

ELECTRICAL AND ELECTRONIC MEASUREMENTS

Chapter 3

MEASUREMENT OF ELECTRIC POWER

Detailed Study Notes for Engineering (Diploma) Students

Notes • Diagrams • Solved Numericals • 15 MCQs with Answer Key • 10 Broad Questions

Unit	Topic covered
3.1	Dynamometer type wattmeter: construction, working principle, multiplying factor, extension of range, errors and their compensation
3.2	Active power in a three-phase circuit (balanced load): one-, two- and three-wattmeter methods, related problems
3.3	Active power in a three-phase circuit (unbalanced load)
3.4	Effect of power factor variation on wattmeter readings in the two-wattmeter method; numerical problems
3.5	Measurement of reactive power in a three-phase circuit
3.6	Maximum demand indicator: construction and working principle

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CONTENTS

Introduction: Electric Power and its Measurement	4
3.1 Dynamometer Type Wattmeter	5
3.1.1 Introduction	5
3.1.2 Construction	5
3.1.3 Working principle	6
3.1.4 Connections of a wattmeter	7
3.1.5 Multiplying factor and extension of range	7
3.1.6 Different types of errors and their compensation	9
3.2 Active Power in a Three-Phase Circuit (Balanced Load)	12
3.2.1 Basics of three-phase power	12
3.2.2 One-wattmeter method	12
3.2.3 Three-wattmeter method	13
3.2.4 Two-wattmeter method	13
3.2.5 Comparison of the three methods	15
3.2.6 Related problems	15
3.3 Active Power in a Three-Phase Circuit (Unbalanced Load)	16
3.3.1 Three-phase four-wire system	16
3.3.2 Three-phase three-wire system	16
3.3.3 Selection of the method	17
3.4 Effect of Power Factor on Wattmeter Readings (Two-Wattmeter Method)	18
3.4.1 Interpretation of the readings	18
3.4.2 Power factor from the ratio of readings	19
3.4.3 Numerical problems	19
3.5 Measurement of Reactive Power in a Three-Phase Circuit	21
3.5.1 Using the two-wattmeter method (balanced load)	21
3.5.2 One-wattmeter method (balanced load)	21
3.5.3 Three-wattmeter method (unbalanced three-wire load)	22
3.5.4 Varmeter	22
3.6 Maximum Demand Indicator (MDI)	23
3.6.1 Introduction and definitions	23

3.6.2 Types of maximum demand indicators	23
3.6.3 Mechanical type: construction and working	23
3.6.4 Thermal type: construction and working	25
3.6.5 Electronic (digital) maximum demand meters	25
Chapter Summary: Important Formulae	26
Multiple Choice Questions (15)	27
Broad Questions (10)	30

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Introduction: Electric Power and its Measurement

Electrical power is the rate at which electrical energy is delivered to, or consumed by, a circuit. Every machine, appliance and industrial plant is rated in terms of power, and every consumer is billed for the energy (power $\times$ time) used. Therefore, the measurement of power is one of the most important measurements in electrical engineering. This chapter explains how power is measured in single-phase and three-phase circuits using the **wattmeter**, how reactive power is obtained, and how the **maximum demand** of a consumer is recorded for tariff purposes.

Power in DC and AC circuits

In a DC circuit the power is simply the product of voltage and current, $P = VI$. In an AC circuit the voltage and current vary with time, so the instantaneous power $p = vi$ also varies. Let $v = V_m \sin \omega t$ and $i = I_m \sin (\omega t - \phi)$, where ϕ is the phase angle between them. Then

$$p = vi = VI \cos \phi - VI \cos (2\omega t - \phi)$$

The first term is constant. The second term is a pulsating component whose frequency is twice the supply frequency and whose average over a cycle is zero. Hence the **average (active) power** is

$$P = VI \cos \phi \text{ watts}$$

where V and I are r.m.s. values and $\cos \phi$ is the **power factor**. A wattmeter is an instrument that responds to this average value.

Active, reactive and apparent power

Quantity	Symbol / unit	Single-phase	Three-phase (balanced)
Active (true) power	P / watt (W)	$VI \cos \phi$	$\sqrt{3} V_L I_L \cos \phi = 3 V_{ph} I_{ph} \cos \phi$
Reactive power	Q / volt-ampere reactive (VAr)	$VI \sin \phi$	$\sqrt{3} V_L I_L \sin \phi$
Apparent power	S / volt-ampere (VA)	VI	$\sqrt{3} V_L I_L$
Power factor	$\cos \phi$ (no unit)	P / S	P / S

The three powers are related by the **power triangle**: $S^2 = P^2 + Q^2$, $\tan \phi = Q/P$ and $\cos \phi = P/S$.

Notation used in this chapter

- V_L, I_L : line voltage and line current; V_{ph}, I_{ph} : phase voltage and phase current.
- ϕ : phase angle between phase voltage and phase current of the load (lagging ϕ is taken as positive, leading ϕ as negative).
- W_1, W_2, W_3 : readings of wattmeters; CC : current coil; PC : pressure (voltage) coil.
- Phase sequence is taken as R - Y - B throughout unless stated otherwise.

3.1 Dynamometer Type Wattmeter

3.1.1 Introduction

A **wattmeter** is an instrument that measures the average electrical power in a circuit directly, in watts. It has two circuits: a **current circuit** that carries (or is proportional to) the load current, and a **pressure (voltage) circuit** that carries a current proportional to the load voltage. The deflection of the pointer is proportional to the product of the two, that is, to the power. The commonly used types are the dynamometer type, the induction type and the electronic (digital) type. The **dynamometer type** is the most widely used one because it can measure power in both DC and AC circuits and can be built with high accuracy (it is often used as a laboratory sub-standard).

3.1.2 Construction

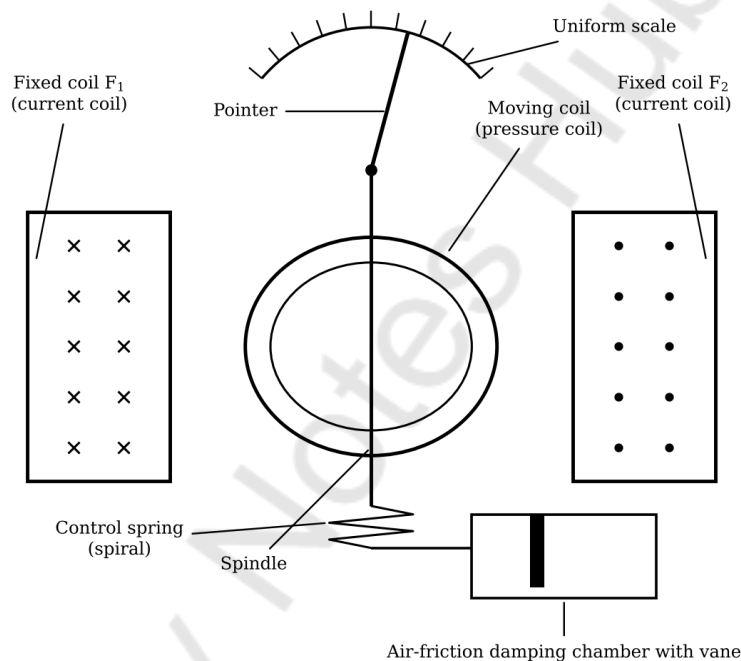


Fig. 3.1 Construction of a dynamometer type wattmeter (schematic)

A dynamometer wattmeter consists of the following parts.

Part	Description	Purpose / reason
Fixed coils (current coil, CC)	Two identical coils wound with thick wire of few turns, placed face to face with a gap between them. Connected in series with the load.	Produce a magnetic field proportional to the load current. Two halves give a nearly uniform field over the moving coil and allow series/parallel connection to change the current range.
Moving coil (pressure coil, PC)	A light coil of many turns of fine wire, mounted on a spindle inside the fixed coils. Connected across the load through a high non-inductive series resistance.	Carries a small current (a few milliamperes) proportional to the load voltage.
Series (multiplier) resistance	High, non-inductive resistance (usually manganin) in series with the moving coil.	Limits the pressure-coil current, keeps its circuit almost purely resistive and sets the voltage range.

Part	Description	Purpose / reason
Controlling system	Two spiral springs (hair springs) of phosphor bronze, which also lead the current in and out of the moving coil.	Provide controlling torque proportional to deflection (spring control).
Damping system	Air-friction damping: light aluminium vanes attached to the spindle move in sector-shaped closed air chambers.	Brings the pointer quickly to rest. Eddy-current damping is avoided because it would distort the weak working field.
Pointer and scale	Light aluminium pointer (knife-edge type) with balance weights; scale fitted with a mirror.	Mirror avoids parallax error. The scale is practically uniform.
Frame and shielding	Air-cored (no iron) construction; instrument enclosed in a shield of high-permeability alloy or laminated iron.	No hysteresis or iron losses; protection from stray magnetic fields.

- The instrument is **air-cored** (no iron), so the readings are free from hysteresis and eddy-current errors in the magnetic circuit. The price paid is a weak field, so the coils consume a comparatively large amount of power to produce sufficient torque.
- Since the field is weak, stray fields can influence the reading. High-grade instruments are therefore shielded or use an **astatic** arrangement (two moving coils on the same spindle, in opposing fields).
- The moving coil is a delicate part; its current is kept small and is brought in through the control springs.

3.1.3 Working principle

The wattmeter works on the principle that *a current-carrying coil placed in the magnetic field of another current-carrying coil experiences a torque*. The fixed coils carry the load current i and set up a field proportional to i . The moving coil carries a current i_p proportional to the load voltage v . The torque on the moving coil is proportional to the product $i \times i_p$ and to the rate of change of mutual inductance between the coils.

Let $v = V_m \sin \omega t$ be the load voltage and $i = I_m \sin (\omega t - \phi)$ the load current. If R_p is the total resistance of the pressure-coil circuit (its inductance being neglected), then the pressure-coil current is $i_p = v/R_p$, in phase with the voltage. The instantaneous deflecting torque is

$$T_i = i \cdot i_p \cdot (dM/d\theta) = (vi / R_p) \cdot (dM/d\theta)$$

Because of the inertia of the moving system, the pointer cannot follow the rapid variations of this torque and takes up a position corresponding to the **average torque** over a cycle:

$$T_d = (1 / R_p) \cdot (dM/d\theta) \cdot (1/T) \int vi dt = (VI \cos \phi / R_p) \cdot (dM/d\theta)$$

At the final steady position the deflecting torque equals the controlling torque $T_c = K\theta$, where K is the spring constant. Hence

$$\theta = (1 / K R_p) \cdot (dM/d\theta) \cdot VI \cos \phi$$

The coils are shaped and placed so that $dM/d\theta$ is nearly constant over the working range. Then $\theta \propto VI \cos \phi = P$: the deflection is proportional to the power, and the **scale is uniform**. For DC, $\phi = 0$ and $\theta \propto VI$. Thus the same instrument can be used on DC and AC.

Feature	Remark
Quantity measured	Average (active) power = $VI \cos \phi$
Scale	Uniform (since $\theta \propto P$, $dM/d\theta$ constant)
Controlling torque	Spring control, $T_c = K\theta$
Damping	Air friction
Suitable for	DC and AC (power-frequency) circuits; single- and three-phase
Power consumption	Relatively high, because the field is weak (air-cored)
Direction of deflection	Reverses if the connections of either coil alone are reversed

3.1.4 Connections of a wattmeter

Each coil has a marked terminal (marked $\pm$, M or with a dot). The two marked terminals must be connected to the **same supply line**, otherwise the pointer deflects below zero. The pressure coil can be connected in two ways, shown in Fig. 3.2.

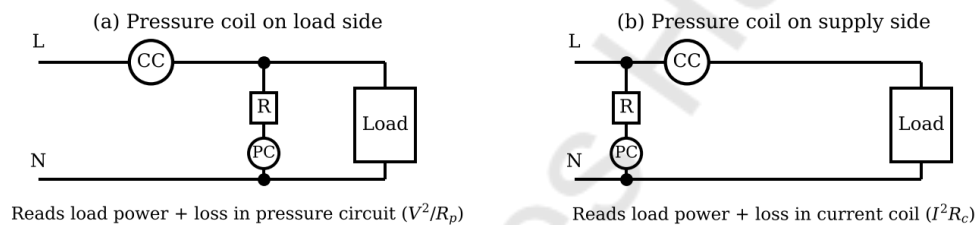


Fig. 3.2 Two ways of connecting a wattmeter (CC = current coil, PC = pressure coil, R = series resistance)

Connection	Pressure coil placed	Wattmeter reading	Suitable for
(a)	On the load side of the current coil	Load power + power loss in pressure circuit (V^2/R_p). Current coil carries load current plus pressure-coil current.	Low voltage, heavy current loads (the pressure-circuit loss is then very small compared with load power).
(b)	On the supply side of the current coil	Load power + power loss in the current coil ($I^2 R_c$). Pressure coil carries the drop of the current coil also.	High voltage, light current loads (the current-coil loss is then very small).

- The marked terminal of the pressure coil is joined to the same line as the marked current terminal. This keeps the potential difference between the fixed and moving coils very small and avoids an **electrostatic force** between them.
- A wattmeter may read backwards (negative) at very low or leading power factors. A **reversing switch** in the pressure circuit is used to reverse the pressure-coil connection (not the current coil) so that the reading can be taken and treated as negative.

3.1.5 Multiplying factor and extension of range

A wattmeter has a voltage range and a current range, for example 300 V and 10 A. The **power range** of a wattmeter (for unity power factor) is the product of these two ranges. The reading of the pointer, in divisions, must be multiplied by a **multiplying factor (MF)** to obtain the actual power in watts.

Multiplying factor

$$MF = (\text{Rated voltage} \times \text{Rated current} \times \text{Rated power factor}) / (\text{Full-scale divisions})$$

$$\text{Power (W)} = MF \times \text{Wattmeter reading (divisions)}$$

Extension of range. The range of a wattmeter can be changed in the following ways.

- **Current range:** The two fixed coils are connected in series (lower current range) or in parallel (current range doubled). For DC or large currents, a shunt may be used. For AC, a **current transformer (CT)** is used.
- **Voltage range:** A different tapping of the series (multiplier) resistance is selected, or an external multiplier is added. For high AC voltages, a **potential (voltage) transformer (PT)** is used.
- With instrument transformers the true power is the product of the wattmeter reading, the CT ratio and the PT ratio.

Power with instrument transformers

$$P = (\text{Wattmeter reading in watts}) \times (\text{CT ratio}) \times (\text{PT ratio})$$

Feature	UPF wattmeter	LPF (low power factor) wattmeter
Designed for	Circuits of normal power factor (near unity)	Circuits of very low power factor, e.g. no-load / open-circuit tests of transformers and induction motors
Full-scale deflection corresponds to	Rated $V \times \text{rated } I \times 1$	Rated $V \times \text{rated } I \times 0.1$ or 0.2 (typical)
Multiplying factor	Large	Small, so low powers can be read accurately
Special features	Usually no compensation	Provides greater torque for small $VI \cos \phi$; compensated for pressure-coil current and inductance because errors are large at low pf

Example 3.1

A wattmeter is rated 250 V, 5 A, unity power factor, with 100 divisions on its scale. Find the power when the pointer reads 64 divisions.

$$MF = (250 \times 5) / 100 = 12.5 \text{ W per division. Power} = 12.5 \times 64 = \mathbf{800 \text{ W}}.$$

Example 3.2

A low power factor wattmeter is rated 300 V, 10 A and 0.2 power factor with 150 divisions. Find its MF and the power for a reading of 90 divisions.

$$MF = (300 \times 10 \times 0.2) / 150 = 4 \text{ W per division. Power} = 4 \times 90 = \mathbf{360 \text{ W}}.$$

Example 3.3

A wattmeter is used with a CT of ratio 100/5 A and a PT of ratio 11000/110 V and reads 450 W. Find the actual power.

$$\text{CT ratio} = 100/5 = 20 \text{ and PT ratio} = 11000/110 = 100. \text{ Power} = 450 \times 20 \times 100 = 900\,000 \text{ W} = \mathbf{900 \text{ kW}}.$$

3.1.6 Different types of errors and their compensation

Even in a well-made dynamometer wattmeter certain errors are present. They are explained below with the usual methods of compensation.

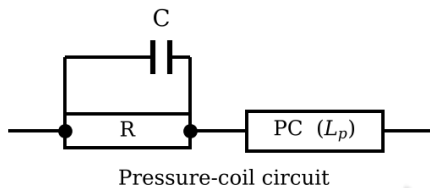
(a) Error due to pressure-coil inductance

The pressure coil has some inductance L_p , so its current lags the voltage by an angle β , where $\tan \beta = \omega L_p / R$ (R is the resistance of the pressure circuit). The angle between the load current and the pressure-coil current becomes $(\phi - \beta)$ instead of ϕ . The pressure-coil current is $V \cos \beta / R$, so

Indicated power $\propto (VI/R) \cos \beta \cos (\phi - \beta)$
True power $\propto (VI/R) \cos \phi$
Correction factor = True / Indicated = $\cos \phi / [\cos \beta \cos (\phi - \beta)] \approx 1/(1 + \tan \phi \tan \beta)$

For a lagging load the wattmeter therefore reads **high**, and for a leading load it reads **low**. The error increases as the power factor falls, since $\tan \phi$ becomes large. **Compensation:** a capacitor C is connected across a part of the series resistance of the pressure circuit (Fig. 3.3a). The circuit then behaves as a pure resistance when $C \approx L_p / R^2$ approximately, so that the pressure-coil current is brought in phase with the voltage at all frequencies of interest.

(a) Capacitor across part of series resistance



(b) Compensating winding

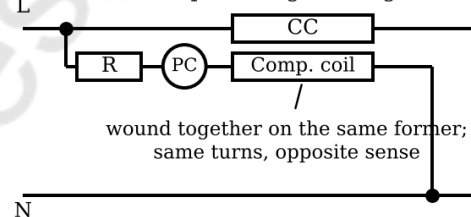


Fig. 3.3 Compensation of errors in a wattmeter

(b) Error due to pressure-coil capacitance

The series resistance and the pressure coil have stray capacitance between turns and to earth. This produces an effect opposite to inductance (the current tends to lead). At power frequency it is negligible; it is reduced further by careful winding of the resistor and coil. When a compensating capacitor is correctly chosen, the net effect of inductance and stray capacitance is cancelled.

(c) Error due to connections (power loss in the instrument)

As shown in Section 3.1.4, one connection makes the instrument read the load power plus the pressure-circuit loss, and the other plus the current-coil loss. **Compensation** is done in two ways: (i) by calculating the loss and subtracting it from the reading, or (ii) by using a **compensating (neutralising) coil**. The compensating coil has the same number of turns as the current coil, is wound in the same space, and is connected in series with the pressure circuit but with its current in the **opposite sense** (Fig. 3.3b). The pressure-coil current thus produces equal and opposite ampere-turns in the compensating coil and the current coil, and the net field is due only to the load current. Hence the reading corresponds to true load power. Compensated wattmeters are normally LPF wattmeters.

(d) Eddy-current error

The alternating field of the current coil induces eddy currents in solid metal parts nearby (case, coil formers, supports). These set up extra fields that change the phase and magnitude of the field acting on the moving coil. The effect depends on the power factor. It is reduced by using non-metallic or high-resistivity frames, by keeping metal parts away from the coils and by using stranded conductors for the coils.

(e) Error due to stray magnetic fields

The working field is weak, so external fields from nearby cables, machines or transformers can cause appreciable error. **Compensation:** enclose the instrument in a high-permeability magnetic shield, or use an **astatic** construction in which the effect on one moving coil is cancelled by the other.

(f) Electrostatic error

A potential difference between the fixed and moving coils produces an electrostatic attraction that disturbs the deflection, particularly at high voltage. **Compensation:** join the marked terminal of the pressure coil to the line carrying the current coil, and place an earthed electrostatic shield between the coils.

(g) Temperature and mechanical errors

The resistance of the pressure circuit and the stiffness of the springs vary with temperature. A low-temperature-coefficient material (manganin) is used for the series resistance. Friction at the bearings and vibration of the moving system are reduced by using jewelled bearings and a light moving system.

Error	Cause	Effect	Remedy
Pressure-coil inductance	PC current lags voltage by β	Reads high (lagging pf), low (leading pf); large at low pf	Capacitor across a part of series resistance, $C \approx L_p/R^2$
Pressure-coil capacitance	Stray capacitance of coil and resistor	Effect opposite to inductance; small at 50 Hz	Careful winding; correct compensating capacitor
Connection (instrument loss)	PC current passes through CC, or CC drop appears across PC	Reads load power + loss	Calculate and subtract; compensating coil
Eddy currents	Induced in nearby solid metal parts	Phase and magnitude error depending on pf	Non-metallic frame, no metal near coils, stranded wire
Stray magnetic field	Weak air-cored working field	Spurious torque	Magnetic shielding; astatic system
Electrostatic force	Potential difference between coils	Spurious deflection at high voltage	Common connection of marked terminals; electrostatic screen
Temperature / mechanical	Resistance and spring changes; friction	Slow drift, sticking pointer	Manganin resistance; jewelled bearings

Example 3.4

(a) A wattmeter is connected with its pressure coil on the supply side of the current coil. It reads 2000 W when the load current is 10 A. The resistance of the current coil is 0.2 Ω . Find the true power. (b) With the pressure coil on the load side, the instrument reads 1500 W at 250 V. The resistance of the pressure circuit is 10 k Ω . Find the true power.

(a) Current-coil loss = $I^2 R_c = 10^2 \times 0.2 = 20$ W. True power = 2000 – 20 = **1980 W**.

(b) Pressure-circuit loss = $V^2/R_p = 250^2/10000 = 6.25$ W. True power = $1500 - 6.25 = \mathbf{1493.75}$ W.

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3.2 Active Power in a Three-Phase Circuit (Balanced Load)

3.2.1 Basics of three-phase power

A three-phase load may be connected in star or in delta. In a **balanced** load, all three phase impedances are equal, so the three phase currents are equal in magnitude and 120° apart. The total power is three times the power of one phase.

Quantity	Star connection	Delta connection
Line voltage V_L	$\sqrt{3} V_{ph}$	V_{ph}
Line current I_L	I_{ph}	$\sqrt{3} I_{ph}$
Power per phase	$V_{ph} I_{ph} \cos \phi$	$V_{ph} I_{ph} \cos \phi$
Total power	$3 V_{ph} I_{ph} \cos \phi = \sqrt{3} V_L I_L \cos \phi$	$3 V_{ph} I_{ph} \cos \phi = \sqrt{3} V_L I_L \cos \phi$

The three common methods of measuring three-phase active power are the one-wattmeter, two-wattmeter and three-wattmeter methods. Note that ϕ in all these formulae is the phase angle of the *load* (angle between phase voltage and phase current), whatever the connection.

3.2.2 One-wattmeter method

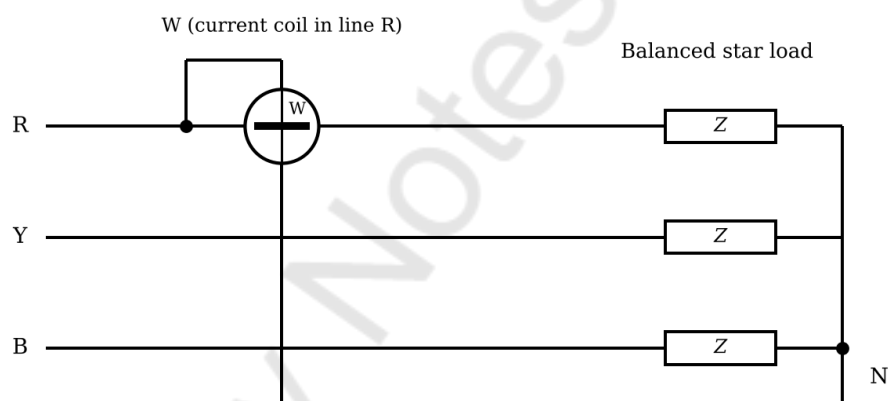


Fig. 3.4 One-wattmeter method (balanced load): $P = 3W$

This method is applicable only to a **balanced** load. The current coil of the wattmeter is connected in one line (say R) and its pressure coil is connected between that line and the neutral point (star point) of the load. The wattmeter then reads the power of one phase:

$$W = V_{ph} I_{ph} \cos \phi \quad \text{Total power } P = 3W$$

- If the star point of the load is not accessible (delta load, or star load with no neutral), an **artificial neutral** is made by two equal resistors, each equal to the resistance of the pressure circuit, connected from the other two lines and joined with the pressure coil in star.
- **Advantages:** only one wattmeter is needed; simple and cheap.
- **Limitations:** valid only for balanced loads; the neutral must be available or created; any unbalance in the load gives a wrong result. The total power is multiplied by 3, so any error in the single reading is also tripled.

3.2.3 Three-wattmeter method

In the three-wattmeter method one wattmeter is placed in each line. The current coil of each wattmeter carries the line current and its pressure coil is connected between that line and the neutral point (see Fig. 3.7 in Section 3.3). The three wattmeters read the powers of the three phases separately, so

$$P = W_1 + W_2 + W_3$$

For a balanced load, all readings are equal and $P = 3W$. The method is valid for any load (balanced or unbalanced) in a three-phase four-wire system and is the most direct method; its drawback is that three wattmeters are needed. For a three-wire star or delta load, an artificial star point made of three equal resistors can be used. Each reading then no longer equals the phase power, but their sum still equals the total power (see Section 3.3).

3.2.4 Two-wattmeter method

This is the most popular method for three-phase, three-wire systems. Two wattmeters are used, whatever the load is (star or delta, balanced or unbalanced), and no neutral point is needed.

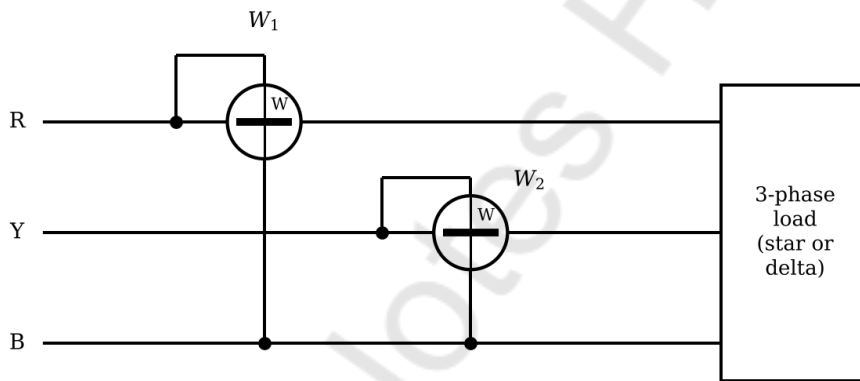


Fig. 3.5 Two-wattmeter method: $P = W_1 + W_2$

Connection:

1. The current coils of the two wattmeters are connected in any two lines (here R and Y).
2. The marked pressure terminal of each wattmeter is connected to the line in which its current coil lies.
3. The other pressure terminals of both wattmeters are joined together to the third line (here B), which has no current coil.

Proof (instantaneous powers). Let v_R, v_Y, v_B be the instantaneous phase voltages and i_R, i_Y, i_B the line currents of a star load. The total instantaneous power is

$$p = v_R i_R + v_Y i_Y + v_B i_B$$

In a three-wire system the line currents add up to zero, $i_R + i_Y + i_B = 0$, so $i_B = -(i_R + i_Y)$. Substituting,

$$p = v_R i_R + v_Y i_Y - v_B (i_R + i_Y) = (v_R - v_B) i_R + (v_Y - v_B) i_Y = v_{RB} i_R + v_{YB} i_Y$$

The term $v_{RB} i_R$ is exactly what wattmeter W_1 responds to (current in line R, voltage between R and B), and $v_{YB} i_Y$ is what W_2 responds to. Taking the average over a cycle, $P = W_1 + W_2$, the **algebraic** sum of the two readings. This proof uses only the fact that the three currents add up to zero, so it holds for unbalanced loads also. (This is a special case of **Blondel's theorem**: in an

n -wire system, $n - 1$ wattmeters are enough.)

Readings for a balanced load (phasor method). Let the load be balanced with a lagging angle ϕ and take V_R as the reference. With phase sequence R - Y - B, $V_R = V \angle 0^\circ$, $V_Y = V \angle -120^\circ$, $V_B = V \angle +120^\circ$. Then

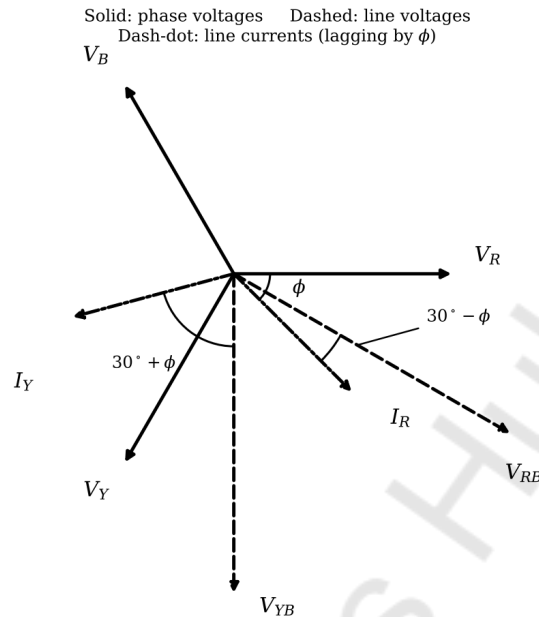


Fig. 3.6 Phasor diagram for the two-wattmeter method (balanced, lagging load)

- $V_{RB} = V_R - V_B$ has magnitude V_L and lies at -30° . The current I_R lies at $-\phi$. The angle between them is $(30^\circ - \phi)$.
- $V_{YB} = V_Y - V_B$ has magnitude V_L and lies at -90° . The current I_Y lies at $(-120^\circ - \phi)$. The angle between them is $(30^\circ + \phi)$.

$$W_1 = V_L I_L \cos (30^\circ - \phi)$$

$$W_2 = V_L I_L \cos (30^\circ + \phi)$$

Adding and subtracting these two readings gives the important results below.

Results of the two-wattmeter method (balanced load)

$$W_1 + W_2 = 2 V_L I_L \cos 30^\circ \cos \phi = \sqrt{3} V_L I_L \cos \phi = P$$

$$W_1 - W_2 = 2 V_L I_L \sin 30^\circ \sin \phi = V_L I_L \sin \phi$$

$$\tan \phi = \sqrt{3} (W_1 - W_2) / (W_1 + W_2)$$

$$\text{Power factor} = \cos \phi = \cos [\tan^{-1} \{ \sqrt{3} (W_1 - W_2) / (W_1 + W_2) \}]$$

$$\text{Reactive power } Q = \sqrt{3} (W_1 - W_2)$$

- For a **lagging** load, W_1 is the larger reading; for a **leading** load the readings interchange. If the phase sequence is reversed, the readings also interchange.
- The ratio method: if $k = W_2/W_1$ (smaller/larger) then $\tan \phi = \sqrt{3} (1 - k)/(1 + k)$.
- **Advantages:** only two wattmeters; no neutral required; works for balanced and unbalanced three-wire loads; gives power factor and reactive power for balanced loads.
- **Limitations:** cannot be used for four-wire systems with neutral current; the sign of each reading must be noted carefully (a reading may be negative); power factor formula valid only

for balanced loads.

3.2.5 Comparison of the three methods

Method	No. of wattmeters	Load / system	Total power
One wattmeter	1	Balanced only; star point (real or artificial) needed	3 W
Two wattmeter	2	Balanced or unbalanced; 3-wire system; no neutral needed	$W_1 + W_2$ (algebraic)
Three wattmeter	3	Balanced or unbalanced; 4-wire (or 3-wire with artificial star)	$W_1 + W_2 + W_3$

3.2.6 Related problems

Example 3.5

A balanced three-phase load takes a line current of 20 A at 415 V line voltage and 0.8 power factor lagging. Find the total power and the readings of two wattmeters connected by the two-wattmeter method.

$$P = \sqrt{3} V_L I_L \cos \phi = 1.732 \times 415 \times 20 \times 0.8 = \mathbf{11\,501\,W \approx 11.5\,kW}.$$

$$\cos \phi = 0.8 \text{ gives } \phi = 36.87^\circ. V_L I_L = 415 \times 20 = 8300 \text{ VA.}$$

$$W_1 = 8300 \cos (30^\circ - 36.87^\circ) = 8300 \cos 6.87^\circ = \mathbf{8240\,W}.$$

$$W_2 = 8300 \cos (30^\circ + 36.87^\circ) = 8300 \cos 66.87^\circ = \mathbf{3260\,W}.$$

Check: $8240 + 3260 = 11\,500\,W$ (verified).

Example 3.6

The readings of two wattmeters connected to a balanced three-phase load are 6 kW and 2 kW. Find the total power, power factor and reactive power.

$$P = 6 + 2 = \mathbf{8\,kW}.$$

$$\tan \phi = \sqrt{3} (6 - 2) / (6 + 2) = 1.732 \times 0.5 = 0.866, \text{ so } \phi = 40.89^\circ \text{ and power factor} = \cos 40.89^\circ = \mathbf{0.756 \text{ lagging.}}$$

$$Q = \sqrt{3} (6 - 2) = \mathbf{6.93\,kVAr}.$$

3.3 Active Power in a Three-Phase Circuit (Unbalanced Load)

When the three phase impedances are not equal, the phase currents differ in magnitude and phase. One wattmeter reading multiplied by three no longer gives the total power, and the power-factor formulae of the balanced case do not apply. The total power must therefore be found from measurements that account for each line.

3.3.1 Three-phase four-wire system

In a four-wire system, the neutral wire carries the unbalance current ($i_R + i_Y + i_B \neq 0$). The two-wattmeter method is not valid here. The **three-wattmeter method** is used: each wattmeter has its current coil in one line and its pressure coil between that line and the neutral wire.

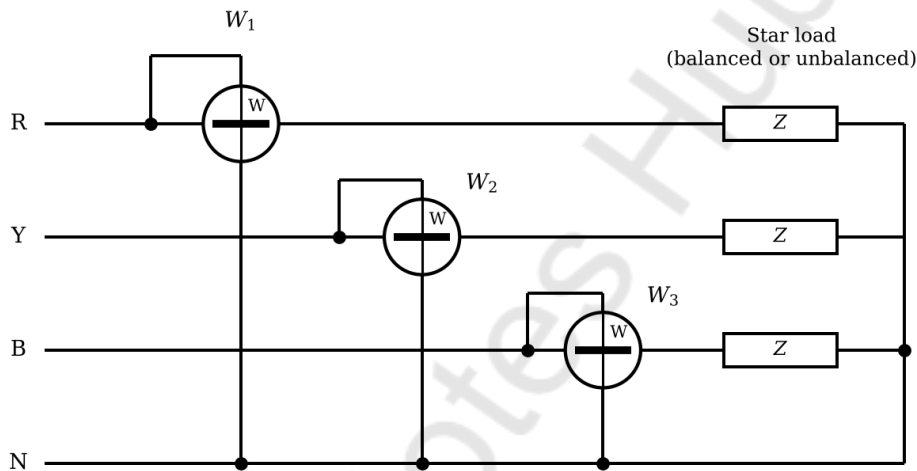


Fig. 3.7 Three-wattmeter method (3-phase 4-wire): $P = W_1 + W_2 + W_3$

$$P = W_1 + W_2 + W_3 = P_R + P_Y + P_B$$

A single **three-element (polyphase) wattmeter**, in which three moving coils act on the same spindle, can be used instead of three separate instruments; its pointer shows the total power directly.

3.3.2 Three-phase three-wire system

In a three-wire system (star without neutral, or delta), the line currents add up to zero, so the **two-wattmeter method** remains valid for any unbalanced load (see the proof in Section 3.2.4):

$$P = W_1 + W_2 \text{ (algebraic sum)}$$

- The individual readings no longer have the simple form $\cos(30^\circ \mp \phi)$, so $\tan \phi = \sqrt{3} (W_1 - W_2) / (W_1 + W_2)$ cannot be used to find a power factor of the load. Only the total power is obtained.
- A three-wattmeter method with an **artificial neutral** may also be used. The artificial neutral is formed by three equal resistors (each equal to the pressure-circuit resistance of a wattmeter) connected in star with the three pressure coils. The potential v_N of this point differs from the real neutral, but $\Sigma(v - v_N)i = \Sigma vi - v_N \Sigma i = \Sigma vi$ because $\Sigma i = 0$. Hence the sum of the three readings is still the total power, though individual readings are not phase powers.

3.3.3 Selection of the method

System	Load	Method	Total power
3-phase, 4-wire	Balanced or unbalanced	Three wattmeters (one per phase) or a polyphase wattmeter	$W_1 + W_2 + W_3$
3-phase, 3-wire	Balanced or unbalanced	Two wattmeters (Aron connection)	$W_1 + W_2$
3-phase, 3-wire	Balanced only	One wattmeter with real or artificial neutral	$3 W$

Example 3.7

In a three-phase four-wire system supplying an unbalanced star load, three wattmeters connected in the lines read 2.4 kW, 1.8 kW and 3.1 kW. Find the total power.

$$P = 2.4 + 1.8 + 3.1 = \mathbf{7.3 \text{ kW}}.$$

Example 3.8

Two wattmeters measure the input to an unbalanced three-phase three-wire load. One reads 2.5 kW and the other, after reversing its pressure-coil connection, reads 0.5 kW. Find the total power.

The second reading is negative: $W_2 = -0.5 \text{ kW}$. $P = 2.5 + (-0.5) = \mathbf{2.0 \text{ kW}}$.

3.4 Effect of Power Factor on Wattmeter Readings (Two-Wattmeter Method)

From Section 3.2.4, for a balanced load the two readings are $W_1 = V_L I_L \cos(30^\circ - \phi)$ and $W_2 = V_L I_L \cos(30^\circ + \phi)$. Their values depend on the load power factor, as the following analysis and Fig. 3.8 show.

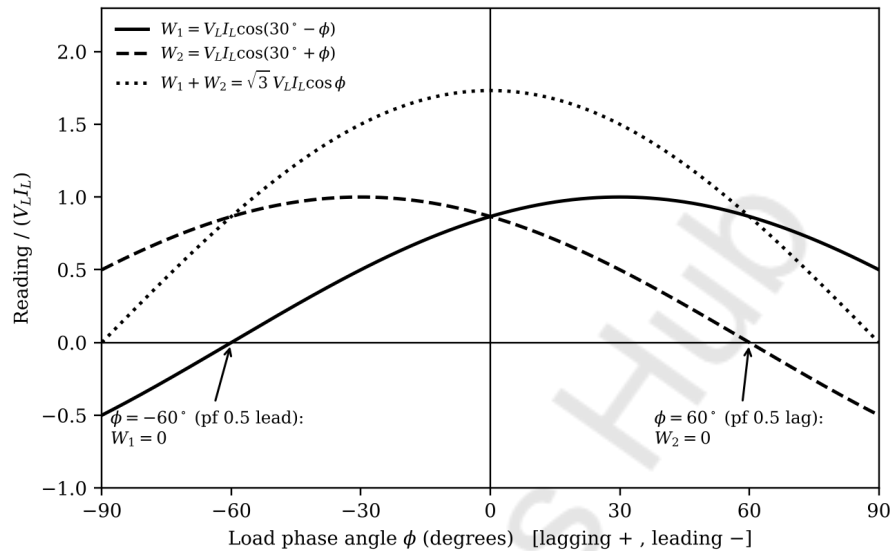


Fig. 3.8 Variation of wattmeter readings with load power factor angle

Power factor (lag)	ϕ	$W_1 / V_L I_L$	$W_2 / V_L I_L$	Observation
1.0 (unity)	0°	0.866	0.866	$W_1 = W_2$; both positive
0.866	30°	1.000	0.500	$W_1 = 2 W_2$; both positive
0.707	45°	0.966	0.259	W_2 is falling; both positive
0.5	60°	0.866	0	$W_2 = 0$; $W_1 =$ total power
0.2	78.5°	0.663	-0.317	W_2 negative; $P = W_1 - W_2 $
0 (zero)	90°	0.500	-0.500	$W_1 = -W_2$; total power = 0

3.4.1 Interpretation of the readings

- **Unity power factor ($\phi = 0$):** both wattmeters read equally, each half of the total power.
- **Power factor 0.866 ($\phi = 30^\circ$):** $W_1 = 2 W_2$.
- **Power factor between 1 and 0.5:** both readings positive but unequal; $W_1 > W_2$ for a lagging load.
- **Power factor 0.5 ($\phi = 60^\circ$):** one wattmeter reads zero (W_2 for lagging) and the other reads the total power.
- **Power factor less than 0.5:** one wattmeter reads negative. Its pointer tends to move backwards, so the pressure coil (or current coil) connection is reversed to get a deflection, and that reading is subtracted from the other: $P = W_1 - W_2$.
- **Zero power factor ($\phi = 90^\circ$):** the readings are equal and opposite, so the total power is zero while $Q = \sqrt{3} (W_1 - W_2) = \sqrt{3} \times 2W_1 = V_L I_L \sqrt{3}$.

- **Leading power factor:** the readings are interchanged: W_2 becomes the larger one and W_1 is the one that becomes zero or negative below 0.5.

3.4.2 Power factor from the ratio of readings

$k = W_2/W_1$ (smaller / larger)	$\tan \phi$	Power factor
1	0	1.000
0.5	0.577	0.866
0.25	1.039	0.693
0 (one reading zero)	1.732	0.500
Negative (below 0.5 pf)	> 1.732	< 0.500

3.4.3 Numerical problems

Example 3.9

A balanced three-phase load takes 20 kW at 0.8 power factor lagging. Find the readings of the two wattmeters.

$\phi = \cos^{-1} 0.8 = 36.87^\circ$, $\tan \phi = 0.75$. Since $W_1 + W_2 = 20$ kW and $W_1 - W_2 = P \tan \phi / \sqrt{3} = 20 \times 0.75 / 1.732 = 8.66$ kW,

$W_1 = (20 + 8.66) / 2 = \mathbf{14.33 \text{ kW}}$ and $W_2 = (20 - 8.66) / 2 = \mathbf{5.67 \text{ kW}}$.

Example 3.10

Two wattmeters measure the power of a balanced three-phase load. One reads 2000 W, and the other, with its connection reversed, reads 500 W. Find the power, power factor and reactive power.

$W_1 = 2000$ W, $W_2 = -500$ W. $P = 2000 - 500 = \mathbf{1500 \text{ W}}$.

$\tan \phi = \sqrt{3} (2000 + 500) / (2000 - 500) = 1.732 \times 2500 / 1500 = 2.887$, $\phi = 70.89^\circ$, power factor = **0.327 lagging**.

$Q = \sqrt{3} (2000 + 500) = \mathbf{4330 \text{ VAr}}$.

Example 3.11

A balanced three-phase load takes 10 kW at power factor 0.5 lagging from a 400 V supply. Find the readings of the two wattmeters and the line current.

At pf 0.5, $\phi = 60^\circ$, so $W_2 = V_L I_L \cos 90^\circ = 0$ and $W_1 = \text{total power}$. Readings: **$W_1 = 10 \text{ kW}$, $W_2 = 0$** .

$I_L = P / (\sqrt{3} V_L \cos \phi) = 10000 / (1.732 \times 400 \times 0.5) = \mathbf{28.87 \text{ A}}$.

Example 3.12

The reading of one wattmeter is double that of the other in a two-wattmeter test of a balanced load (both positive). Find the power factor.

$W_1 / W_2 = 2$. $\tan \phi = \sqrt{3} (2 - 1) / (2 + 1) = 0.5774$, so $\phi = 30^\circ$ and power factor = **0.866**.

Example 3.13

A 415 V three-phase motor takes 30 kW at 0.4 power factor lagging. Find the two wattmeter readings and the line current.

$\phi = \cos^{-1} 0.4 = 66.42^\circ$, $\tan \phi = 2.291$. $W_1 - W_2 = 30 \times 2.291 / 1.732 = 39.68$ kW.

$W_1 = (30 + 39.68) / 2 = \mathbf{34.84 \text{ kW}}$; $W_2 = (30 - 39.68) / 2 = \mathbf{-4.84 \text{ kW}}$ (negative, as pf < 0.5).

$$I_L = 30000 / (1.732 \times 415 \times 0.4) = \mathbf{104.3 \text{ A.}}$$

Poly Notes Hub

3.5 Measurement of Reactive Power in a Three-Phase Circuit

Reactive power is the part of the apparent power that does no useful work but is necessary to set up the magnetic or electric fields of the load. It is measured in volt-ampere reactive (VAr) and is important for assessing power factor correction. For a balanced three-phase load

$$Q = \sqrt{3} V_L I_L \sin \phi$$

3.5.1 Using the two-wattmeter method (balanced load)

From Section 3.2.4, $W_1 - W_2 = V_L I_L \sin \phi$. Multiplying by $\sqrt{3}$ gives

$$Q = \sqrt{3} (W_1 - W_2)$$

Thus the same two wattmeters give both active power ($W_1 + W_2$) and reactive power. The apparent power is $S = \sqrt{P^2 + Q^2}$.

3.5.2 One-wattmeter method (balanced load)

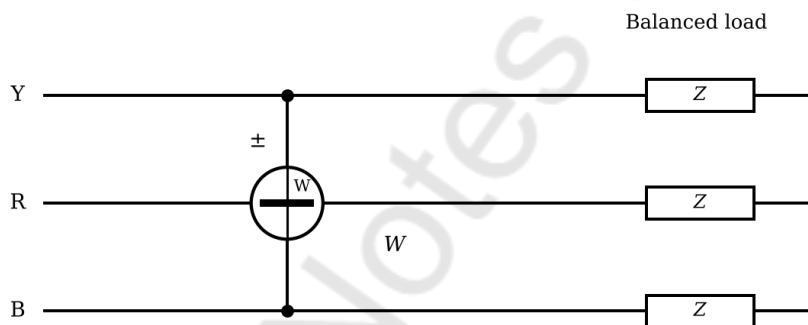


Fig. 3.9 Reactive power by one wattmeter (current in line R, pressure coil across Y-B): $Q = \sqrt{3} W$

The current coil of the wattmeter is connected in one line (R) and its pressure coil is connected across the **other two lines** (Y and B). With the phase sequence R - Y - B, the voltage across the pressure coil is V_{YB} , which lies at -90° (see Fig. 3.6), while the current I_R lies at $-\phi$. The angle between them is $(90^\circ - \phi)$. The wattmeter reading is

$$W = V_{YB} I_R \cos (90^\circ - \phi) = V_L I_L \sin \phi$$

$$Q = \sqrt{3} W$$

- The pressure coil is connected to the two lines *other* than the current-coil line, so that its voltage is at 90° to the line-to-neutral voltage of the current-coil line.
- For a lagging load the reading is positive; for a leading load the pointer reverses and the reading is taken as negative, so the sign of the reading also tells whether the load is inductive or capacitive (with R - Y - B sequence).
- The method is valid for balanced loads with sinusoidal supply and the correct phase sequence. If the sequence is reversed, the sign of the reading is reversed.
- Power factor can be found from $\tan \phi = Q/P$ if a second reading of active power is also available.

3.5.3 Three-wattmeter method (unbalanced three-wire load)

For an unbalanced load, three wattmeters are used. Each has its current coil in one line and its pressure coil across the other two lines ($W_1: I_R$ with V_{YB} , $W_2: I_Y$ with V_{BR} , $W_3: I_B$ with V_{RY}). The total reactive power is

$$Q = (W_1 + W_2 + W_3) / \sqrt{3}$$

For a balanced load each reading is $V_L I_L \sin \phi$, so $Q = 3 V_L I_L \sin \phi / \sqrt{3} = \sqrt{3} V_L I_L \sin \phi$, which agrees with the balanced formula.

3.5.4 Varmeter

A varmeter is a wattmeter-type instrument in which the voltage applied to the pressure coil is shifted by exactly 90° (by an inductor-capacitor network or electronic phase shifter). The deflection is then proportional to $VI \sin \phi$ and the scale can be calibrated directly in VAR.

Method	Connection	Reactive power
Two wattmeters (balanced)	As for active power	$\sqrt{3} (W_1 - W_2)$
One wattmeter (balanced)	CC in one line, PC across the other two lines	$\sqrt{3} \times W$
Three wattmeters (unbalanced, 3-wire)	Each CC in one line, PC across other two	$(W_1 + W_2 + W_3) / \sqrt{3}$
Varmeter	Pressure-coil voltage shifted by 90°	Direct reading in VAR

Example 3.14

In a balanced three-phase system, a wattmeter with its current coil in line R and pressure coil across lines Y and B reads 2.4 kW. If the active power is 9 kW, find the reactive power, apparent power and power factor.

$Q = \sqrt{3} \times 2.4 = \mathbf{4.157 \text{ kVAR}}$. $S = \sqrt{(9^2 + 4.157^2)} = \mathbf{9.914 \text{ kVA}}$. Power factor = $9/9.914 = \mathbf{0.908}$ lagging.

Example 3.15

For the load of Example 3.5 (415 V, 20 A, 0.8 pf lag, $W_1 = 8240 \text{ W}$, $W_2 = 3260 \text{ W}$), find the reactive power by the two-wattmeter method and check it with the formula.

$Q = \sqrt{3} (8240 - 3260) = 1.732 \times 4980 = \mathbf{8626 \text{ VAR}}$.

Check: $\sqrt{3} V_L I_L \sin \phi = 1.732 \times 415 \times 20 \times 0.6 = 8626 \text{ VAR}$ (verified).

3.6 Maximum Demand Indicator (MDI)

3.6.1 Introduction and definitions

An electricity supply authority must provide generating, transmission and distribution capacity to meet the highest load of each consumer, even if that load occurs only for a short time. A consumer with a very peaky load therefore costs the supplier more than one with a steady load of the same energy consumption. For this reason, industrial consumers are charged under a **two-part tariff**: a charge for the maximum demand (per kW or kVA) plus a charge for the energy used (per kWh). The instrument that records the maximum demand is the **maximum demand indicator**.

Term	Meaning
Demand	The average power (kW or kVA) required by the load over a specified short period (the demand interval).
Demand interval	Fixed period over which the average is taken, usually 15 or 30 minutes (sometimes 60 minutes).
Maximum demand (MD)	The highest value of demand recorded during a billing period (normally one month).
Connected load	The sum of the ratings of all appliances and machines connected to the supply.
Demand factor	Maximum demand / Connected load.
Load factor	Average demand / Maximum demand (over the billing period).

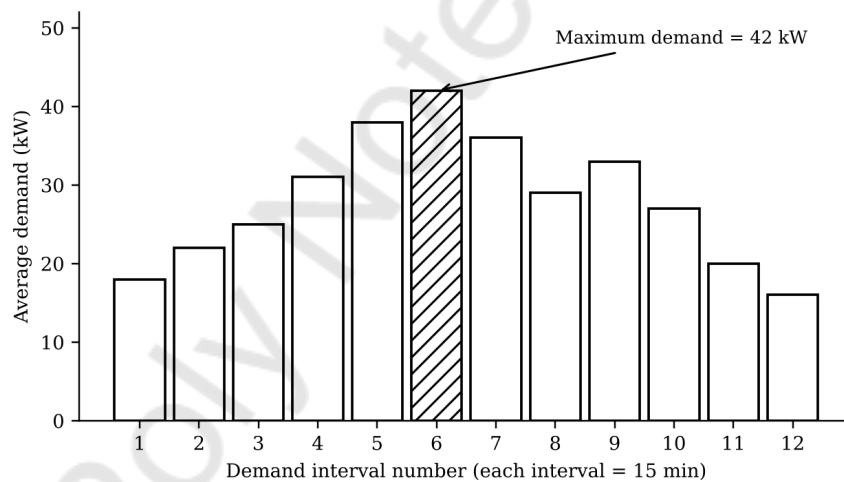


Fig. 3.11 Average demand in successive intervals (illustrative)

Fig. 3.11 shows the average demand in successive 15-minute intervals. Although the instantaneous load may peak higher, the maximum demand is the highest *average* value, 42 kW in the interval 6, because supply equipment is stressed mainly by sustained loads.

3.6.2 Types of maximum demand indicators

- **Mechanical (integrating) type:** works with a watt-hour (energy) meter and a clock mechanism.
- **Thermal type:** uses the heating effect of the load current on bimetallic strips.
- **Electronic (digital) type:** a microprocessor-based meter that calculates and stores the maximum demand.

3.6.3 Mechanical type: construction and working

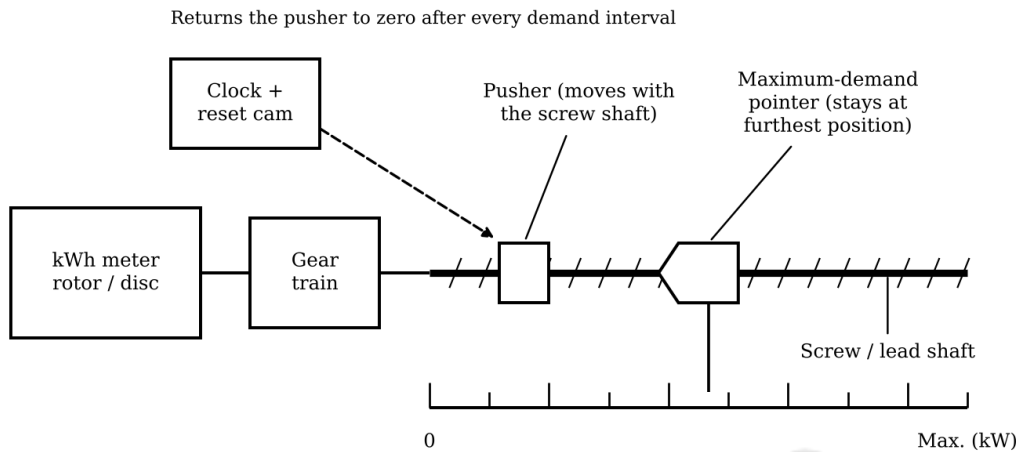


Fig. 3.10 Schematic of a mechanical (integrating) maximum demand indicator

Construction. The mechanical MDI is an attachment to a kWh meter and consists of the following parts.

- The rotor (disc) of the energy meter, whose speed is proportional to the power and whose revolutions are proportional to energy.
- A **gear train** and a screw (lead) shaft driven by the meter spindle. The **pusher** is carried on this screw shaft and moves along it.
- A **maximum-demand pointer** that moves freely on a track in front of the pusher and is moved forward only by the pusher.
- A **clock mechanism with a reset cam**, which returns the pusher to the zero (starting) position after every demand interval.
- A scale calibrated in kW (or kVA) over which the pointer moves.

Working.

1. During a demand interval, the energy-meter disc drives the pusher forward through the gear train. The distance moved is proportional to the energy consumed in that interval, which is proportional to the average power (demand) in that interval.
2. The pusher advances and, if the distance moved is greater than the previous maximum, it pushes the maximum-demand pointer forward. If the demand is smaller than the previous maximum, the pusher stops short of the pointer and does not move it.
3. At the end of each demand interval the clock-driven reset cam returns the pusher to zero, while the pointer remains where it was left.
4. The pointer is therefore always at the maximum value of average demand reached so far.
5. At the end of the billing period the meter reader records the pointer reading and resets the pointer to zero, normally through a sealed reset mechanism.

3.6.4 Thermal type: construction and working

A thermal MDI has two bimetallic spiral elements (or strips) heated by heater coils carrying the load current (or a current proportional to it). As the load current flows, the elements heat up and deflect; the deflection follows an exponential curve whose time constant is chosen so that the response reaches about 90 % in the demand interval (for example 15 or 30 minutes), as in the real averaging process. A **maximum pointer**, pushed forward by the indicating pointer, stays at the highest position reached; it is reset by hand at the end of the billing period. The scale is calibrated in amperes, kW or kVA. This type has the advantage of being robust, but its calibration depends on ambient temperature and on the heating time constant.

3.6.5 Electronic (digital) maximum demand meters

In modern static meters, the energy pulses are counted by a microprocessor. The microprocessor computes the average power for every demand interval, compares it with the stored maximum, and keeps the larger one together with the date and time. Many such meters can also reset the MD automatically at the end of the billing cycle and send data to a remote centre.

Feature	Mechanical MDI	Thermal MDI	Electronic MDI
Principle	Integration of energy over fixed interval	Heating of bimetallic element	Digital integration of energy pulses
Used with	Induction kWh meter	Current circuit directly	Static (electronic) meter
Accuracy	Good	Moderate (temperature dependent)	High
Reset	Pusher by clock; pointer manually	Pointer manually	Automatic / manual
Data stored	Only maximum demand	Only maximum demand	MD with date, time and other data

Example 3.16

The energy recorded by a consumer's meter in a 30-minute demand interval is 18 kWh. Find the demand for that interval.

$$\text{Demand} = \text{Energy} / \text{Time} = 18 \text{ kWh} / 0.5 \text{ h} = \mathbf{36 \text{ kW}}.$$

Example 3.17

A factory has a connected load of 50 kW. In a 30-day month its maximum demand is 35 kW and the energy consumed is 12 600 kWh. Find the average demand, load factor and demand factor.

$$\text{Average demand} = 12\,600 / (30 \times 24) = 17.5 \text{ kW. Load factor} = 17.5 / 35 = \mathbf{0.5} \text{ (50 \%). Demand factor} = 35 / 50 = \mathbf{0.7}.$$

Chapter Summary: Important Formulae

Topic	Formula / key point
Average power (AC)	$P = VI \cos \phi$
Wattmeter deflection	$\theta \propto VI \cos \phi$ (uniform scale)
Multiplying factor	$MF = (V_{\text{rated}} \times I_{\text{rated}} \times \text{pf}_{\text{rated}}) / \text{full-scale divisions}$
Power with CT and PT	$P = \text{Reading} \times \text{CT ratio} \times \text{PT ratio}$
Connection errors	Reads load power + V^2/R_p (PC on load side) or + $I^2 R_c$ (PC on supply side)
PC inductance correction	Correction factor = $\cos \phi / [\cos \beta \cos (\phi - \beta)]$; compensating $C \approx L_p/R^2$
3-phase power (balanced)	$P = \sqrt{3} V_L I_L \cos \phi$
One-wattmeter method	$P = 3 W$
Three-wattmeter method	$P = W_1 + W_2 + W_3$
Two-wattmeter readings	$W_1 = V_L I_L \cos (30^\circ - \phi)$; $W_2 = V_L I_L \cos (30^\circ + \phi)$
Total power	$P = W_1 + W_2$ (algebraic)
Power factor	$\tan \phi = \sqrt{3} (W_1 - W_2) / (W_1 + W_2)$
Special power factors	pf 1: $W_1 = W_2$; pf 0.5: one reading is zero; pf < 0.5: one reading negative
Reactive power	$Q = \sqrt{3} (W_1 - W_2) = \sqrt{3} \times (\text{one-wattmeter reading with PC across other two lines})$
Unbalanced 3-wire reactive	$Q = (W_1 + W_2 + W_3)/\sqrt{3}$
Maximum demand	Highest average demand (kW/kVA) over a fixed interval in the billing period
Load factor	Average demand / Maximum demand

Multiple Choice Questions (15)

Choose the correct answer. The answer key is given after the questions.

Q1. In a dynamometer type wattmeter, the fixed coils are used as the

- (a) pressure coil
- (b) current coil
- (c) damping coil
- (d) compensating coil only

Q2. The pressure-coil circuit of a dynamometer wattmeter contains

- (a) a low resistance in series
- (b) a high non-inductive resistance in series
- (c) a low inductive resistance in parallel
- (d) a large capacitor in series

Q3. The scale of a dynamometer wattmeter is approximately

- (a) uniform
- (b) square-law
- (c) logarithmic
- (d) cramped at the upper end

Q4. The average deflecting torque of a dynamometer wattmeter is proportional to

- (a) VI
- (b) $VI \sin \phi$
- (c) $VI \cos \phi$
- (d) V^2/I

Q5. A wattmeter of 250 V, 10 A, unity power factor has 100 divisions. Its multiplying factor is

- (a) 2.5 W/div
- (b) 10 W/div
- (c) 25 W/div
- (d) 250 W/div

Q6. The error due to inductance of the pressure coil is compensated by

- (a) a capacitor connected across a part of the series resistance
- (b) a resistor in parallel with the current coil
- (c) a shunt across the current coil
- (d) an iron shield around the instrument

Q7. A compensating coil in a wattmeter is used to correct the error due to

- (a) stray magnetic field
- (b) eddy currents
- (c) pressure-coil inductance
- (d) power loss in the pressure-coil circuit

- Q8. Two wattmeters connected to a balanced three-phase load read 4000 W and 1000 W. The total power is**
- (a) 3000 W
 - (b) 5000 W
 - (c) 4000 W
 - (d) 2500 W
- Q9. For a balanced load at unity power factor, the readings of the two wattmeters are**
- (a) equal
 - (b) one is double the other
 - (c) one is zero
 - (d) equal and opposite
- Q10. At a power factor of 0.5 lagging, in the two-wattmeter method**
- (a) both readings are equal
 - (b) one reading is twice the other
 - (c) one wattmeter reads zero
 - (d) both wattmeters read zero
- Q11. In a balanced load, the larger wattmeter reading is twice the smaller (both positive). The power factor is**
- (a) 0.5
 - (b) 0.707
 - (c) 0.8
 - (d) 0.866
- Q12. The reactive power of a balanced three-phase load from the two-wattmeter readings is**
- (a) $W_1 + W_2$
 - (b) $\sqrt{3} (W_1 - W_2)$
 - (c) $(W_1 - W_2)/\sqrt{3}$
 - (d) $\sqrt{3} (W_1 + W_2)$
- Q13. A wattmeter has its current coil in line R and pressure coil across lines Y and B of a balanced load and reads W . The reactive power is**
- (a) W
 - (b) $W/\sqrt{3}$
 - (c) $\sqrt{3} W$
 - (d) $3 W$
- Q14. The number of wattmeters required to measure power in a three-phase four-wire unbalanced load is**
- (a) three
 - (b) two
 - (c) one
 - (d) four

Q15. A maximum demand indicator records

- (a) the total energy consumed in a month
- (b) the peak instantaneous current
- (c) the highest average power demand over a fixed interval
- (d) the lowest power factor of the load

Answer Key

Q	Ans	Reason
1	(b)	Fixed coils are of heavy wire and carry the load current.
2	(b)	High non-inductive series resistance limits the current and makes the circuit almost purely resistive.
3	(a)	Deflection is proportional to power, so the scale is uniform.
4	(c)	Torque $\propto$ average of $vi = VI \cos \phi$.
5	(c)	$MF = 250 \times 10 / 100 = 25$ W per division.
6	(a)	The capacitor brings the pressure-coil current in phase with the voltage.
7	(d)	It cancels the extra ampere-turns due to the pressure-coil current in the current coil.
8	(b)	$P = W_1 + W_2 = 5000$ W.
9	(a)	At $\phi = 0$, $W_1 = W_2 = V_L I_L \cos 30^\circ$.
10	(c)	At $\phi = 60^\circ$, $W_2 = V_L I_L \cos 90^\circ = 0$.
11	(d)	$\tan \phi = \sqrt{3} (2 - 1) / (2 + 1) = 1/\sqrt{3}$, so $\phi = 30^\circ$.
12	(b)	$W_1 - W_2 = V_L I_L \sin \phi$.
13	(c)	$W = V_L I_L \sin \phi$, so $Q = \sqrt{3} W$.
14	(a)	By Blondel's theorem, $n - 1 = 3$ wattmeters for four wires.
15	(c)	MD is the highest average demand over the demand interval (15/30 min).

Broad Questions (10)

- Q1. Describe, with a neat sketch, the construction of a dynamometer type wattmeter. Explain its working principle and show that the deflection is proportional to the power in the circuit.
- Q2. Explain the different types of errors in a dynamometer wattmeter. How are the errors due to pressure-coil inductance and due to the pressure-coil current compensated?
- Q3. What is the multiplying factor of a wattmeter? Explain how the current and voltage ranges of a wattmeter are extended. A wattmeter rated 300 V, 5 A, unity pf has 150 divisions and reads 90 divisions with a CT of ratio 50/5 and PT of ratio 3000/300. Find the actual power. (*Ans: MF = 10 W/div; reading 900 W; power = 900 × 10 × 10 = 90 kW*)
- Q4. Explain the one-wattmeter and three-wattmeter methods of measuring power in a three-phase circuit with neat connection diagrams. State the conditions under which each can be used.
- Q5. With a neat circuit diagram, explain the two-wattmeter method of measuring three-phase power. Prove that the readings are $W_1 = V_L I_L \cos(30^\circ - \phi)$ and $W_2 = V_L I_L \cos(30^\circ + \phi)$, and derive expressions for the power factor and reactive power.
- Q6. Explain how the active power of an unbalanced three-phase load is measured in (i) a four-wire system and (ii) a three-wire system. Why is the two-wattmeter method not suitable for a four-wire system?
- Q7. Discuss the effect of load power factor on the readings of the two wattmeters in a balanced three-phase circuit. Draw the variation of the readings with power factor angle and state the condition for one wattmeter to read zero or negative.
- Q8. Two wattmeters connected to a balanced three-phase load read 7.5 kW and 2.5 kW. Find the total power, power factor and reactive power. If the load is 415 V, find also the line current. (*Ans: 10 kW; 0.756 lagging; 8.66 kVar; 18.4 A*)
- Q9. Explain with diagrams how reactive power is measured in a three-phase circuit by (i) the two-wattmeter method and (ii) the one-wattmeter method. Derive the relation between the wattmeter reading and reactive power in the second case.
- Q10. What is a maximum demand indicator and why is it needed? Explain the construction and working of a mechanical MDI with a neat sketch, and briefly compare it with the thermal type.