposes the direction of rotation

Q5. For a DC motor with supply voltage V , armature resistance R_a and armature current I_a , the back emf is:

- (a) $V + I_a R_a$
- (b) $V - I_a R_a$
- (c) $V I_a$
- (d) V / R_a

Q6. For the same volume, the energy that can be stored in a magnetic field of 1 T compared with an electric field at the breakdown strength of air (about 3 MV/m) is approximately:

- (a) 10 times smaller
- (b) Equal
- (c) 10^4 times greater
- (d) 100 times smaller

Q7. In a linear magnetic system with inductance L carrying current i , the stored field energy is:

- (a) $\frac{1}{2} L i^2$
- (b) $L i^2$
- (c) $\frac{1}{2} L^2 i$
- (d) L / i

Q8. In motor mode, the electrical input is converted into:

- (a) Mechanical output only
- (b) Heat only
- (c) Stored field energy only
- (d) Mechanical output, losses and any change in stored field energy

Q9. For a singly excited linear magnetic system, the electromagnetic force is given by:

- (a) $\frac{1}{2} i^2 L$
- (b) $i^2 dL/dx$
- (c) $\frac{1}{2} i^2 dL/dx$
- (d) $\frac{1}{2} L di/dx$

Q10. Which of the following statements about a DC machine is correct?

- (a) It can work both as a generator and as a motor (reversible)
- (b) It can work only as a generator
- (c) It can work only as a motor
- (d) It cannot convert energy

Q11. The mechanical power developed by the armature of a DC motor is equal to:

- (a) $V I_a$
- (b) $E_b I_a$
- (c) $I_a^2 R_a$
- (d) V^2 / R_a

Q12. A DC machine works as a generator when:

- (a) Induced emf is less than terminal voltage
- (b) Induced emf equals zero
- (c) Terminal voltage is zero
- (d) Induced emf is greater than terminal voltage

Q13. The magnitude of motional emf in a conductor of length l moving with velocity v at an angle θ to a field B is:

- (a) $B I l$
- (b) $B v / l$
- (c) $B l v \sin \theta$
- (d) $B^2 l v$

Q14. A conductor 0.5 m long moves at 10 m/s perpendicular to a uniform field of 1.2 T. The induced emf is:

- (a) 6 V
- (b) 12 V
- (c) 0.6 V
- (d) 60 V

Q15. A DC motor on a 200 V supply has an armature resistance of 0.4Ω and draws 25 A. The back emf is:

- (a) 210 V
- (b) 190 V
- (c) 200 V
- (d) 10 V

Answer Key for MCQs

Table 1.13 Answer key with short explanation

Q.	Ans.	Brief explanation
1	(b)	Magnetic field stores far more energy per unit volume than an electric field.
2	(c)	Faraday's law gives $e = -d\lambda/dt$; for a moving conductor $e = Blv$.
3	(a)	Motor action $\rightarrow$ Fleming's left-hand rule (force).
4	(d)	By Lenz's law the electromagnetic torque opposes the prime mover.
5	(b)	$V = E_b + I_a R_a \Rightarrow E_b = V - I_a R_a$.
6	(c)	$\approx 4 \times 10^5 \text{ J/m}^3$ versus $\approx 40 \text{ J/m}^3$, a ratio of about 10^4 .
7	(a)	$W_f = \int i d\lambda$ with $\lambda = Li$ gives $\frac{1}{2} Li^2$.
8	(d)	Input = mechanical output + losses + change in stored field energy.
9	(c)	$f_e = \partial W_f / \partial x = \frac{1}{2} i^2 dL/dx$.
10	(a)	Every rotating machine is reversible.
11	(b)	Electrical power developed in the armature equals $T \cdot \omega$.
12	(d)	Current flows out of the machine only when $E_g > V$.
13	(c)	$e = B l v \sin \theta$.
14	(a)	$e = Blv = 1.2 \times 0.5 \times 10 = 6 \text{ V}$.
15	(b)	$E_b = 200 - 25 \times 0.4 = 190 \text{ V}$.

B. Broad Questions (10 Questions)

- Q1. Explain the concept of electromechanical energy conversion with a neat block diagram. Why is the magnetic field preferred as the coupling medium? Support your answer with energy-density expressions.
- Q2. Write the energy balance equation for a lossless coupling field. Derive the expressions for electromagnetic force in terms of field energy and in terms of coenergy for a singly excited system.
- Q3. Explain with a neat diagram the mechanism of electromechanical energy conversion in **generator mode**. State the laws involved and show how mechanical power is converted into electrical power.
- Q4. Explain with a neat diagram the mechanism of electromechanical energy conversion in **motor mode**. Describe the role of back emf in the working of a motor.
- Q5. Differentiate between generator mode and motor mode of a rotating machine in a tabular form (at least eight points). Explain why a rotating machine is called reversible.
- Q6. State and explain Faraday's laws of electromagnetic induction, Lenz's law and Fleming's right-hand and left-hand rules with reference to electrical machines.
- Q7. Derive the power equations of a DC generator and a DC motor. Draw the power-flow diagrams and define the efficiency in each mode.

- Q8. Show that for a linear magnetic system the field energy and coenergy are equal to $\frac{1}{2}Li^2$ and obtain the expressions $f_e = \frac{1}{2}i^2(dL/dx)$ and $T_e = \frac{1}{2}i^2(dL/d\theta)$. Explain the direction of the force.
- Q9. Classify electrical machines. Describe the three categories of electromechanical energy conversion devices with two examples each.
- Q10. Numerical: A DC motor on a 240 V supply has an armature resistance of $0.25\ \Omega$ and draws an armature current of 40 A at 1200 rpm. Calculate (i) back emf, (ii) armature copper loss, (iii) power developed, (iv) torque developed, and (v) electrical power input to the armature.

End of Topic 1.1

Revise the laws, the two voltage equations ($E_g = V + I_a R_a$ and $V = E_b + I_a R_a$), the energy-balance equation and the comparison table. These are the most frequently asked areas in university and polytechnic examinations.