

DC MACHINES AND TRANSFORMERS

Chapter 3

D.C. MOTOR

Detailed Study Notes for Engineering Students

Unit	Topic
3.1	Working principle, back e.m.f., speed and torque equations (Numerical)
3.2	Characteristics of series, shunt and compound motors
3.3	Methods of speed control of D.C. motors (Numerical)
3.4	Starting methods of D.C. motor – 3-point and 4-point starter
3.5	Losses and efficiency (Numerical)
3.6	Braking methods – regenerative, counter-current and dynamic braking
3.7	Applications of different types of D.C. motor
3.8	Brushless D.C. motors: construction, working principle and applications
–	Formula summary, 15 MCQs with answer key and 10 broad questions

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Chapter Overview: The D.C. Motor

A **D.C. motor** is an electromechanical energy converter that changes direct-current electrical energy into mechanical rotational energy. Structurally it is identical to a D.C. generator; the same machine can work as a generator when driven mechanically or as a motor when supplied with electrical power. D.C. motors are still widely used where **wide and smooth speed control**, **high starting torque** or **fast reversal and braking** are needed, for example in traction, cranes, rolling mills, lifts and machine tools.

The behaviour of a D.C. motor is mainly decided by the way its field winding is connected to the armature. Based on the method of excitation, D.C. motors are classified as follows.

Type of motor	Field winding connection	Main feature
Separately excited	Field fed from an independent D.C. source	Flux is independent of armature current; excellent speed control
Shunt	Field in parallel with the armature (many turns of thin wire)	Nearly constant speed from no load to full load
Series	Field in series with the armature (few turns of thick wire)	Very high starting torque; speed falls heavily with load
Compound	Both shunt and series fields; <i>cumulative</i> (fields aid) or <i>differential</i> (fields oppose); <i>long-shunt</i> or <i>short-shunt</i> connection	High starting torque with reasonably steady speed (cumulative)
Permanent magnet (PMDC)	Permanent magnets replace the field winding	Compact; used in small motors and automobile accessories

3.1 Working Principle, Back E.M.F., Speed and Torque Equations

A. Working principle

A D.C. motor works on the principle that **a current-carrying conductor placed in a magnetic field experiences a mechanical force**. If a conductor of length l metres carrying a current I amperes is placed at right angles to a magnetic field of flux density B tesla, the force on it is

$$F = B \cdot I \cdot l \text{ newton}$$

The direction of this force is found by **Fleming's left-hand rule**:

- Stretch the **forefinger**, **middle finger** and **thumb** of the left hand mutually at right angles to one another.
- Forefinger – direction of the magnetic **field** (north pole to south pole).
- Middle finger – direction of the **current** in the conductor.
- Thumb – direction of the **force** (motion) on the conductor.

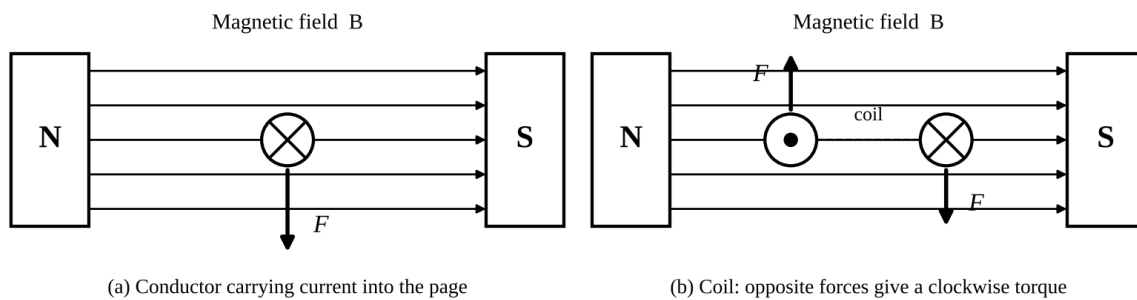


Fig. 1: Force on a current-carrying conductor in a magnetic field and the torque on a coil

In a motor the armature carries a large number of conductors placed in slots. The conductors under a north pole carry current in one direction and those under a south pole carry current in the opposite direction. The forces on all the conductors then act in the same rotational sense, and their combined turning moment is the **torque** that rotates the armature. The **commutator** reverses the current in each coil as it passes from one pole to the next, so that the torque always remains in the same direction. In this way the commutator acts as a mechanical inverter, converting the D.C. supply into alternating current in the armature conductors.

B. Construction of a D.C. motor

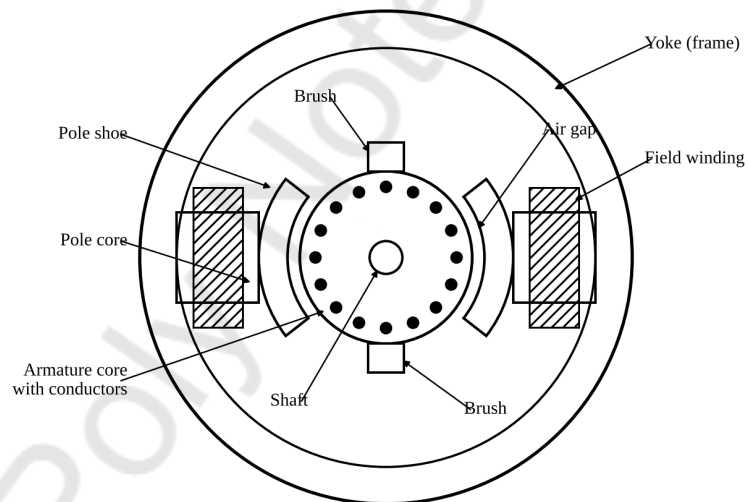


Fig. 2: Cross-section of a two-pole D.C. machine showing the main parts

Part	Function
Yoke (frame)	Outer cast-iron or steel body; gives mechanical support and provides a path for the magnetic flux
Pole core and pole shoe	Laminated structure that carries the field winding; the shoe spreads the flux uniformly over the air gap
Field winding	Coils on the poles that produce the main magnetic flux when energised
Armature core	Laminated silicon-steel cylinder with slots; lamination reduces eddy-current loss
Armature winding	Copper conductors in the slots, connected as lap ($A = P$; for low voltage, high current) or wave ($A = 2$; for high voltage, low current); the torque is produced here

Part	Function
Commutator	Copper segments insulated from one another by mica; reverses the current in the armature coils
Brushes and brush gear	Carbon or graphite blocks that carry current between the external circuit and the rotating commutator
Shaft and bearings	Transmit the mechanical power to the load and support the rotating armature

C. Back e.m.f. (counter e.m.f.)

When the armature of a motor rotates, its conductors cut the magnetic flux of the field poles. By Faraday's law an e.m.f. is induced in them. By Lenz's law this e.m.f. acts in a direction that **opposes the cause producing it**, which is the applied voltage. It is therefore called the **back e.m.f.** or **counter e.m.f.** and is denoted by E_b . Its magnitude is given by the same expression as the generated e.m.f. of a generator:

$$E_b = (P \cdot \phi \cdot Z \cdot N) / (60 \cdot A) \text{ volt}$$

- P – number of poles; ϕ – flux per pole in weber; Z – total number of armature conductors.
- N – speed of the armature in r.p.m.; A – number of parallel paths in the armature winding ($A = P$ for lap winding, $A = 2$ for wave winding).

The back e.m.f. acts against the supply voltage V , so the net voltage that drives current through the armature resistance is $(V - E_b)$. This gives the **voltage equation** of a motor:

$$V = E_b + I_a R_a \text{ (+ brush drop, usually about 2 V)}$$

$$I_a = (V - E_b) / R_a$$

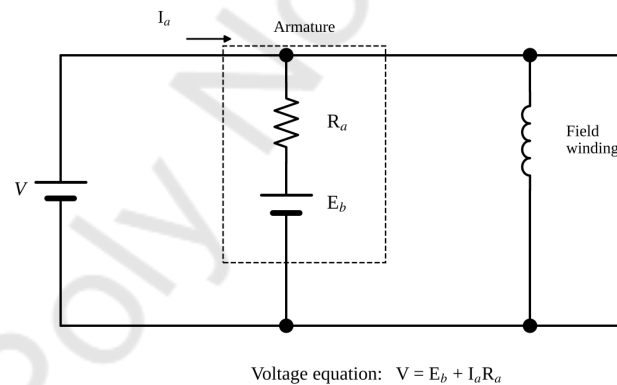


Fig. 3: Equivalent circuit of a D.C. shunt motor

Significance of back e.m.f.

- **Self-regulation:** if the load increases, the speed falls, E_b falls and $I_a = (V - E_b) / R_a$ rises automatically, supplying the extra torque required. Back e.m.f. therefore acts like a governor.
- **Starting:** at standstill $E_b = 0$, so the starting current is V / R_a , which is very large. This is why a starter is essential (see 3.4).
- **Energy conversion:** the electrical power converted into mechanical power is $E_b \cdot I_a$. Without a back e.m.f. there would be no mechanical power output.

Condition	Speed N	Back e.m.f. E_b	Armature current I_a
At starting	Zero	Zero	Maximum = V / R_a (dangerously high)

Condition	Speed N	Back e.m.f. E_b	Armature current I_a
No load	Maximum	Almost equal to V	Very small (only to meet losses)
Full load	Slightly less	A little less than V	Rated value
Overload	Falls further	Falls further	Increases above rated value

D. Power equation and condition for maximum power

Multiplying the voltage equation by I_a gives the power equation:

$$V \cdot I_a = E_b \cdot I_a + I_a^2 R_a$$

Here $V \cdot I_a$ is the electrical power supplied to the armature, $E_b \cdot I_a$ is the gross mechanical power developed (P_m) and $I_a^2 R_a$ is the copper loss in the armature.

Condition for maximum power: $P_m = V \cdot I_a - I_a^2 R_a$. Differentiating with respect to I_a and equating to zero gives $V - 2I_a R_a = 0$, that is, $I_a R_a = V/2$. Hence at maximum power $E_b = V/2$ and $P_{m(max)} = V^2/(4R_a)$. At this point half of the input is lost as armature copper loss, so the efficiency cannot exceed 50 per cent. Practical motors therefore work with E_b close to V.

E. Torque equation

Torque is the turning moment of a force about an axis. For a motor, the **armature torque** T_a is the torque developed by the armature. The power developed is $T_a \cdot \omega$, where $\omega = 2\pi N/60$ rad/s. Equating this to the electrical equivalent $E_b \cdot I_a$:

$$\begin{aligned}
 T_a \times (2\pi N/60) &= E_b I_a = (P\phi ZN/60A) \cdot I_a \\
 T_a &= (P \cdot \phi \cdot Z \cdot I_a) / (2\pi \cdot A) \text{ N-m} \\
 T_a &= 0.159 \cdot \phi \cdot Z \cdot I_a \cdot (P/A) \text{ N-m} \\
 T_a &= 9.55 \cdot E_b I_a / N \text{ N-m}
 \end{aligned}$$

For a given motor P, Z and A are constant, so $T_a \propto \phi \cdot I_a$. The useful torque available at the shaft is smaller than T_a because of iron and mechanical losses:

$$T_{sh} = P_{out} / \omega = 9.55 \cdot P_{out} / N \text{ N-m} \quad (P_{out} \text{ in watt, } N \text{ in r.p.m.})$$

F. Speed equation

From the back e.m.f. equation $E_b = P\phi ZN/60A$ and from $V = E_b + I_a R_a$:

$$\begin{aligned}
 N &= K \cdot E_b / \phi = K \cdot (V - I_a R_a) / \phi, \text{ where } K = 60A / (PZ) \\
 N_2 / N_1 &= (E_{b2} / E_{b1}) \times (\phi_1 / \phi_2)
 \end{aligned}$$

Hence the speed of a D.C. motor is **directly proportional to the back e.m.f. and inversely proportional to the flux per pole**. Weakening the field raises the speed; strengthening it lowers the speed.

Motor	Flux ϕ	Torque T_a	Speed N
Shunt (ϕ constant)	Constant	$\propto I_a$	$\propto E_b$ (nearly constant)
Series (unsaturated)	$\propto I_a$	$\propto I_a^2$	$\propto E_b / I_a$

Numerical problems

Example 1. A 4-pole, lap-wound D.C. motor has 600 armature conductors and a flux of 0.03 Wb per pole. Calculate the back e.m.f. when it runs at 1000 r.p.m.

Solution:

For lap winding $A = P = 4$.

$$E_b = P\phi ZN / 60A = (4 \times 0.03 \times 600 \times 1000) / (60 \times 4) = 72000 / 240 = 300 \text{ V}$$

Answer: $E_b = 300 \text{ V}$

Example 2. A 220 V D.C. shunt motor has an armature resistance of 0.5Ω and takes an armature current of 20 A at 1000 r.p.m. Find the back e.m.f., the gross mechanical power developed and the armature torque.

Solution:

$$E_b = V - I_a R_a = 220 - (20 \times 0.5) = 210 \text{ V}$$

$$P_m = E_b I_a = 210 \times 20 = 4200 \text{ W}$$

$$T_a = P_m / (2\pi N/60) = 4200 / (2\pi \times 1000/60) = 4200 / 104.72 = 40.11 \text{ N-m}$$

Answer: $E_b = 210 \text{ V}$, $P_m = 4200 \text{ W}$, $T_a \approx 40.1 \text{ N-m}$

Example 3. A 4-pole, wave-wound D.C. motor has 500 armature conductors and a flux of 25 mWb per pole. Find the armature torque when the armature current is 20 A.

Solution:

Wave winding: $A = 2$. $\phi = 25 \text{ mWb} = 0.025 \text{ Wb}$.

$$T_a = P\phi ZI_a / (2\pi A) = (4 \times 0.025 \times 500 \times 20) / (2\pi \times 2) = 1000 / 12.566 = 79.58 \text{ N-m}$$

Answer: $T_a \approx 79.6 \text{ N-m}$

Example 4. A 220 V D.C. shunt motor with armature resistance 0.4Ω runs at 800 r.p.m. with an armature current of 30 A. Find its speed when the armature current is 60 A and the flux is weakened to 90 % of its original value.

Solution:

$$E_{b1} = 220 - (30 \times 0.4) = 208 \text{ V}; E_{b2} = 220 - (60 \times 0.4) = 196 \text{ V}$$

$$N_2 / N_1 = (E_{b2} / E_{b1}) \times (\phi_1 / \phi_2) = (196 / 208) \times (1 / 0.9) = 1.0470$$

$$N_2 = 800 \times 1.0470 = 837.6 \text{ r.p.m.}$$

Answer: $N_2 \approx 838 \text{ r.p.m.}$

3.2 Characteristics of Series, Shunt and Compound Motors

The performance of a motor is judged from its **characteristic curves**, which show how one quantity varies with another when the supply voltage is constant. Three characteristics are important:

- **Torque – armature current** (T_a vs I_a) – the electrical characteristic.
- **Speed – armature current** (N vs I_a).
- **Speed – torque** (N vs T_a) – the mechanical characteristic, derived from the first two.

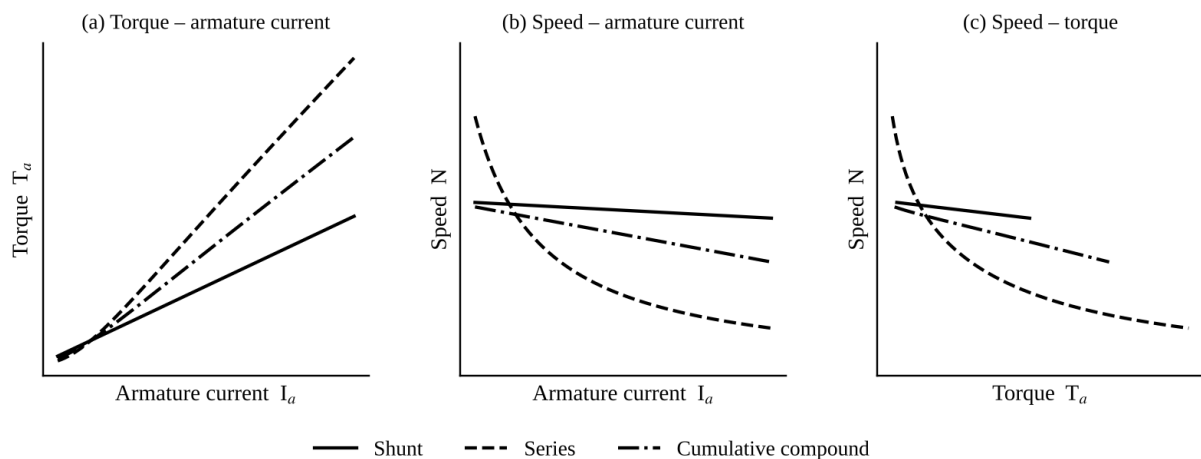


Fig. 4: Characteristics of D.C. shunt, series and cumulative compound motors

A. D.C. shunt motor

The field winding is connected directly across the supply, so the field current and the flux remain practically constant (apart from a small armature-reaction effect).

- $T_a - I_a$: since ϕ is constant, $T_a \propto I_a$; the curve is a straight line through the origin.
- $N - I_a$: $N \propto E_b$. From no load to full load E_b falls only by the small drop $I_a R_a$, so the speed drops by only about 2 to 5 per cent. A shunt motor is therefore called a **constant-speed motor**.
- **Starting torque** is moderate; the motor can be started on a reasonable load.
- **Precaution:** the shunt field circuit must never be opened while the motor runs. The flux would fall to the small residual value and the speed would rise dangerously.

B. D.C. series motor

The field winding is in series with the armature, so the whole armature current flows through it and the flux depends on the load current.

- $T_a - I_a$: before magnetic saturation $\phi \propto I_a$ and hence $T_a \propto I_a^2$ (a parabola). After saturation ϕ is almost constant and $T_a \propto I_a$ (straight line).
- $N - I_a$: $N \propto E_b / \phi \propto 1/I_a$. The speed falls rapidly as the load current rises and becomes very high at light load.
- **High starting torque** – a series motor develops a torque proportional to the square of the current, so it can start very heavy loads.
- **Danger of runaway:** a series motor must never be started or run without load, and never connected to its load through a belt, because at no load the flux is tiny and the speed can rise to a destructive value. It is therefore coupled directly or through gears.
- It is a **variable-speed motor**: speed adjusts itself to the load.

C. D.C. compound motor

A compound motor carries both a shunt field and a series field on the same poles. The series field may *aid* the shunt field (**cumulative compound**) or *oppose* it (**differential compound**). The shunt field may be connected across the armature alone (short-shunt) or across the armature plus series field (long-shunt).

Feature	Cumulative compound	Differential compound
Resultant flux	Increases with load (fields aid)	Decreases with load (fields oppose)
Starting torque	High (between shunt and series)	Low; may be poor

Feature	Cumulative compound	Differential compound
Speed with load	Falls moderately; never runs away at no load (shunt field present)	Speed may rise with load; unstable operation
Use	Widely used	Rarely used (special cases only)

D. Comparison of the three motors

Feature	Shunt	Series	Cumulative compound
Field flux	Constant	Proportional to I_a (until saturation)	Partly constant, partly varies with I_a
T_a versus I_a	$\propto I_a$ (straight line)	$\propto I_a^2$, later $\propto I_a$	Between shunt and series
Speed regulation	Very good (2–5 % drop)	Very poor	Moderate
Starting torque	Moderate	Very high	High
No-load operation	Safe	Dangerous (runaway)	Safe
Type of speed	Nearly constant	Variable	Fairly steady

Example 5. A D.C. series motor develops a torque of 60 N-m when it takes 25 A. Find the torque when the current is 40 A (i) assuming the field is unsaturated, (ii) assuming the field is fully saturated.

Solution:

(i) Unsaturated: $T_a \propto I_a^2$, so $T_2 = 60 \times (40/25)^2 = 60 \times 2.56 = 153.6$ N-m

(ii) Saturated: ϕ constant, $T_a \propto I_a$, so $T_2 = 60 \times (40/25) = 96$ N-m

Answer: 153.6 N-m (unsaturated); 96 N-m (saturated)

3.3 Methods of Speed Control of D.C. Motors

A great advantage of the D.C. motor is that its speed can be varied smoothly over a wide range. From the speed equation

$$N = K(V - I_a R_a) / \phi$$

it is clear that speed can be controlled by three basic means: (i) varying the **flux ϕ** (field control), (ii) varying the **resistance in the armature circuit** (armature-resistance control) and (iii) varying the **voltage applied to the armature** (voltage control). The *base speed* is the speed at rated voltage and rated flux.

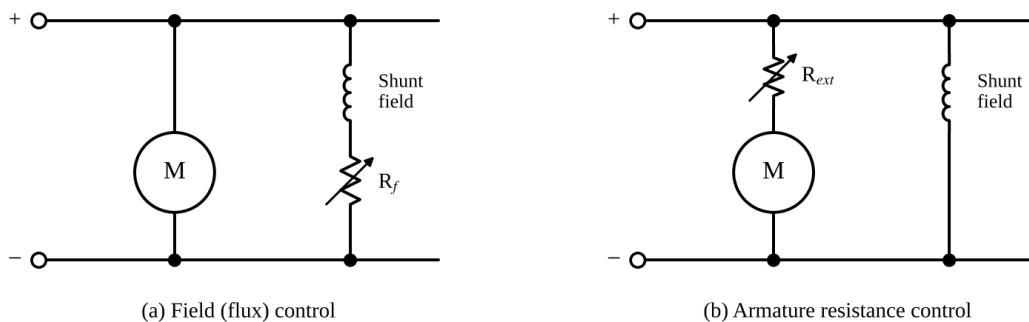


Fig. 5: Field control and armature-resistance control of a D.C. shunt motor

A. Speed control of D.C. shunt motors

1. Field (flux) control

A variable resistance R_f (shunt-field rheostat) is connected in series with the shunt field. Increasing R_f reduces the field current and flux, so the speed **rises above the base speed** ($N \propto 1/\phi$). Because I_a is small compared with the field power, the loss in the rheostat is small and the method is economical. When the flux is reduced the armature current increases for a given torque, so the motor is a *constant-power drive* in this region. The maximum speed is limited by commutation and mechanical strength.

2. Armature-resistance control

A variable resistance is connected in series with the armature. The voltage across the armature falls, so E_b and hence the speed fall: speed is controlled **below the base speed**. The method is simple but the series resistance carries the full armature current, causing large I^2R loss, poor efficiency and poor speed regulation (speed varies greatly with load). It is used only for small motors or for short-time duty.

3. Armature-voltage control

The voltage applied to the armature is varied while the field is kept constant. Speed is then roughly proportional to the armature voltage and can be controlled from zero up to the base speed, with constant torque capability.

- **Multiple-voltage control:** the armature is connected to one of several fixed voltages. Speed changes in steps.
- **Ward–Leonard system:** a separately excited D.C. generator supplies the motor armature; the generator voltage is varied by its field rheostat. Smooth control, described below.
- **Static converters (thyristor rectifier or chopper):** modern equivalent of the Ward–Leonard system; the D.C. voltage is varied electronically.

Ward–Leonard system

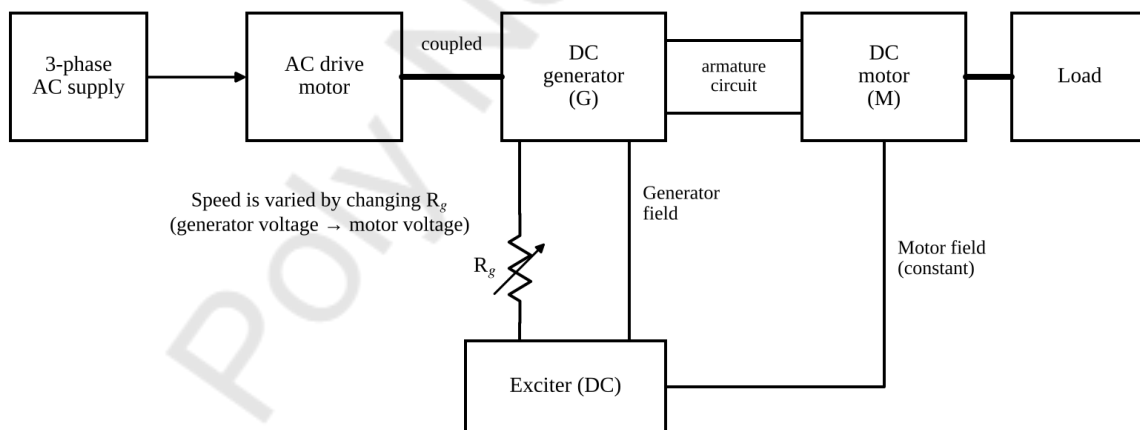


Fig. 6: Ward–Leonard system of speed control

A constant-speed A.C. motor drives a D.C. generator G. The generator field is excited by a small D.C. exciter through a rheostat R_g , while the field of the D.C. motor M is kept constant. Varying R_g changes the generator e.m.f., and hence the voltage applied to the armature of M, so the motor speed varies smoothly from zero to base speed. Reversing the generator field reverses the generator voltage and therefore the direction of rotation of M. Speeds above base speed are obtained by weakening the motor field. During deceleration the motor acts as a generator and returns energy (regenerative braking).

- **Advantages:** very smooth and wide speed range in both directions, excellent speed regulation, easy reversal and regenerative braking.

- **Disadvantages:** three machines are required, so the initial cost, floor space and maintenance are high, and overall efficiency is lower.
- **Uses:** lifts, mine hoists, rolling mills, paper machines and elevators.

B. Speed control of D.C. series motors

- **Flux control:** (i) *field diverter* – a resistance in parallel with the series field reduces the field current and increases speed; (ii) *tapped field* – the number of series-field turns in circuit is changed; (iii) *series-parallel connection of field coils* – used for fans.
- **Armature diverter:** a variable resistance across the armature changes the armature current for a given torque.
- **Armature-resistance control:** a resistance in series with the motor reduces the armature voltage and so the speed (below normal speed); widely used in traction and cranes.
- **Series-parallel control of two motors:** in traction, two motors are first connected in series (each gets $V/2$, low speed) and then in parallel (each gets V , higher speed).
- **Variation of supply voltage** by thyristor or chopper control.

Method	Range	Merits	Demerits
Field control	Above base speed	Simple, cheap, efficient; small loss	Limited range; poor commutation at weak field
Armature resistance	Below base speed	Simple and cheap	Low efficiency; speed depends on load
Ward-Leonard / converter	Zero to base (and above with field control)	Smooth, wide, reversible, regenerative	High cost (M-G set) or harmonics (converters)

Numerical problems

Example 6. A 220 V D.C. shunt motor with $R_a = 0.4 \Omega$ runs at 1000 r.p.m. taking an armature current of 25 A. The flux is reduced by 20 % by field control while the load torque stays constant. Find the new speed.

Solution:

Torque constant: ϕI_a constant, so $I_{a2} = 25 / 0.8 = 31.25 \text{ A}$

$E_{b1} = 220 - (25 \times 0.4) = 210 \text{ V}$; $E_{b2} = 220 - (31.25 \times 0.4) = 207.5 \text{ V}$

$N_2 = N_1 \times (E_{b2}/E_{b1}) \times (\phi_1/\phi_2) = 1000 \times (207.5/210) \times (1/0.8) = 1235 \text{ r.p.m.}$

Answer: $N_2 \approx 1235 \text{ r.p.m.}$

Example 7. A 220 V D.C. shunt motor, $R_a = 0.5 \Omega$, runs at 900 r.p.m. with a full-load armature current of 20 A. What resistance must be inserted in series with the armature to reduce the speed to 600 r.p.m. at constant torque?

Solution:

Constant torque and constant flux: I_a stays 20 A.

$E_{b1} = 220 - (20 \times 0.5) = 210 \text{ V}$; $E_{b2} = 210 \times (600/900) = 140 \text{ V}$

$220 = 140 + 20(0.5 + R) \rightarrow 0.5 + R = 80/20 = 4 \Omega \rightarrow R = 3.5 \Omega$

Answer: External series resistance $R = 3.5 \Omega$

Example 8. A 240 V D.C. series motor has a total resistance (armature + series field) of 0.5Ω . It runs at 1000 r.p.m. taking 20 A. Assuming flux proportional to current, find the speed when the current falls to 10 A.

Solution:

$$E_{b1} = 240 - (20 \times 0.5) = 230 \text{ V}; E_{b2} = 240 - (10 \times 0.5) = 235 \text{ V}$$

$$N_2 / N_1 = (E_{b2} / E_{b1}) \times (I_1 / I_2) = (235 / 230) \times (20 / 10) = 2.043$$

$$N_2 = 1000 \times 2.043 = 2043 \text{ r.p.m.}$$

Answer: $N_2 \approx 2043$ r.p.m. (speed doubles when the load is halved)

3.4 Starting Methods of D.C. Motor – 3-Point and 4-Point Starter

A. Need for a starter

At the instant of starting the armature is at rest, so $E_b = 0$ and the armature current is limited only by the small armature resistance:

$$I_{a(\text{start})} = V / R_a$$

For example, with $V = 220 \text{ V}$ and $R_a = 0.5 \Omega$ the starting current would be 440 A, about 22 times a full-load current of 20 A. Such a current would burn the armature winding, damage the commutator and brushes, cause a heavy voltage dip on the supply and produce a severe mechanical shock. A **starter** is therefore a variable resistance connected in series with the armature at the start and cut out in steps as the motor gains speed and E_b builds up. The starting resistance required to limit the current to I_{max} is

$$R_s = V / I_{\text{max}} - R_a$$

Example 9. A 220 V shunt motor has $R_a = 0.4 \Omega$ and a rated armature current of 25 A. Find the starting resistance needed to limit the starting current to 1.5 times the rated current.

Solution:

$$I_{\text{max}} = 1.5 \times 25 = 37.5 \text{ A}$$

$$R_{\text{total}} = V / I_{\text{max}} = 220 / 37.5 = 5.867 \Omega$$

$$R_s = 5.867 - 0.4 = 5.467 \Omega$$

Answer: $R_s \approx 5.47 \Omega$

B. Three-point starter

The three-point starter is used for shunt motors and compound motors. It takes its name from its three terminals: **L** (line), **A** (armature) and **F** (field).

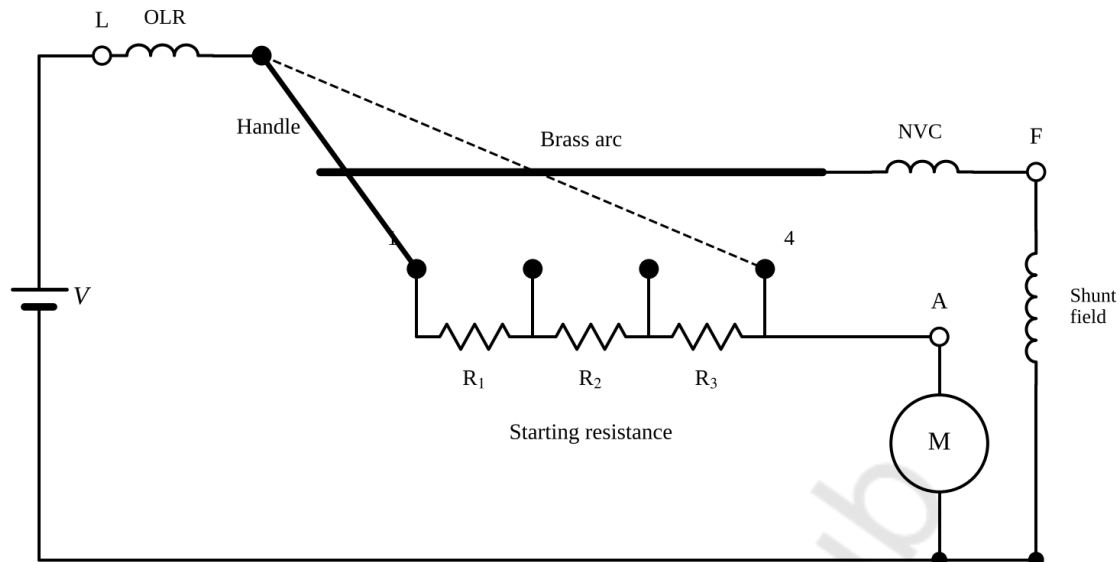


Fig. 7: Three-point starter connected to a D.C. shunt motor

Main parts

- **Starting resistance** divided into sections R_1 , R_2 , R_3 , connected between a series of copper studs.
- **Handle (starter arm)** pivoted at one end; its other end slides over the studs. A spring tends to return it to the OFF position.
- **Brass arc** that connects the field circuit to the handle through the no-volt coil.
- **No-volt coil (NVC) or hold-on coil** – an electromagnet that holds the handle in the ON position by attracting a soft-iron piece fixed to it.
- **Overload release (OLR)** – an electromagnet in the line circuit that trips the starter on overload.

Working

- 1 With the handle in the OFF position, the motor is disconnected. The supply is switched on.
- 2 The handle is moved to stud 1. The armature is now connected to the supply through the **whole** starting resistance, while the shunt field (via the brass arc and NVC) receives full voltage immediately, giving the maximum flux and torque.
- 3 As the motor accelerates, E_b builds up. The handle is moved slowly from stud to stud, cutting out sections of resistance step by step.
- 4 At the last stud (ON position) all the resistance is out and the armature is at full supply voltage. The NVC, energised by the field current, holds the handle against the spring.
- 5 If the supply fails or the field circuit opens, the NVC loses its magnetism and the spring returns the handle to OFF. The motor cannot restart at full voltage when the supply returns.
- 6 On overload the OLR magnet lifts a plunger that short-circuits the NVC; the handle is released and the motor is disconnected.

Limitation: the NVC is in series with the shunt field. When the speed is raised by weakening the field with a rheostat, the field current and the NVC current also fall, which may release the handle and stop the motor.

C. Four-point starter

The four-point starter removes the drawback of the three-point starter by a fourth terminal **N**. The no-volt coil, in series with a protective resistance R_p , is connected **directly across the supply**, independent of the field circuit. The field winding is connected to terminal F through the brass arc.

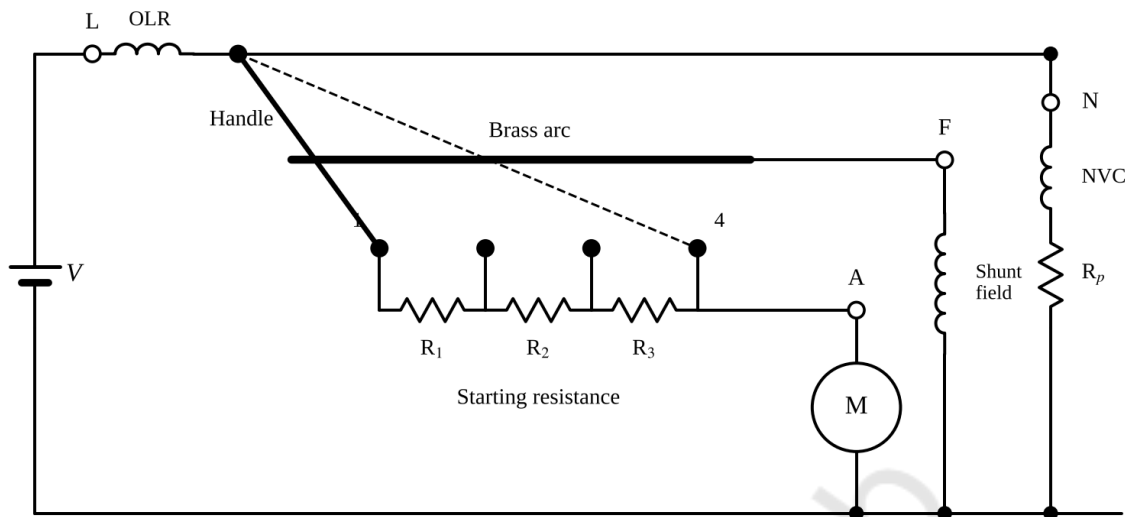


Fig. 8: Four-point starter connected to a D.C. shunt motor

- The NVC current is constant and does not depend on the setting of the field rheostat, so the handle is not released when the field current is reduced for speed control.
- The NVC still releases the handle on failure of the supply voltage. The overload release is retained.
- A drawback is that the NVC does not respond if the field circuit alone opens, so the motor may run away; in practice a field-failure relay can be added.

Feature	3-point starter	4-point starter
Terminals	L, A, F	L, A, F, N
NVC connection	In series with shunt field	Across the supply with series resistor R_p
Effect of field rheostat on NVC	NVC current changes; handle may be released	No effect on NVC current
Suitable for	Constant-speed shunt/compound motors	Shunt/compound motors with field speed control
Protection on field open	Handle released (NVC de-energised)	Handle is not released

Note: for D.C. series motors a simpler *two-point starter* is used, since there is no separate shunt field circuit.

3.5 Losses and Efficiency

A. Losses in a D.C. motor

Loss	Formula	Remarks
Armature copper loss	$I_a^2 R_a$	Variable; depends on load current
Field copper loss	$V \cdot I_{sh}$ (shunt motor); $I_a^2 R_{se}$ (series motor)	Constant for a shunt motor
Brush contact loss	$I_a \times$ (brush drop, about 2 V total)	Small; variable
Hysteresis loss	$W_h = \eta B_{max}^{1.6} f V$	Constant; due to repeated magnetisation reversal of the armature core
Eddy-current loss	$W_e = K B_{max}^2 f^2 t^2 V$	Constant; reduced by laminating the core
Mechanical loss	Friction (bearing, brush) and windage	Constant at constant speed
Stray load loss	—	Small; due to distortion of flux; usually neglected

Losses are also grouped as **constant losses** (iron loss, mechanical loss and shunt field copper loss, which do not change with load) and **variable losses** (armature copper loss and brush contact loss, which change with load current).

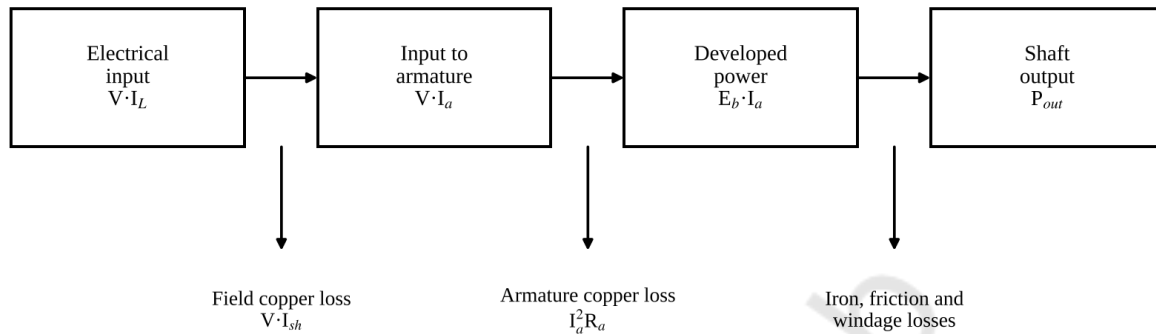


Fig. 9: Power flow in a D.C. shunt motor

B. Efficiency

Efficiency	Expression
Electrical efficiency	$\eta_e = E_b I_a / V I_a = E_b / V$
Mechanical efficiency	$\eta_m = \text{Shaft output} / E_b I_a$
Overall (commercial) efficiency	$\eta = \text{Output} / \text{Input} = (\text{Input} - \text{Total losses}) / \text{Input}$

Condition for maximum efficiency

For a shunt motor with input $V \cdot I_L$ and approximately constant field current, the efficiency is maximum when the **variable loss equals the constant loss**:

$$I_a^2 R_a = W_c \text{ (constant losses)}$$

$$I_a \text{ (for maximum } \eta) = \sqrt{(W_c / R_a)}$$

C. Swinburne's (no-load) test

This is an **indirect** test for a shunt motor. The motor is run on no load at rated voltage and speed; the no-load line current I_0 and shunt field current I_{sh} are noted. At no load the armature current is $I_{a0} = I_0 - I_{sh}$, and the input $V \cdot I_0$ supplies all constant losses plus a small armature copper loss:

$$W_c = V \cdot I_0 - I_{a0}^2 R_a$$

(W_c here includes the shunt-field loss.) The efficiency at any load is then calculated by adding the calculated armature copper loss to W_c . **Advantages:** convenient, economical and needs little power. **Limitations:** it does not account for the change in iron loss caused by armature reaction on load, cannot be used for series motors (which cannot run on no load) and does not give the temperature rise.

Numerical problems

Example 10. A 220 V D.C. shunt motor takes a line current of 40 A at full load. The shunt-field resistance is 110 Ω , the armature resistance is 0.5 Ω and the iron and mechanical (rotational) losses are 400 W. Find the efficiency and the output in horse-power (1 hp = 746 W).

Solution:

$$\text{Input} = 220 \times 40 = 8800 \text{ W}$$

$$I_{sh} = 220/110 = 2 \text{ A}; I_a = 40 - 2 = 38 \text{ A}$$

$$\text{Armature copper loss} = 38^2 \times 0.5 = 722 \text{ W}; \text{field copper loss} = 220 \times 2 = 440 \text{ W}$$

$$\text{Total loss} = 722 + 440 + 400 = 1562 \text{ W}; \text{Output} = 8800 - 1562 = 7238 \text{ W}$$

$$\eta = 7238 / 8800 \times 100 = 82.25 \%; \text{output} = 7238 / 746 = 9.70 \text{ hp}$$

Answer: $\eta \approx 82.3 \%$, output $\approx 7238 \text{ W}$ (9.7 hp)

Example 11. For the motor in the previous example, find the armature current at which the efficiency is maximum.

Solution:

$$\text{Constant losses } W_c = \text{field loss} + \text{rotational loss} = 440 + 400 = 840 \text{ W}$$

$$\text{Condition: } I_a^2 R_a = W_c \rightarrow I_a = \sqrt{(840 / 0.5)} = \sqrt{1680} = 40.99 \text{ A}$$

$$\text{Line current} = 40.99 + 2 = 42.99 \text{ A}$$

Answer: $I_a \approx 41 \text{ A}$ (line current $\approx 43 \text{ A}$)

Example 12. In a Swinburne test on a 220 V shunt motor ($R_a = 0.5 \Omega$) the no-load line current is 4 A and the field current is 1 A. Find the efficiency when the motor takes 40 A from the supply.

Solution:

$$\text{No load: } I_{a0} = 4 - 1 = 3 \text{ A}; W_c = 220 \times 4 - 3^2 \times 0.5 = 880 - 4.5 = 875.5 \text{ W}$$

$$\text{Load: } I_a = 40 - 1 = 39 \text{ A}; \text{armature copper loss} = 39^2 \times 0.5 = 760.5 \text{ W}$$

$$\text{Total loss} = 875.5 + 760.5 = 1636 \text{ W}; \text{input} = 220 \times 40 = 8800 \text{ W}; \text{output} = 8800 - 1636 = 7164 \text{ W}$$

$$\eta = 7164 / 8800 \times 100 = 81.4 \%$$

Answer: $\eta \approx 81.4 \%$

3.6 Braking Methods of D.C. Motor

Braking means stopping a motor quickly or controlling its speed on a down-going load. Besides mechanical friction brakes, a D.C. motor can be braked **electrically**. Electrical braking is smooth, produces no brake-wear and can return energy to the supply. The three methods are regenerative, counter-current (plugging) and dynamic (rheostatic) braking.

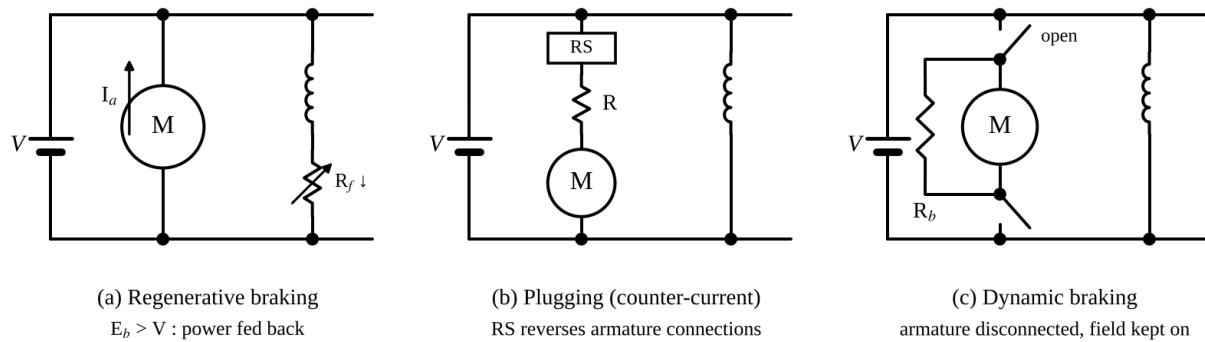


Fig. 10: Electrical braking of a D.C. motor

A. Regenerative braking

In this method the back e.m.f. is made **greater than the supply voltage** ($E_b > V$). The armature current then reverses ($I_a = (E_b - V)/R_a$) and the machine works as a generator, returning the kinetic energy of the load to the supply. A braking torque is produced because the reversed current gives a torque opposing rotation.

- E_b can exceed V when the speed rises above the no-load speed, for example by an over-hauling load such as a train going downhill or a lowering hoist, or by strengthening the field.
- **Advantages:** energy is saved by returning it to the supply; no heat is wasted in resistors.
- **Limitation:** it works only above a certain speed ($E_b > V$) and cannot bring the motor to a complete stop. It is mainly used with shunt and separately excited motors.

B. Counter-current braking (plugging or reverse-current braking)

The connections of the armature (or the field, but not both) are suddenly **reversed** while the motor is running. The supply voltage V and the back e.m.f. E_b now act in the same direction, so a very high current would flow. A resistance R is therefore inserted in series with the armature. The armature current and torque are reversed and the motor is braked rapidly.

$$I_a = (V + E_b) / (R_a + R)$$

$$R = (V + E_b) / I_{max} - R_a$$

- The motor must be disconnected from the supply at zero speed, otherwise it will start to run in the opposite direction.
- **Advantages:** very quick stopping; simple.
- **Disadvantages:** large energy loss as heat in the resistance (the supply also feeds the motor during braking); needs a speed-sensing switch; severe mechanical stress.
- **Uses:** rolling mills, printing presses, machine tools that need quick stopping and reversing.

Example 13. A 220 V motor has $R_a = 0.5 \Omega$ and a back e.m.f. of 200 V at the instant of plugging. Find the series resistance needed to limit the braking current to 40 A.

Solution:

$$R_{total} = (V + E_b) / I_{max} = (220 + 200) / 40 = 10.5 \Omega$$

$$R = 10.5 - 0.5 = 10 \Omega$$

Answer: $R = 10 \Omega$

C. Dynamic braking (rheostatic braking)

The armature is **disconnected from the supply** and immediately connected across a resistance R_b , while the field remains excited from the supply (shunt motor) or is connected so that its current flows in the same direction (series motor). The rotating armature, driven by its kinetic energy, acts as a generator and sends a current through R_b . The kinetic energy is dissipated as heat in the resistance and a braking torque is produced.

- **Advantages:** simple, reliable, smooth braking and can be applied down to very low speeds; does not depend on the supply.
- **Disadvantages:** braking torque falls as speed falls; energy is wasted as heat.
- **Uses:** lifts, hoists, cranes, electric trains and trams.

Method	Armature connection	Energy during braking	Effective down to
Regenerative	Connected to supply; $E_b > V$	Returned to supply	Only above no-load speed
Plugging	Supply terminals reversed, series R	Dissipated in R (supply also feeds)	Zero speed (must be switched off)
Dynamic	Disconnected; across R_b	Dissipated in R_b	Very low speed

3.7 Applications of Different Types of D.C. Motor

Type of motor	Characteristic used	Typical applications
Shunt motor	Nearly constant speed; moderate starting torque; good speed control by field	Lathes, drilling and milling machines, centrifugal pumps, fans and blowers, printing machines, textile and paper machinery, conveyors, reciprocating pumps under light starting load
Series motor	Very high starting torque; speed varies inversely with load; must be loaded	Electric traction (trains, trams, trolley buses), electric cranes and hoists, lifts, vehicle starter motors, winches, conveyors
Cumulative compound motor	High starting torque and fairly constant speed; no runaway on no load	Rolling mills, shears and punches, presses, elevators, reciprocating compressors, heavy planers, intermittent heavy-load machines (with flywheel)
Differential compound motor	Almost constant or rising speed with load; poor starting	Rarely used; experimental and some special constant-speed applications
Separately excited motor	Independent control of flux and armature voltage; wide, smooth speed control	Ward-Leonard drives, steel mills, paper machines, high-precision speed-controlled drives
Permanent-magnet D.C. motor	Compact, no field supply, good efficiency in small ratings	Toys, car wipers and window lifts, small fans, robots, medical devices, household appliances

Selection of a motor: a series motor is chosen when high starting torque is needed and the load is always connected; a shunt motor is chosen when almost constant speed is needed; a compound motor is chosen when the load has sudden peaks, because the cumulative series field gives a high torque while the shunt field limits the no-load speed.

3.8 Brushless D.C. (BLDC) Motors

A **brushless D.C. motor** has the torque–speed behaviour of a D.C. motor, but the mechanical commutator and brushes are replaced by **electronic commutation**. It is essentially a permanent-magnet synchronous machine whose stator is energised from a D.C. supply through an inverter, switched according to the rotor position.

A. Construction

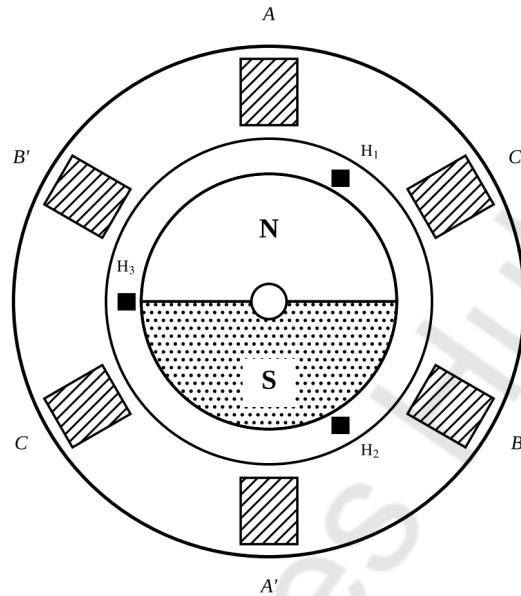


Fig. 11: Cross-section of a three-phase BLDC motor (A, B, C = stator phases; H_1, H_2, H_3 = Hall sensors)

- **Stator:** laminated silicon-steel core with slots carrying a **three-phase winding**, normally star-connected. The windings are fixed, so no brushes are needed to carry current.
- **Rotor:** permanent magnets (ferrite, or high-energy neodymium and samarium-cobalt magnets) mounted on the shaft, forming alternate north and south poles. There are no windings on the rotor, hence no rotor copper loss.
- **Position sensors:** usually three **Hall-effect sensors** spaced 120 electrical degrees apart in the stator, which detect the rotor magnet polarity. (Optical encoders or sensorless back-e.m.f. detection can also be used.)
- **Electronic controller and inverter:** a six-switch three-phase bridge (MOSFET or IGBT) and a controller that uses the Hall signals to decide which two phases to energise.

B. Working principle

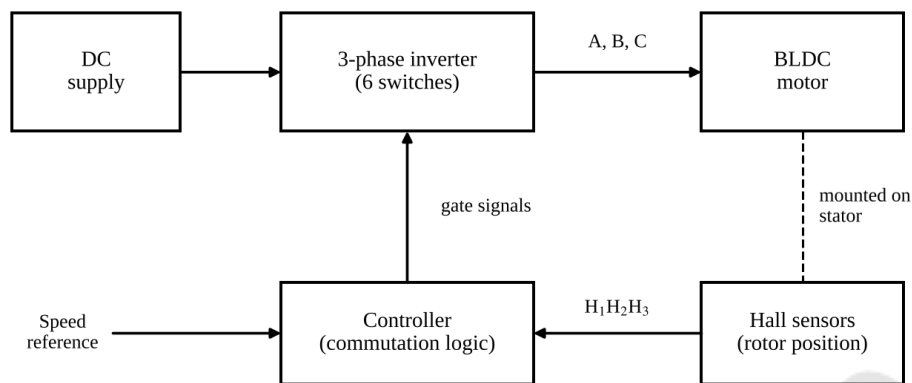


Fig. 12: Block diagram of a BLDC motor drive

At any instant the controller energises **two of the three stator phases**, one with current entering and one with current leaving, while the third phase is left unenergised. The resulting stator magnetic field interacts with the field of the permanent-magnet rotor and produces torque. As the rotor turns, the Hall sensors change state and the controller advances the energised pair so that the stator field stays ahead of the rotor field. One full electrical cycle consists of **six commutation steps**, each 60 electrical degrees long. This is called **six-step (trapezoidal) commutation**.

Step	Phase A	Phase B	Phase C
1	+	–	Off
2	+	Off	–
3	Off	+	–
4	–	+	Off
5	–	Off	+
6	Off	–	+

- **Speed control:** by varying the average voltage with PWM on the inverter switches; speed is roughly proportional to the applied voltage.
- **Torque control:** torque is proportional to the stator current, which is regulated by the controller.
- **Reversal:** by reversing the commutation sequence.

C. Comparison with brushed D.C. motor

Feature	Brushed D.C. motor	BLDC motor
Commutation	Mechanical (commutator and brushes)	Electronic (Hall sensors and inverter)
Maintenance	Brush and commutator wear; sparking	Very low; no brushes
Efficiency	Lower (brush loss, rotor copper loss)	Higher (no brush loss, no rotor winding)
Speed range and life	Limited by brushes	High speed, long life
Noise and EMI	Sparking, electrical noise	Quiet; low electromagnetic noise
Cost and control	Cheap and simple	Higher cost; needs a controller

Advantages: high efficiency, long life, compact with high power-to-weight ratio, silent, good heat dissipation (heat is in the stator), suitable for hazardous environments (no sparks). **Disadvantages:** higher initial cost, complex electronic controller, and torque ripple in trapezoidal drives.

D. Applications of BLDC motors

- Computer hard-disk drives, CD/DVD drives and CPU cooling fans.
- Electric vehicles, e-bikes and e-scooters; drones and model aircraft.
- Washing machines, air-conditioner compressors, refrigerators and vacuum cleaners (energy-efficient appliances).
- Robotics, CNC machines, 3-D printers and industrial automation servo drives.
- Medical equipment such as infusion and blood pumps, and aerospace actuators.

Important Formulas at a Glance

Quantity	Formula
Force on a conductor	$F = B I l$
Back e.m.f.	$E_b = P\phi ZN / 60A$
Voltage equation	$V = E_b + I_a R_a$
Power equation	$V I_a = E_b I_a + I_a^2 R_a$
Armature torque	$T_a = P\phi Z I_a / 2\pi A = 9.55 E_b I_a / N$
Shaft torque	$T_{sh} = 9.55 P_{out} / N$
Speed	$N = K E_b / \phi; N_2/N_1 = (E_{b2}/E_{b1})(\phi_1/\phi_2)$
Condition for maximum power	$E_b = V/2$
Starting resistance	$R_s = V / I_{max} - R_a$
Plugging current	$I_a = (V + E_b) / (R_a + R)$
Maximum efficiency	Variable loss = constant loss ($I_a^2 R_a = W_c$)
Efficiency	$\eta = (\text{Input} - \text{losses}) / \text{Input}$

MULTIPLE CHOICE QUESTIONS

Choose the correct answer. The answer key is given at the end of this section.

Q1. The back e.m.f. in a D.C. motor

- (a) aids the applied voltage
- (b) opposes the applied voltage
- (c) is zero at all speeds
- (d) equals the supply voltage at starting

Q2. A 220 V D.C. motor has $R_a = 0.5 \Omega$ and an armature current of 20 A. The back e.m.f. is

- (a) 230 V
- (b) 220 V
- (c) 210 V
- (d) 200 V

Q3. The torque developed by a D.C. motor is directly proportional to

- (a) $\phi \cdot I_a$
- (b) ϕ / I_a
- (c) I_a / ϕ
- (d) $\phi \cdot N$

Q4. In an unsaturated D.C. series motor, the torque is proportional to

- (a) I_a
- (b) $1 / I_a$
- (c) I_a^2
- (d) $\sqrt{I_a}$

Q5. Which of the following motors should never be started without load?

- (a) Shunt motor
- (b) Separately excited motor
- (c) Cumulative compound motor
- (d) Series motor

Q6. The speed of a D.C. shunt motor above its rated speed can be obtained by

- (a) increasing the armature resistance
- (b) reducing the supply voltage
- (c) increasing the field current
- (d) decreasing the field flux with a field rheostat

Q7. In the Ward–Leonard method, the motor speed is controlled by varying

- (a) the generator field current, which changes the armature voltage
- (b) the field current of the motor only
- (c) the frequency of the A.C. supply
- (d) the load torque

Q8. In a three-point starter, the no-volt coil is connected in series with the

- (a) armature
- (b) supply line
- (c) shunt field
- (d) starting resistance

Q9. The main advantage of a four-point starter over a three-point starter is that

- (a) it is cheaper
- (b) it gives a lower starting current
- (c) it needs no overload protection
- (d) the no-volt coil current is independent of the field current

Q10. The efficiency of a D.C. motor is maximum when

- (a) variable losses = constant losses
- (b) armature copper loss = field copper loss
- (c) iron loss = friction loss
- (d) output = input / 2

Q11. In dynamic braking, the motor armature is

- (a) connected to the supply with reversed polarity
- (b) disconnected from the supply and connected across a resistor
- (c) connected to the supply with a weak field
- (d) short-circuited with the field reversed

Q12. Regenerative braking of a D.C. motor is possible when

- (a) $E_b < V$
- (b) $E_b = 0$
- (c) $E_b > V$
- (d) the supply is disconnected

Q13. During plugging, $V = 220 \text{ V}$, $E_b = 200 \text{ V}$, $R_a = 0.5 \Omega$ and the external resistance is 10Ω . The initial armature current is

- (a) 20 A (b) 40 A
(c) 42 A (d) 84 A

Q14. D.C. series motors are preferred for electric traction mainly because they have

- (a) constant speed (b) low cost
(c) no maintenance (d) high starting torque

Q15. Commutation in a BLDC motor is done by

- (a) carbon brushes (b) a mechanical commutator
(c) electronic switching based on rotor position (d) slip rings

Answer Key

Q.	Ans.	Reason
1	(b)	Back e.m.f. opposes the supply voltage (Lenz's law).
2	(c)	$E_b = V - I_a R_a = 220 - 10 = 210 \text{ V}$.
3	(a)	$T_a = (PZ / 2\pi A) \phi I_a$.
4	(c)	$\phi \propto I_a$ so $T_a \propto I_a^2$.
5	(d)	At no load the series-motor flux is very small and the speed becomes dangerously high.
6	(d)	$N \propto 1/\phi$; weakening the field raises the speed above base speed.
7	(a)	Generator field current changes generator voltage and hence motor armature voltage.
8	(c)	The NVC is in series with the shunt field through the brass arc.
9	(d)	In the 4-point starter the NVC is across the supply, so the field rheostat does not affect it.
10	(a)	Condition for maximum efficiency: $I_a^2 R_a = W_c$.
11	(b)	The armature is switched from the supply to a braking resistor R_b ; the field stays excited.
12	(c)	When $E_b > V$ the armature current reverses and the machine acts as a generator.
13	(b)	$I_a = (220 + 200)/(0.5 + 10) = 420/10.5 = 40 \text{ A}$.
14	(d)	Series motor: $T_a \propto I_a^2$ gives a very high starting torque.
15	(c)	Hall sensors tell the controller which phases to energise (electronic commutation).

BROAD (LONG ANSWER) QUESTIONS

- Q1.** Explain the working principle of a D.C. motor using Fleming's left-hand rule. Derive the expression for the back e.m.f. and state its significance.
- Q2.** Derive the torque equation of a D.C. motor. Show that the torque of a shunt motor is proportional to the armature current and that of a series motor is proportional to the square of the armature current (unsaturated).
- Q3.** A 4-pole, 220 V D.C. shunt motor has a lap-wound armature with 540 conductors and a flux of 0.025 Wb per pole. The armature resistance is $0.6\ \Omega$ and the armature current is 25 A. Calculate the back e.m.f., the speed and the gross torque developed.
- Q4.** Draw and explain the torque–armature current, speed–armature current and speed–torque characteristics of D.C. shunt, series and cumulative compound motors. Why must a series motor not be started on no load?
- Q5.** Explain the methods of speed control of a D.C. shunt motor. Compare field control and armature-resistance control. A 220 V shunt motor with armature resistance $0.5\ \Omega$ runs at 1000 r.p.m. with an armature current of 20 A; find the series resistance needed to reduce the speed to 700 r.p.m. at constant torque.
- Q6.** Describe the Ward–Leonard method of speed control with a neat diagram. State its advantages, disadvantages and applications.
- Q7.** Why is a starter necessary for a D.C. motor? With a neat diagram explain the construction and working of a three-point starter. How does the four-point starter overcome its drawback?
- Q8.** Name the various losses in a D.C. motor and explain how they vary with load. State and prove the condition for maximum efficiency. A 220 V shunt motor takes 30 A on full load; $R_{sh} = 110\ \Omega$, $R_a = 0.4\ \Omega$ and rotational losses are 350 W. Calculate its efficiency.
- Q9.** Explain regenerative braking, plugging and dynamic braking of a D.C. motor with circuit diagrams. State the advantages and limitations of each method.
- Q10.** Describe the construction, working principle and applications of a brushless D.C. motor. Compare it with a conventional brushed D.C. motor.

— End of Chapter 3: D.C. Motor —